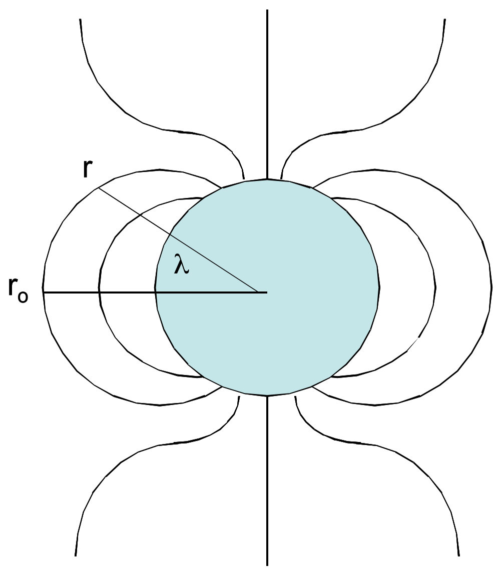
Homework #3
Handed out October 28, 2010
Due November 16, 2009

Read Chapters 2 and 3.
For this homework you will use the following formulas.

The formula for a magnetic field line is: $r = r_o \cos^2 \lambda$



Assume for all of these calculations that the Earth's dipole is centered and the dipole moment is along the z axis. You can ignore rotation as well.

The magnitude of the magnetic field for a centered dipole with the dipole moment pointing in the –z direction is

$$B(r,\lambda) = \frac{\mu_o M}{4\pi r^3}(1 + 3\sin^2\lambda)^{\frac{1}{2}} \qquad (2)$$

where M = 8.0 x 10^{22} A m^2.

The formula for the path along a magnetic fieldline is

$$\frac{ds}{d\lambda} = r_o \cos\lambda(1 + 3\sin^2\lambda)^{\frac{1}{2}} \qquad (3)$$

so the complete path length will be

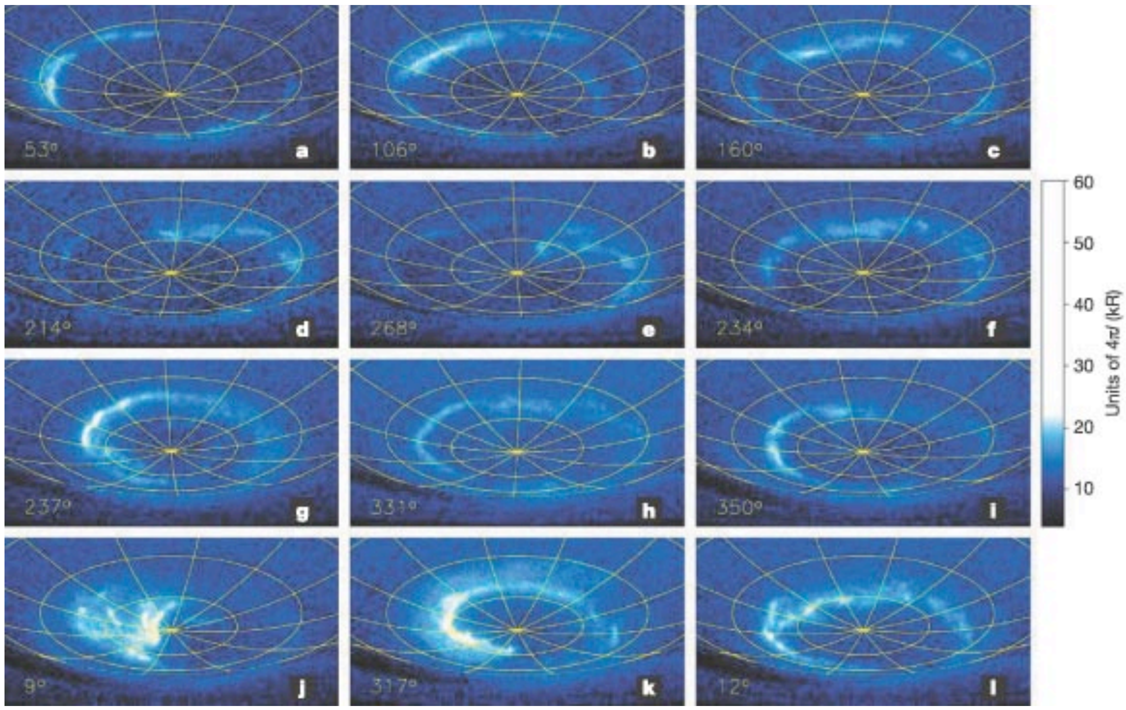
$$\int ds = \int r_o \cos\lambda(1 + 3\sin^2\lambda)^{\frac{1}{2}} d\lambda \qquad (4)$$

1) Find the pitch angle of a 1 keV proton at the equator at $5R_E$ that mirrors at an altitude of 100 km.
2) For this same particle, calculate the gyrofrequency and gyroradius at the equator. What is the period of gyration?
3) For the same particle, calculate the bounce period between the two mirror points. You can simplify the calculation by assuming that $v = v_{\|}$ during the entire trip. You will also need the path length along a fieldline between two latitudes. Determine that by using the formula #4 above, and integrate to solve for s (hint: use variable substitution and a table of integrals). You will also need
$$\sinh^{-1}(x) = \ln(x + \sqrt{(x^2 + 1)})$$

4) Write down the formula for the drift velocity due to the gradient and curvature drifts. Use dimensional analysis to simplify the formula and remove the cross products, what is the formula now? Use this approximation to calculate the drift period for the particle above. Compare the time you calculate to the time you determine from the following equation:
$$T_D \approx \begin{cases} \dfrac{43.8}{L\varepsilon} & for \quad \alpha = 90^0 \\ \dfrac{62.7}{L\varepsilon} & for \quad \alpha = 0^0 \end{cases}$$
where T_D is the time in minutes, L is the L-shell (ie distance in R_E) and ε is the energy of the particle in MeV.

5) Jupiter has footprint aurora (emissions at the base of fieldlines in the polar region) due to interactions with some of its moons. Assuming a dipole field that is aligned with the rotation axis (a very good approximation for Saturn), at what latitudes would footprint aurora appear in Saturn's polar cap for the moons Titan and Enceladus, and the rings? Do you see any evidence for footprint aurora in the attached image? You can assume that the moons/rings are in the equator and that the emission you see in the following image is coming from and altitude of 1000 km You will need the following information:

Orbital radius of Titan: 20 R_S
" Enceladus: 4 R_S
" the rings: 1.5 – 2 R_S



Ultraviolet images of Saturn's southern hemisphere from *Clarke et al.* (Nature, 2005). The longitude and latitude lines are plotted every 10^{o} and 30^{o} respectively. So the first yellow circle at the pole indicates a latitude of 80^{o} and the second one is 70^{o}.