What is a Plasma?
Characteristic lengths and frequencies

Plasma: *gas of charged particles*
+ and - charges, but overall usually neutral having kinetic energy of particles much larger than potential energy due to nearest neighbor.

Consider a gas with n particles/volume and Temperature T (or energy kT, where k is Boltzman's Constant k=1.38*10^{-23} J/^{o}K)
(Actually the overall energy of particles is

$$3kT \equiv \frac{1}{2} m\langle v^2\rangle$$

Mean interparticle distance = $\frac{1}{n^{1/3}}$ so we can determine the ratio of the Kinetic Energy to Potential Energy:= $\frac{kT}{e\Phi} = \frac{kT}{e^2 n^{1/3}}$ where $e\Phi \approx e\frac{e}{r} \approx e^2 n^{1/3}$

so $KE/PE = \left(\frac{kT}{1ev}\right)\left(\frac{1\ cm^{-3}}{n}\right) 7\times 10^6 \quad >> 1$ for nearly all plasmas
(opposite for solids)
Note: DeBroglie wavelength

$$\lambda_{DB} \sim \hbar / p = \hbar / \sqrt{2mkT} \sim \left(\frac{1ev}{kT}\right)^{1/2} 2\times 10^{-8} \text{cm for electrons ...}$$

Therefore Quantum effects are negligible in a plasma

Temperature

Consider 1-D gas in thermal equilibrium
most probable distribution is maxwellian (WHY?)

$$f(v_x) = A\exp\left[-\left(\frac{1}{2}mv_x^2\right)/kT\right]$$

f(u) is the probability function of finding poarticles with velocity u.

$f(\vec{u})d\vec{u}$ is the probability of finding a particle with velocity $\vec{u}$ in a volume $d\vec{u}$ of velocity space.

Normalization for f(u) comes from

$n = \int_{-\infty}^{\infty} f(u)du$ which is the density. Now integrate over all velocities to find total density at a point $n(\vec{r})$.

Note, if $\int_{-\infty}^{\infty} f(u)du = A\int \exp\left[-\left(\frac{1}{2}mv_x^2\right)/kT\right]du = n$

then $A = n\left(\frac{m}{2\pi kT}\right)^{1/2}$

Thermal Eqilibrium:

Why is a Maxwellian distribution the right one to describe thermal equilibrium?

(from http://silas.psfc.mit.edu/introplasma/chap1.html#tth_sEc1.1)

1.2.1 Elementary Derivation of the Boltzmann Distribution

Basic principle of Statistical Mechanics:
Thermal Equilibrium ↔ *Most Probable State* i.e. State with large number of possible arrangements of micro-states.

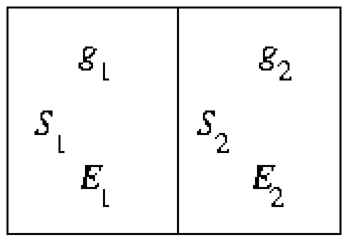


Figure 1.4: Statistical Systems in Thermal Contact

Consider two weakly coupled systems S_1, S_2 with energies E_1, E_2. Let g_1, g_2 be the number of microscopic states which give rise to these energies, for each system. Then the total number of micro-states of the *combined* system is (assuming states are independent)

$$g = g_1 \, g_2 \qquad (1.10)$$

If the total energy of combined system is fixed $E_1 + E_2 = E_t$ then this can be written as a function of E_1:

$$g = g_1(E_1)\, g_2(E_t - E_1) \qquad (1.11)$$

and

$$\frac{dg}{dE_1} = \frac{dg_1}{dE} g_2 - g_1 \frac{dg_2}{dE} \; . \qquad (1.12)$$

The most probable state is that for which $[dg/(dE_1)] = 0$ i.e.

$$\frac{1}{g_1}\frac{dg_1}{dE} = \frac{1}{g_2}\frac{dg_2}{dE} \quad \text{or} \quad \frac{d}{dE}\ln g_1 = \frac{d}{dE}\ln g_2 \qquad (1.13)$$

Thus, in equilibrium, states in thermal contact have equal values of $[d/dE] \ln g$.
One defines $\sigma \equiv \ln g$ as the *Entropy*.
And $[\, [d/dE] \ln g\,]^{-1} = T$ the *Temperature*.
Now suppose that we want to know the relative probability of 2 *micro*-states of system 1 in equilibrium. There are, in all, g_1 of these states, for each specific E_1 but we want to know how many states of the *combined* system correspond to a *single* microstate of S_1.

