

## Lecture 21 Notes from Prof. F. S. Mozer, UC Berkeley

Physics 142

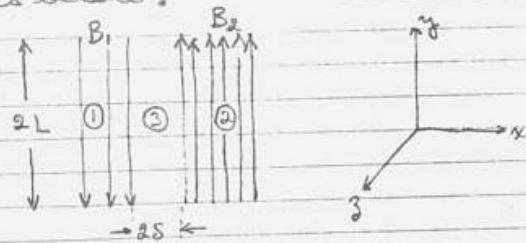
### MAGNETIC FIELD ANNIHILATION OR RECONNECTION

The purposes of this section are to discuss:

a) How open magnetic field lines get created in the magnetosphere discussed earlier.

b) To describe the process of magnetic field annihilation or reconnection

The open field lines occurred in the system having interplanetary field lines flowing past the magnetosphere. Idealize this problem in 2-dimensions as below:



Region ①

Interplanetary field lines flowing to right.  $T = \infty$   
Magnetic field =  $B_1$  and in  $-y$  direction

Region ②

Magnetospheric field lines that go to the solid body.  
 $T = \infty$ . Magnetic field =  $B_2$  and in  $+y$  direction

Region ③

Region of small width  $2S$  whose properties will be deduced below. Clearly something unusual happens in this region since there is a steady state flow of plasma and field into it from the left. The frame of reference is selected so that region 3 is stationary.

Why do the field lines / no solid body, move?  
What kind of force have to act?

The field lines in region ① are flowing to the right at speed  $v_1$   
 ∴ in region ①

$$E_{31} = \frac{v_1}{c} B_1 \quad \text{from ohm's law } \delta = \theta$$

Since the tangential component of  $E$  is continuous across the boundary from region ① to ③

$$E_{33} = E_{31}$$

if  $\Delta z \ll$  length of the interface in the  $z$ -direction

Then the electric field in region ③ is roughly constant

At the interface between region ② and region ③

Tangential is continuous

∴ just inside region ③

$$E_{32} = E_{31}$$

∴ THE ELECTRIC FIELD IS CONSTANT IN AND NEAR THE INTERFACE

#### CONSEQUENCES OF THIS FACT

$$\text{I in region ② } V_2 = \frac{S \vec{E} \times \vec{B}_2}{B_2^2} = -\hat{x} \frac{V_1 B_1}{B_2}$$

So field lines flow towards the interface from region ② also.

Flux of field lines flowing toward interface  
 = density of field lines  $\times$  velocity

$$= B_2 \times \frac{V_1 B_1}{B_2} = V_1 B_1$$

∴ NUMBER OF FIELD LINES/SEC FLOWING TOWARDS INTERFACE IS THE SAME ON BOTH SIDES

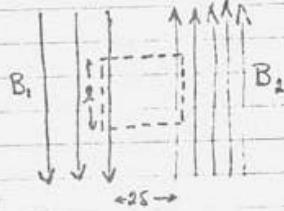
II In region ③  $B$  must be nearly equal to zero by symmetry

$\therefore j_{3z} = \sigma E_{3z}$  and we can't assume  $\sigma = \infty$  or an infinite current would flow.  
 $\therefore$  MHD equations not valid in region ③

The current density in region ③ is computed from integrating  $\nabla \times \vec{B} = \frac{4\pi}{c} \vec{j}$  over the surface below

This gives

$$\begin{aligned} (B_1 + B_2)l &= \frac{4\pi}{c} j_{33} \times 2s l \\ &\text{using } \text{sym} \quad \checkmark \text{ ③} \\ &= \frac{4\pi}{c} \times 2s \sigma E_{3z} \\ &= \frac{4\pi}{c} \times 2s \sigma \frac{V_1}{c} B_1 \end{aligned}$$



Thus  $V_1 = \left(1 + \frac{B_2}{B_1}\right) \frac{c^2}{8\pi s \sigma}$

In region ③  $\vec{j} \cdot \vec{E} \neq 0$

$\therefore$  Energy is dissipated in region ③ and the plasma is heated as it flows into the region.

WHERE DOES THIS ENERGY COME FROM?

Since  $B_3$  is small, the magnetic energy  $\frac{B_1^2}{8\pi}$  flowing in at speed  $V_1$  and  $\frac{B_2^2}{8\pi}$  flowing in at speed  $V_1 \frac{B_1}{B_2}$  must disappear

$\therefore$  MAGNETIC ENERGY  $\rightarrow$  PARTICLE ENERGY

This is why the process is called magnetic field annihilation

WHERE DOES THE HEATED PLASMA GO?

It can't flow in the  $x$ -direction because that opposes the inward flow of plasma and field line flow.

