Lecture 24

Plasma Waves (Plane waves, phase velocity and group velocity)
MHD Waves review

Waves in plasmas

assume timevarying quantities vary as plane waves

$$n = \tilde{n} e^{i(\bar{k}\cdot\bar{r} - \omega t)}$$

where $\bar{k}\cdot\bar{r} = k_x x + k_y y + k_z z$ (cartesian coords)

a point of constant phase moves so that

$$\frac{d}{dt}(kx - \omega t) = 0 \qquad (1-\text{Dim})$$

or $\frac{dx}{dt} = \frac{\omega}{k} \equiv v_\phi$ phase velocity

Similarly $E = \bar{E} e^{i(kx - \omega t + \delta)}$ allowing a different phase for E

$$= \underbrace{\bar{E} e^{i\delta}}_{\text{complex } \bar{E}_c} e^{i(kx - \omega t)}$$

Phase velocity can exceed c → does not violate relativity because only long wave train of constant amplitude carries no information.

Group Velocity:
Carrier of a radio wave carries no information until it is modulated. The modulation travels slower than c. To see this imagine two waves of nearly equal frequencies →

$$E_1 = E_0 \cos\left[(k+\Delta k)x - (\omega+\Delta\omega)t\right]$$
$$E_2 = E_0 \cos\left[(k-\Delta k)x - (\omega-\Delta\omega)t\right]$$

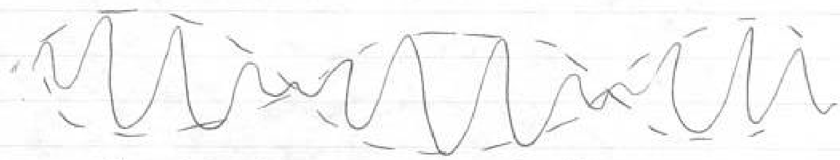
frequency difference $= 2\Delta\omega$
wave# difference $= 2\Delta k$

let $a \equiv kx - \omega t$
$b \equiv (\Delta k)x - (\Delta\omega)t$

add the waves

$$\begin{aligned} E_1 + E_2 &= E_0 \cos(a+b) + E_0 \cos(a-b) \\ &= E_0(\cos a \cos b - \sin a \sin b + \cos a \cos b + \sin a \sin b) \\ &= 2E_0 \cos a \cos b \\ &= 2E_0 \cos\left[(\Delta k)x - (\Delta\omega)t\right](\cos kx - \omega t) \end{aligned}$$

this is a sinusoidally modulated wave



the envelope of the wave is given by
$\cos\left[(\Delta k)x - (\Delta\omega)t\right]$ carries
the information with the group velocity $\frac{\Delta\omega}{\Delta k}$

take limit as $\Delta\omega \to 0$ and define

Group velocity $v_g \equiv \frac{d\omega}{dk}$

Plasma Oscillation

Look at situation where
1. No magnetic field
2. Cold $(kT = 0)$
3. ions fixed in space
4. ∞ plasma
5. motion of e^- only in 1-Dim $(\hat{x})$

so $\vec{\nabla} \rightarrow \hat{x}\frac{\partial}{\partial x}$ $\quad \vec{E} = E\hat{x}$ $\quad \vec{\nabla}\times\vec{E} = 0 \Rightarrow \vec{E} = -\vec{\nabla}\phi$

from electron equations of motion

$$m n_e \cdot \frac{\partial v_e}{\partial t} + (\bar{v}_e \cdot \bar{\nabla})\bar{v}_e = -e n_e \vec{E} \qquad \text{Force}$$

$$\frac{\partial n_e}{\partial t} + \bar{\nabla}\cdot(n_e \bar{v}_e) = 0 \qquad \text{continuity}$$

$$\epsilon_0 \bar{\nabla}\cdot\bar{E} = \epsilon \frac{\partial \bar{E}}{\partial x} = e(n_i - n_e) \qquad \text{Gauss's Law}$$

