Problems for all students:

The first two problems illustrate that the Eulerian description of motion is usually simpler than the Lagrangian. In these problems, \( \mathbf{r} \) is the current position of a fluid particle that was at \( \mathbf{x} \) at time \( t=0 \) and \( \mathbf{u} \) is the velocity at \( \mathbf{r} \). Bold face denotes a vector quantity and the subscripts 1, 2 and 3 refer to components along 3 perpendicular directions.

1. The Lagrangian description \( \mathbf{r} = \mathbf{r}(\mathbf{x},t) \) of the motion of a certain lump of jello is

\[
\begin{align*}
\mathbf{r}_1 &= x_1 \cos(\omega t) + x_2 \sin(\omega t) \\
\mathbf{r}_2 &= -x_1 \sin(\omega t) + x_2 \cos(\omega t) \\
\mathbf{r}_3 &= x_3
\end{align*}
\]

Find the Eulerian description \( \mathbf{u} = \mathbf{u}(\mathbf{r},t) \). Describe the motion geometrically.

2. The Eulerian description of the motion of another lump of jello is

\[
\begin{align*}
\mathbf{u}_1 &= \mathbf{r}_1 \\
\mathbf{u}_2 &= -\mathbf{r}_2 \\
\mathbf{u}_3 &= 0
\end{align*}
\]

Derive the Lagrangian description \( \mathbf{r} = \mathbf{r}(\mathbf{x},t) \). Draw a picture of the paths of the fluid particles.

Problem for 514 students only

3. In the figure below, the cylinder on the left is stationary and the fluid velocity is \( \mathbf{u}_0 \) at great distance. On the right, the fluid far away is stationary and the cylinder moves to the left with velocity \( \mathbf{u}_0 \). Show that this change of reference frame results in changes to both \( \partial \mathbf{u} / \partial t \) and \( \mathbf{u} \cdot \nabla \mathbf{u} \), but leaves \( D\mathbf{u} / Dt \) unchanged.

Hints: What does \( D\mathbf{u} / Dt \) mean physically? What is \( D\mathbf{u}_0 / Dt \) equal to on the left? If \( \mathbf{u}_1 \) is the fluid velocity at point A on the left, what is \( \mathbf{u}_2 \) on the right?