

**ESS 415/515 Problem set 6**  
**Due Friday Feb 14**

Read Parks Chapter 5.

1. Parks Chapter 5 prob 2, page 208.

2. An isothermal plasma with temperature  $T$  is confined between the planes  $x = \pm a$  in a magnetic field  $\mathbf{B} = B\hat{z}$ . The density distribution is given by  $n = n_0(1 - (\sin(\pi x/2a))^2)$ .

a. **Derive** an expression for the magnetization current density  $\mathbf{j}_M$  as a function of  $x$ .

b. **Draw** a diagram showing the density profile and the direction of  $\mathbf{j}_M$  on both sides of the midplane if  $\mathbf{B}$  is out of the paper.

c. **Evaluate**  $\mathbf{j}_M$  at  $x=a/2$  if  $B_0 = 0.4 \times 10^{-4}$  nT,  $kBT_e = 3$  eV and  $a=100$  km (e.g. in the ionosphere).

3. **Determine** the electron partial pressure tensor  $\hat{\mathbf{P}}_e$  components for an isotropic

plasma with  $\mathbf{B} = B\hat{z}$  with electron temperatures of  $T_{e\parallel} = 1$  keV,  $T_{e\perp} = 10$  keV and density  $n_e = 10^4$  cm $^{-3}$ . **Show** specifically that the off diagonal elements are zero.

4. A cylindrically symmetric plasma column in a uniform  $B$  field has

$$n(r) = n_0 \exp(-r^2/r_0^2) \text{ and } n_i = n_e = n_0 \exp(e\phi/KTe)$$

(a) **Show** that the  $V_B \equiv \frac{\vec{E} \times \vec{B}}{B^2}$  drift and the electron Diamagnetic drift  $V_{De}$  are equal and

opposite, where  $V_{De} \equiv -\frac{\nabla P \times \vec{B}}{qnB^2}$ .

(b) **Show** that the plasma rotates as a solid body.

(c) In the frame which rotates with velocity  $V_E$ , some plasma waves (drift waves) propagate with a phase velocity  $v_\phi = 0.5V_{De}$ . What is  $v_\phi$  in the lab frame? On a diagram of the  $r - \theta$  plane, **draw** arrows indicating the relative magnitudes and directions of  $V_B$ ,  $V_{De}$ , and  $V_\phi$  in the lab frame.