Lorentz Transform of $\vec{E}$ and $\vec{B}$ fields

S' coordinate system moving along $\hat{x}$ relative to S at speed v.

$$\vec{E}' = \vec{E} + \gamma\, \vec{v}\times\vec{B} + \frac{\gamma - 1}{v^2}\vec{v}\times(\vec{E} \times \vec{v})$$

$$\vec{E}'_{||} = \vec{E}_{||} \quad \text{while} \quad \vec{E}'_{perp} = \gamma\,(\vec{E}_{perp} + \vec{v}\times\vec{B})$$

(note || with respect to motion)

$$\vec{B}'_{||} = \vec{B}_{||}\ , \text{while} \quad \vec{B}'_{perp} = \gamma\,(\vec{B}_{perp} - \frac{1}{c^2}\vec{v}\times\vec{E}\ )$$

where $\gamma = \dfrac{1}{\sqrt{(1 - v^2/c^2)}}$ and c = speed of light

Note: $\vec{E}$ and $\vec{B}$ are very different, even when γ --> 1 (i.e. nonrelavistic speeds).

Example --> measuring $\vec{E}_{perp}$ in ionosphere.

Magnetic Fields

$$\nabla \times B = \cancel{\mu_0 J} + \cancel{\mu_0 \epsilon_0 \frac{\partial E}{\partial t}}$$ Maxwell

$\mu_0 J \to 0$ in vacuum

$\mu_0 \epsilon_0 \frac{\partial E}{\partial t} \to 0$ in steady state

$$\nabla \times B = 0 \Rightarrow B = -\nabla \psi$$

$$\nabla \times (\nabla \psi) \equiv 0$$

$$\bar{\nabla} \cdot \bar{B} = 0$$ Maxwell

$$-\nabla \cdot \nabla \psi = 0 \quad = -\nabla^2 \psi$$

$$\Rightarrow \nabla^2 \psi = 0$$ Scalar potential satisfies Laplace's Eqtn.

from electrostatics, should know solution in 3-D is given by $\psi \propto \frac{1}{r^2}$

IN Space

show examples
- Solar corona, photosphere
- interplanetary Fields
- Earth's dipole field
- Magnetospheric magnetic Field configuratio

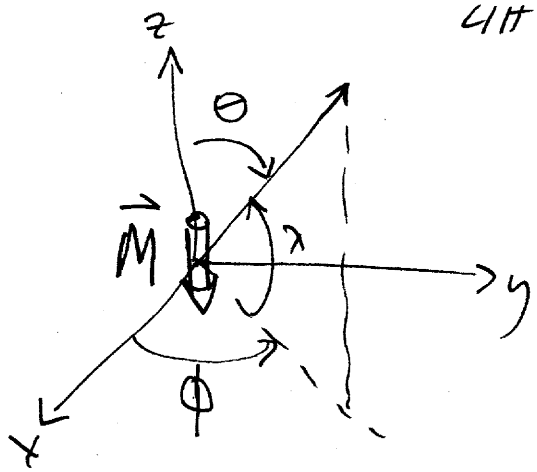
For Earth's dipole field

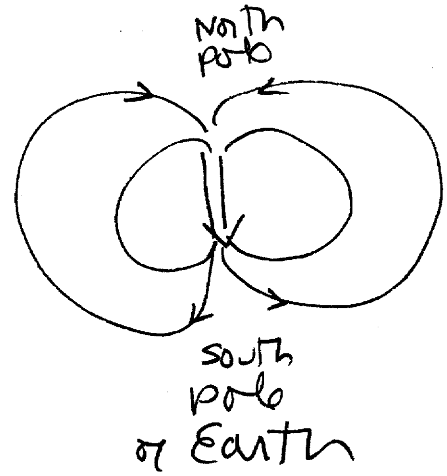
$$\psi = -\frac{\mu_0}{4\pi}\, \vec{m} \cdot \underbrace{\nabla \frac{1}{r}}$$

gradient in spherical coords (see sheet passed out in class, day 3)

$$= -\frac{\mu_0}{4\pi} M \frac{\cos\theta}{r^2} \qquad \theta = \text{colatitude}$$

$$= -\frac{\mu_0}{4\pi} M \frac{\sin\lambda}{r^2} \qquad \lambda = \text{latitude}$$





$$B_r = -\frac{\partial \psi}{\partial r} = -\frac{\mu_0 M}{2\pi} \frac{\sin\lambda}{r^3}$$

$$B_\lambda = -\frac{1}{r}\frac{\partial \psi}{\partial \lambda} = -\frac{\mu_0 M}{4\pi}\frac{\cos\lambda}{r^3}$$

$$B_\phi = -\frac{1}{r}\frac{1}{\cos\lambda}\frac{\partial \psi}{\partial \phi} = 0$$

at equator $\lambda = 0$ $\qquad B_r = 0 \quad B = B_\lambda \hat{z} = \frac{\mu_0 M}{4\pi r^3}$

B at 1 Re at equator = B_{eq}

$$B_{eq} = \frac{\mu_0 M}{4\pi R_e^3} \quad \text{so} \quad |M| = \frac{4\pi R_e^3 B_{eq}}{\mu_0}$$

at $r > R_e$ at $\lambda = 0$

$$|B| = \frac{\mu_0 M}{4\pi r^3} = B_{eq}\left(\frac{R_e}{r}\right)^3$$

B_{eq} 31,000 nT or 0.31 Gauss

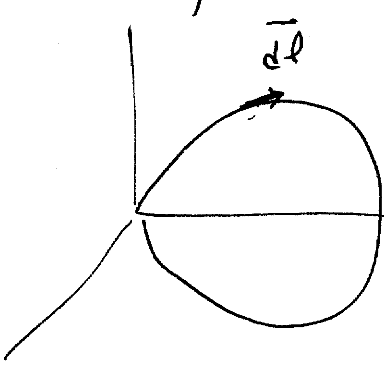
1 Gauss = 10^{-4} tesla

$$|B| = \sqrt{B_r^2 + B_\lambda^2 + B_\phi^2}$$

$$= \frac{\mu_0 M}{4\pi r^3}\left(1 + 3\sin^2\lambda\right)^{1/2}$$

$\propto \frac{1}{r^3}$ dipole field falls off as $\frac{1}{r^3}$

Equation for a field line



$\overline{dl} \times \overline{B} = 0$ definition of a flux tube

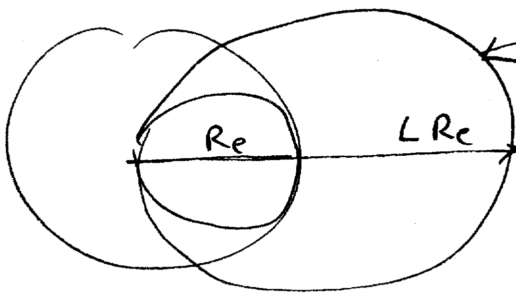
$$\frac{dr}{B_r} = \frac{r\,d\theta}{B_\theta} = \frac{r\sin\theta\,d\phi}{B_\phi}$$

$d\phi = 0$ plugin for B_r, B_θ

integration
Gives $\phi = \phi_0$ and $\boxed{r = r_0 \cos^2 \lambda}$

equation for a dipole field line

Label for dipole field lines based on equator crossing distance



$r = L R_e \cos^2 \lambda$

defines the "L" shell
(Used in particle dynamics in magnetosphere

Organization of text
Ch. 3 → E, B fields with NO particles
4 → " " " " 1 particle
5 → collective effect of many particles