Obviously that is just equal to the number of states of system 2. So, denoting the two values of the energies of S_1 for the two microstates we are comparing by E_A, E_B the ratio of the number of combined system states for S_{1A} and S_{1B} is

$$\frac{g_2\,(E_t - E_A)}{g_2\,(E_t - E_B)} = \exp[\,\sigma(E_t - E_A) - \sigma(E_t - E_B)\,] \tag{1.14}$$

Now we suppose that system S_2 is large compared with S_1 so that E_A and E_B represent very small changes in S_2's energy, and we can Taylor expand

$$\frac{g_2\,(E_t - E_A)}{g_2\,(E_t - E_A)} \cong \exp\left[\, - E_A \frac{d\sigma}{dE} + E_B \frac{d\,\sigma}{dE} \,\right] \tag{1.15}$$

Thus we have shown that the ratio of the probability of a system (S_1) being in any two micro-states A, B is simply

$$\exp\left[\frac{-\,(E_A - E_B)}{T}\right], \tag{1.16}$$

when in equilibrium with a (large) thermal "reservoir". This is the well-known "Boltzmann factor".

You may notice that Boltzmann's constant is absent from this formula. That is because of using natural thermodynamic units for entropy (dimensionless) and temperature (energy). Boltzmann's constant is simply a conversion factor between the *natural units of temperature* (energy, e.g. Joules) and (e.g.) degrees Kelvin. Kelvins are based on $^{\circ}$C which arbitrarily choose melting and boiling points of water and divide into 100.

Plasma physics is done almost always using energy units for temperature. Because Joules are very large, usually electron-volts (eV) are used.

$$1\ \text{eV} = 11600\ \text{K} = 1.6 \times 10^{-19}\ \text{Joules.} \tag{1.17}$$

One consequence of our Botzmann factor is that a gas of moving particles whose energy is $^1/_2\, mv^2$ adopts the Maxwell-Boltzmann (Maxwellian) distribution of velocities $\propto \exp[\,-[(mv^2)/2T]\,]$.

Average Energy

$$\varepsilon_{av} = \frac{\int_{-\infty}^{\infty} 1/2\, mu^2 f(u)du}{\int_{-\infty}^{\infty} f(u)du}$$

See Chen (p.5) for details of integrating a function

$\int y^2 e^{-y^2} dy$

$$\varepsilon_{av} = \frac{1}{2}kT \text{ in 1-D}$$

for multiple dimensions

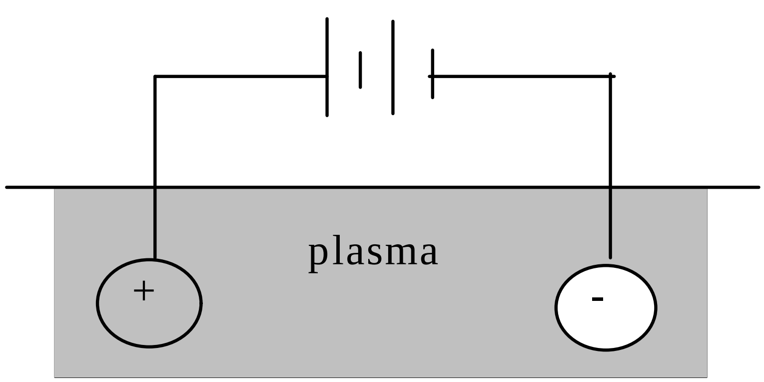
$$f \rightarrow f(v_x, v_y, v_z) = \exp\left[-\frac{1}{2}m\left(v_x^2 + v_y^2 + v_z^2\right)/kT\right]$$