Now Linearize by assuming small amplitude oscillation

"0" → time independent

$$n_e = n_0 + n_1 e^{i(kx-\omega t)} \qquad v_e = v_0 + v_1 e^{i(kx-\omega t)} \qquad E = E_0 + E_1 e^{ikx}$$

but Let $v_0 = E_0 = 0$ and $\nabla n_0 = 0$

second order

Force equation becomes $m\frac{\partial v_1}{\partial t} + \cancel{v_1 \cdot \nabla v_1} = -eE_1$

continuity eqn $\frac{\partial n_1}{\partial t} + \nabla\cdot(n_0 v_1 - \cancel{n_1} v_0) = 0$

stationary

in equilibrium $n_{0e} = n_{0i}$ and we assumed $n_{1i} = 0$ (ions stationary)
so Gauss's law is

$$\epsilon_0 (\nabla \cdot E_1) = -e n_1$$

so we have 3 equations

$$-i m \omega v_1 = -e E_1$$

$$-i \omega n_1 = -n_0 i k v_1$$

$$i k \epsilon_0 E_1 = -e n_1$$

eliminate n_1 and E_1 to get

$$-i m \omega v_1 = -e \frac{-e}{ik\epsilon_0} \frac{-n_0 i k v_1}{-i\omega} = -i \frac{n_0 e^2}{\epsilon_0 \omega} v_1$$

if v_1 does not go to zero we must have

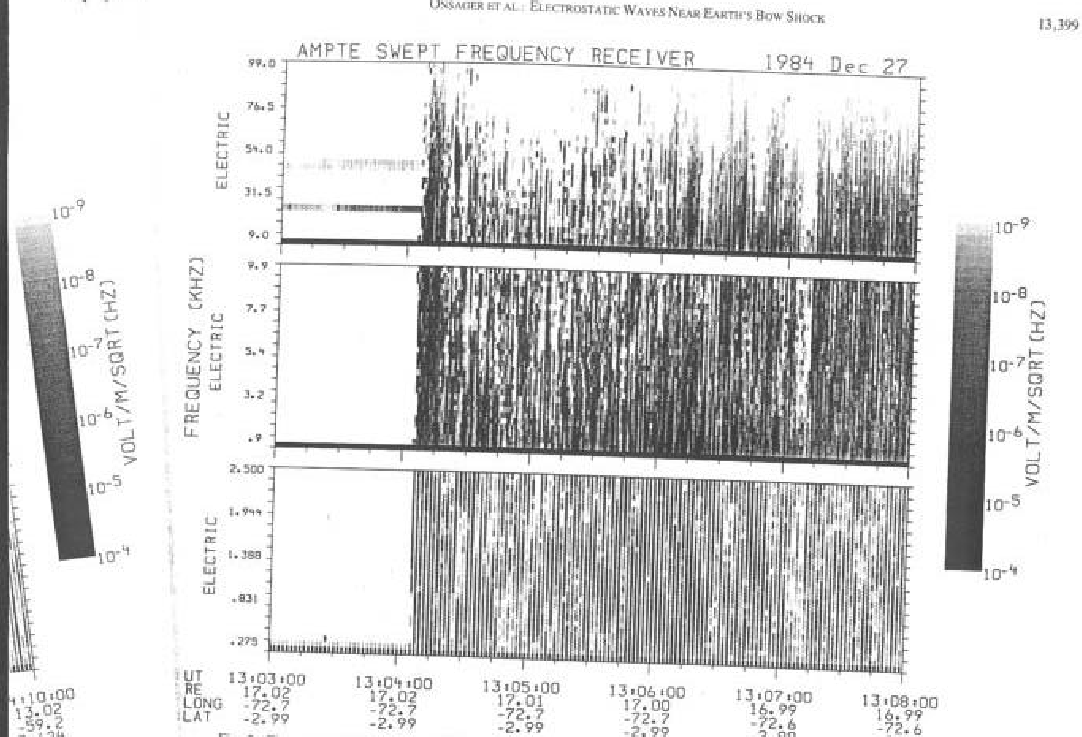
$$\boxed{\omega_p^2 = \frac{n_0 e^2}{m \epsilon_0}}$$