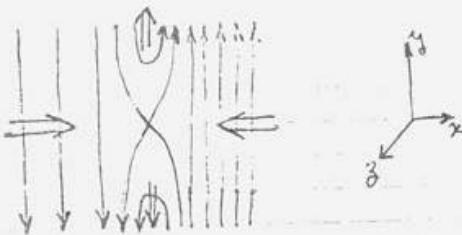
It can't go in the  $y$ -direction because of the assumption that the process is occurring at all  $y$ -values (2-DIMENSIONAL)

So the heated plasma must flow away in the  $z$ -direction

Thus this process can't occur simultaneously at all  $y$ -values but only at one single location.

Thus the geometry of the problem really is

Double arrows indicate flow directions. Same number of field lines flow to the boundary per second from both sides (see page 146)



#### CONCLUSIONS

1. The annihilation process occurs only at one point or along a line in the  $z$ -direction, at which  $B=0$

This is called the neutral point or line.

~~There are various field geometries that can produce points or lines where  $B=0$ . These are called an X-type neutral point or line for obvious reasons.~~

2. In the central region far from the neutral line,  $B \neq 0$  and the assumptions of MHD might again become valid. Since the plasma is flowing up or down in those regions, there is the magnetic field. The  $B$  lines flow in from left and right, annihilate each other at the neutral point, connect with each other, and flow up or down out of the picture.

At the end of the pencil upper halves of field lines initially on the right and left, become connected to each other, as do lower halves. This is how interplanetary field lines can connect to magnetospheric field lines to produce "open" field lines.

The speed of the post-reconnection flow away from the neutral line may be computed as follows:

Magnetic field energy density flowing to neutral line from left per second

$$= \frac{B_1^2}{8\pi} l_y l_z V_1 \quad \text{where } l_y \text{ and } l_z \text{ are small lengths in the } y \text{ and } z \text{ directions}$$

Number of plasma particles flowing to neutral line from left per second

$$= n l_y l_z V_1 \quad \text{where } n = \text{plasma density}$$

∴ Energy gain per particle

$$= \frac{B_1^2}{8\pi m}$$

Thus, if  $V_y$  is the post-reconnection velocity in the  $y$ -direction in region (2) of those particles that cross into region (2) from region (1)

$$\frac{1}{2} m V_y^2 = \frac{1}{2} m V_1^2 + \frac{1}{2} m V_{T_H}^2 + \frac{B_1^2}{8\pi m}$$

where  $V_{T_H}$  is the thermal velocity of particles in region (1)

or

$$V_y^2 = V_1^2 + V_{T_H}^2 + \frac{B_1^2}{4\pi m} \quad (1)$$

A second equation for  $V_y$  may be obtained from conservation of mass as

mass flowing from left. = mass flowing out above and below

## PHYSICS 142

From here on assume  $B_1 = B_2$  only to simplify the equations (which could be solved for an arbitrary ratio of  $B_1$  to  $B_2$ )

In this case, by symmetry, the flow up and down in region ③ of particles entering from the left, occurs over a region of width  $\delta$ .

So

$$\rho_1 \times 2L \times l_3 \times V_1 = \rho_3 \times 2 \times \delta \times l_3 V_y$$

where  $l_3$  is a length in the  $z$ -direction

$2L$  is the scale length in the  $y$ -direction  
(see figure on page 145)

$\rho_1, \rho_3$  are mass densities in regions ① and ③

$V_y$  = post-reconnection flow in the  $y$ -direction in region ③

assuming for simplicity that the flow is incompressible (better assumption give more complex equations but don't change the physical idea) then

$$\rho_1 \equiv \rho_3$$

and 
$$V_y = \frac{L}{\delta} V_1 \quad (2)$$

Combining equation (1) (Page 150), the equation for  $V_1$  on page 147, and equation (2) gives (with the assumptions that  $L/\delta \gg 1$ , and  $\frac{B_1^2}{4\pi\rho_1} \gg \eta_{T4}^2$ , to simplify the equations)

## PHYSICS 142

$$\frac{\delta}{L} = \left( \frac{c^2}{4\pi\sigma L V_A} \right)^{1/2}$$

$$V_y = V_A$$

$$V_t = c \left( \frac{V_A}{4\pi\sigma L} \right)^{1/2}$$

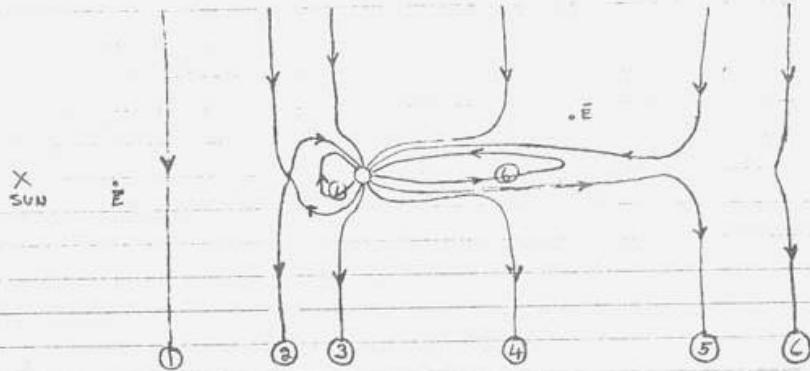
where  $V_A \equiv \text{Alfvén speed} \equiv \left( \frac{B_0^2}{4\pi\rho_0} \right)^{1/2}$

