ESS415/515 Problem Set #3
Due Friday Jan 31, 2014

1. Consider an electron beam (not a plasma!) with circular cross section, uniform density n over that cross section, and initial radius r_o.

a) Show that it's radius after a time t is:

$r = r_o\,(1 + \frac{1}{4}\,\omega_p^2\, t^2\,)$ provided $r - r_o << r_o$
where ω_p is the plasma frequency corresponding to density n.

b) Hence show that the maximum length of the beam, before it becomes dispersed by the elecrostatic repulsion, is of the order of a suitably defined Debye Length.

c) Estimate roughly the maximum length possible for the beam in a Television Tube ('cathode ray tube') before it's spreading becomes appreciable. How does this compare to the actual length

2. Consider a particle moving non-relativistically in uniform ***E*** and ***B*** fields perpendicular to each other ($\boldsymbol{E} \cdot \boldsymbol{B} = 0$). Solve the equations of motion directly (i.e. without using reference frame transformations) for a particle with initial velocity ***v*** perpendicular to ***B*** and thus show that the motion is a superposition of the usual gyration plus constant drift $\boldsymbol{V_B} = \boldsymbol{E} \times \boldsymbol{B}/B^2$. Sketch the particle trajectory for the three cases $\boldsymbol{v} >> \boldsymbol{V_B}$, $\boldsymbol{v} \sim \boldsymbol{V_B}$ and $\boldsymbol{v} << \boldsymbol{V_B}$, for both positive and negative particles.

3. A particle moves in a slowly varying magnetic field $\boldsymbol{B} = \boldsymbol{z}_{\mathrm{hat}}\, B_z(x)$, and a uniform small electric field $\boldsymbol{E} = \boldsymbol{y_{hat}}\, E_y$. Calculate the drifts and the rate of change of $\frac{1}{2}\, m\, v_{perp}^2$. Compare this change with the particle's change of potential energy. ($\boldsymbol{z}_{\mathrm{hat}}$ and $\boldsymbol{y_{hat}}$ are unit vectors in the ***z*** and ***y*** axis directions, respectively).

4. Consider a particle starting in the equatorial plane of a dipole magnetic field with $v_{perp} = 0$. Show that $v_{parallel}$ can be chosen sufficiently small that the change of the magnetic field strength over the Larmor radius (i.e. gyroradius) or along the trajectory during the Larmor period (i.e. gyroperiod) is arbitrarily small. Thus that, the first invariant μ should be conserved.

From the first invariant μ, compute the particle pitch angle at various points along it's trajectory. Are these pitch angles consistent with the force required to make the particle follow it's non-rectilinear trajectory? How can you explain this apparent dilemma? (the pitch angle is the angle between ***B*** and ***v*** .

5. Find the motion of a charged particle in a toroidal static magnetic field under the guiding center approximation. Decide whether the particle is indefinitely trapped in the field.

[Note, in this problems set, vectors are identified, such as for ***B***, ***E*** and ***v*** , where the letter is **bold** and *italicized*.]