Homework #2
Handed out October 14, 2010
Due October 28, 2010

1) Read sections 2.1, 3.1 and Chapter 7.
2) Show that $\mathbf{v} = \dfrac{\mathbf{E}\times\mathbf{B}}{\mathrm{B}^2}$ when $\mathbf{E} = -\mathbf{v}\times\mathbf{B}$.

3) An imbalance of electrical charges in a system of charged particles gives rise to static electric fields. Estimate the strength of the electric field for a 1% deviation from neutrality for a system containing 10^{12} particles/m^3, assuming a spherical geometry of 0.01 m radius.

4) A way to write that magnetic flux is conserved is:
$$\frac{\partial B}{\partial t} + \nabla \bullet (\mathbf{u}B) = 0$$
Use this equation and the equation of continuity of particles to show that
$$\frac{d}{dt}\left(\frac{n}{B}\right) = 0$$
where n is the particle density.

5) Use Ohm's Law J = σ (E + U x B) and the equation of motion to show that:
   a) the momentum equation for the perpendicular motion can be written as
   $$\rho_m \frac{d\mathbf{U}_\perp}{dt} = \mathbf{F}_\perp + \sigma B^2\left(\frac{\mathbf{E}\times\mathbf{B}}{B^2} - \mathbf{U}_\perp\right)$$
   where $F_\perp$ is the non-electromagnetic force perpendicular to B.
   b) the limiting value attained by $U_\perp$ is
   $$\mathbf{U}_\perp = \frac{\mathbf{F}_\perp}{\sigma B^2} + \frac{\mathbf{E}\times\mathbf{B}}{B^2}$$
   Note that in the limit
   $$\sigma \to \infty, \mathbf{U}_\perp \to \frac{\mathbf{E}\times\mathbf{B}}{B^2}$$
   c) the time it takes to reach the limiting velocity in (b) is
   $$\tau \approx \frac{\rho_m}{\sigma B^2}$$

6) Energetic solar particles were detected by a spacecraft in the vicinity of the Earth following a large solar flare. Protons of various energies, alpha particles and medium mass nuclei were all detected at the same time. Explain how this is possible.

7) Show that the arc length of a dipole line is given by:
$$\frac{ds}{d\lambda} = r_0 \cos\lambda(1 + 3\sin^2\lambda)^{1/2}$$

where ds is the differential arc length and r_0 is the equatorial radius .

8) The total magnetic energy contained in a volume V is obtained from

$$\int_V \frac{B^2}{2\mu_0} dV$$

Determine the amount of energy of the Earth's dipole field external to the solid Earth. Use the following formula for the magnetic field magnitude:

$$B(r,\lambda) = \frac{\mu_0 M}{4\pi r^3}(1 + 3\sin^2\lambda)^{1/2}$$

9) Determine the amount of magnetic energy for a magnetosphere that included the dipole and IMF. Do this problem for both northward and southward IMF. Is the difference between these two configurations sufficient to produce the aurora, which typically dissipate 10^{14} J into the atmosphere? Use the following equations given in class:

Northward IMF: $B_T^2(r,\lambda) = \frac{\mu_0^2 M^2}{16\pi^2 r^6}(1 + 3\sin^2\lambda) + B_0^2 + \frac{\mu_0 M B_0}{2\pi r^3}(1 - 3\sin^2\lambda)$

Southward IMF: $B_T^2(r,\lambda) = \frac{\mu_0^2 M^2}{16\pi^2 r^6}(1 + 3\sin^2\lambda) + B_0^2 - \frac{\mu_0 M B_0}{2\pi r^3}(1 - 3\sin^2\lambda)$

ESS 515 students:

10) Show that the dipole field equations satisfy ∇•B = 0 and ∇xB = 0.

11) Show that if **E** = -**U** x **B**

$$\nabla \times \mathbf{E} = \mathbf{n}\nabla \cdot (\mathbf{U}B)$$

where **n** is a unit vector along **B**. State what assumptions are required for the above to be true.