## CONCLUSIONS

1. The reconnection region is thin if  $L$  is big enough
2. The post-reconnection flow occurs at the Alfvén speed
3. The flow into the reconnection region depends on the conductivity,  $\sigma$ , in the reconnection region. Therefore the importance of this process depends on  $\sigma$  and not much happens if  $\sigma$  is too big. When this mechanism was first applied to solar flares, it was concluded that magnetic field annihilation was not the cause of solar flares because  $\sigma$ , based on Coulomb collisions, was too large. Later someone estimated  $\sigma$  based on wave-particle interaction (as will be discussed later in the course) and  $\sigma$  became small enough. Now it seems likely that inertial terms alone (neglected in previous derivation) suffice to make  $\sigma$  small enough. So reconnection is probably important for solar flares and larger flares (pulsars? quasars?) and certainly dominates other mechanisms for putting energy into the terrestrial magnetosphere.

## ENERGY INPUT INTO THE TERRESTRIAL MAGNETOSPHERE

Consider a terrestrial and an interplanetary magnetic field line at several successive points in time; 1, 2, 3, etc. The interplanetary medium flows from left to right.



Reconnection occurs at the X-type neutral line at time ②

Terrestrial magnetic field lines are pulled away from the polar caps by the solar wind flow at times ③ & ④

Reconnection occurs again between times ⑤ and ⑥ after which the interplanetary field line flows to the right and the terrestrial field line flows ~~outward~~<sup>forward</sup>. The kink in the interplanetary field line created at both reconnection points (near times 2, 3, 5 and 6) propagates away at the Alfvén speed.

Terrestrial field lines on the dayside extend to distances  $\leq 10$  earth radii from Earth. These lines are stretched by the solar wind flow to make a tail in the anti-solar direction whose length is  $\sim 500$  earth radii.

THIS STRETCHING OF FIELD LINES  
PROVIDES THE ENERGY INPUT THAT DRIVES  
MAGNETOSPHERIC PROCESSES (particle acceleration,  
radiation belts, auroras, ionospheric currents, etc.)

17

## PHYSICS 142

The energy input to the tail can be computed from the Poynting flux into the tail.

The electric field out of the paper in the figure of page 153 is due to the solar wind flow  $V_i$  in the interplanetary medium where the field strength is  $B_i$ .

$$\therefore E = \frac{V_i}{c} B_i$$

The Poynting vector  $\frac{c}{4\pi} \vec{E} \times \vec{B}$  points toward the equatorial plane in the tail

Thus

$$\text{Energy input/second} = 2 \times \frac{c}{4\pi} E \langle B_{\text{TAIL}} \rangle \times \underset{\uparrow}{\text{width of tail}} \times \underset{\text{length of tail}}{\text{length}}$$

Two halves  
of tail into which  
energy flows

$$= \frac{c}{2\pi} \frac{V_i}{c} B_i \langle B_{\text{TAIL}} \rangle \times \text{width} \times \text{length}$$

From measurement

$$V_i \approx 300 \text{ kilometers/second}$$

$$B_i \approx 3 \times 10^{-5} \text{ gauss}$$

$$\langle B_{\text{TAI}} \rangle \approx 20 \times 10^{-5} \text{ gauss}$$

$$\text{width} \approx 40 \text{ earth radii}$$

$$\text{length} \approx 500 \text{ earth radii}$$

$\therefore$  Potential across magnetopause  $\approx 60$  kilovolts  
and Energy input  $\approx 10^{20}$  ergs/second

THIS IS ENOUGH ENERGY TO DRIVE ALL KNOWN MAGNETOSPHERIC PROCESSES

The real solution of the terrestrial magnetosphere reconnection problem involves:

1. A 3-dimensional reconnection calculator.  
The interplanetary magnetic field fluctuates about its mean "garden hose" direction and is rarely exactly anti-parallel to the earth's field. So a calculation for arbitrary directions between the two fields must be performed.
2. The solar wind flow is supersonic.  
Thus the correct description of its interaction with the magnetosphere involves shock fronts and inclusion of discontinuities.