and $A = n\left(\frac{m}{2\pi kT}\right)^{3/2}$ note exponent

$$\boxed{\varepsilon_{av} = \frac{3}{2}kT}$$

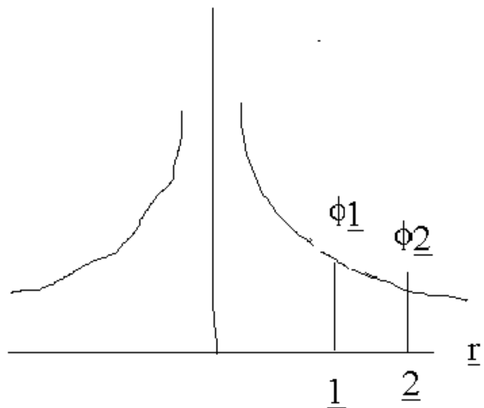
(for average velocity - see homework)

HOW do modify f when we introduce +Q charge into Plasma?



in free space $\vec{E}(\vec{r}) = \frac{q}{4\pi\varepsilon_o}\frac{1}{r^2}\hat{r}$

electrostatic potential $\Phi = \frac{q}{4\pi\varepsilon_o r}$ (from $\vec{E} = -\nabla\Phi$)



Electron gains energy $e(\Phi_1 - \Phi_2)$

energy = distance times force = $\Delta reE = q\Phi$

$\nabla \cdot \vec{E} = \frac{\rho_c}{\varepsilon_o}$ $\qquad \varepsilon_o \nabla^2 \Phi = -\rho_c = e(n_i - n_e)$

in 1-D: $\frac{d^2}{dx^2}\Phi = -e(n_+ - n_-)$

now assume ions are fixed (too slow to move or respond)

so $n_i = n_\infty$ but near the probe, e^- distribution is changed

so: electron energy $= 1/2\, mv^2 + q\Phi$ (where q=-e)

so $f(u) = A\int \exp\left[-\left(\frac{1}{2}mv_x^2 + \mathbf{q\ddot{O}}\right)/kT\right]du$

Now, let $n_e \to \infty$ at large distances (so $\rho \to 0$ at ∞)

$n_e = \int f(u)du = n_\infty e^{e\Phi/kT}$

So, plug into Poisson: $\frac{d^2\Phi}{dx^2} = en_\infty([e^{e\Phi/kT}] - 1)$

Now we showed $e\Phi/kT \ll 1$ so then Taylor series expand

$$\varepsilon_o \frac{d^2\Phi}{dx^2} = en_\infty[\frac{e\Phi}{kT} + \frac{1}{2}(\frac{e\Phi}{kT})^2 + \cdots]$$

$$\frac{d^2\Phi}{dx^2} \approx \frac{ne^2\Phi}{\varepsilon_o kT_e} \text{ or } \Phi = \Phi_o e^{|x|/\lambda_D}$$

where $\lambda_D = (\frac{\varepsilon_o kT_e}{ne^2})^{1/2}$

$$\lambda_D = 740(\frac{kT}{1\text{ eV}})^{1/2}(\frac{1\text{ cm}^3}{n})^{1/2}\text{ cm}$$

~~Problem set has several examples.~~

So, $\Phi = \Phi_o e^{|x|/\lambda_D} \approx (\text{const})\frac{q}{r}e^{|r|/\lambda_D}$, which falls off FASTER than 1/r

This is Debye Shielding

λ_D is Fundamental to plasma physics: Dont Forget It

$\lambda_D \ll L$ scale of the plasma variations

Next, we need lots of plasma particles for shielding to work

Debye Sphere: $\frac{4}{3}\pi\lambda_d^3 n_o = N_D$

we need N_D very large.

Note: Parks defines Plasma parameter $g = \frac{1}{N_D} \ll 1$

Others call N_D the plasma parameter.

Next: How fast does it happen? What is the response time of the charge distribution to the introduction of a charge distribution?