note that the plasma frequency $f_p = \frac{\omega}{2\pi} \sim 9\sqrt{n}$
where n is in mks ($\frac{\#}{m^3}$)

eq. ionosphere peak densities $n \sim 10^{10}/m^3 - 10^{12}/m^3$

so $f_p \sim 1-10$ MHz peak frequency

Solar wind example



ember 14, 1984. The
pper panel). Upstream
e shock (after 0406:30
pproximately 78 kHz.

Fig. 2. Five-minute electric field spectrogram during a transition from solar wind to magnetosheath on December 27, 1984. The format is the same as in Figure 1. Upstream (before 1304:05 UT), Langmuir oscillations and oscillations at twice the plasma frequency are detected at approximately 24 kHz and 45–48 kHz. Downstream from the shock (after 1304:10 UT), broadband bursty waves are detected up to between approximately 40 kHz and 80 kHz.

ccounted for by either thstic waves and the plasma frequency with amplitudes as
1 as an order of magnitude or more above the instrument
spectra and wave polarihold. At the highest-β_e and highest-M_A shocks, electrostatic
netic field) we tentativelys in this frequency range typically have very low ampli-
as the electron beam mod or are not detected at all. The results of this study are
quencies is a possible indiscussed in terms of the possible wave generation mechan-
even though the beams nand the implications regarding the processes occurring at
rements. This point is dhock. Three beam generation mechanisms are discussed:
s. Since the electron bearation by the cross-shock electric field, acceleration by
asma frequency to above thybrid frequency waves, and a magnetosheath "time of
possible that much of themechanism.
wed range for Doppler-shi
ectron beam mode.

2. SURVEY OF WAVES OBSERVED AT THE EARTH'S BOW SHOCK

n study between the high-f
ous upstream solar wind and section electric field measurements from the Earth's bow
ameters are then presentd magnetosheath are presented. The observed wave fre-
nd are anticorrelations betare compared with the plasma normal mode frequencies
s and Alfven Mach number from the measured parameters. It is demonstrated that
arameters M_A and β_e are tic waves are present at frequencies above the maximum
nd; therefore any correlation for Doppler-shifted ion acoustic waves yet below the
. It is shown that at thequency. Whereas previous researchers have identified
≤ 6) shocks, electrostatic hock and magnetosheath electrostatic waves as either
mum frequency for Doppleric or Langmuir waves [*Rodriguez and Gurnett*, 1975, 1976; *Gallagher*, 1985], the analysis presented here shows that these wave modes alone cannot account for the entire measured spectra. From the measured wave spectra and polarization, the electrostatic waves with frequencies above the maximum frequency for Doppler-shifted ion acoustic waves and below the plasma frequency are tentatively identified as electron beam mode waves.

The center frequencies of the three SFRs range from 275 Hz to 99 kHz. The low-frequency SFR measures the frequency range of 275 Hz to 2525 Hz in 32 evenly spaced frequency steps every 2 s. The medium- and high-frequency SFRs measure the frequency ranges 0.9 kHz to 9.9 kHz and 9.0 kHz to 99 kHz, with each SFR sampling 32 evenly spaced frequency steps each second. The channel bandwidths for the low-, medium-, and high-frequency instruments are 100 Hz, 300 Hz, and 3 kHz, respectively. The electric field antenna is a single dipole, 47 m tip to tip. This instrument has been described by *Häusler et al.* [1985]. The vector magnetic field is obtained from three orthogonal flux gate sensors [*Lühr et al.*, 1985]. Moments of the electron and ion distribution functions are obtained from the three-dimensional plasma instrument [*Paschmann et al.*, 1985].

Five-minute electric field spectrograms from two bow shock crossings are shown in Figures 1 and 2. These spectrograms show