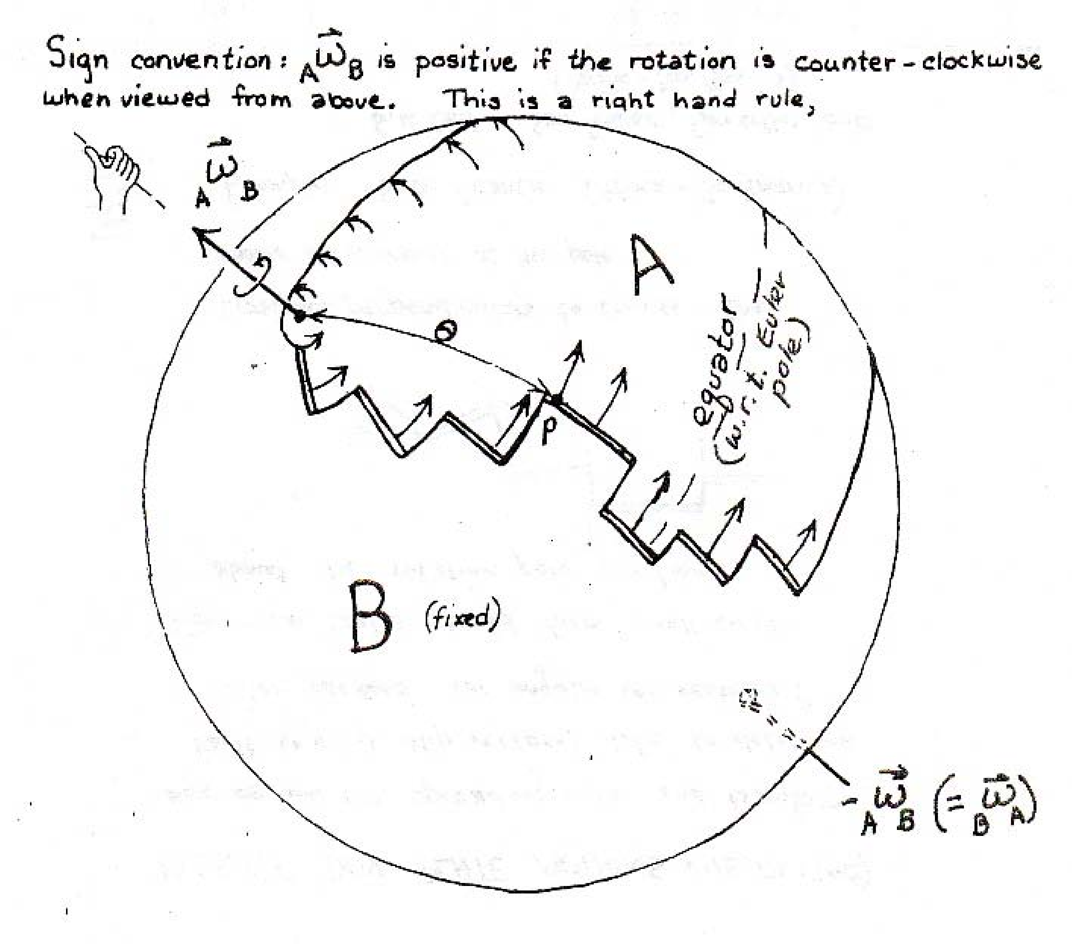
# Plate motions on a sphere

Euler's Theorem, 1776 ("Oiler")

The motion of a rigid body (e.g. a plate) across the surface of a sphere can be described as a rotation about some pole that passes through the center of the sphere.

Plates cannot be translated, only rotated.

Also, any combinations of rotations can be described as some equivalent single rotation.



Sign convention: $_A\vec{\omega}_B$ is positive if the rotation is counter-clockwise when viewed from above. This is a right hand rule, Two versions of Euler poles:

RELATIVE PLATE VELOCITIES are described by
"instantaneous poles" or "Euler vectors" or "angular velocity vectors"

For each plate pair, need
(a) pole position and (b) angular rate
(equivalent to vector direction {thru center of earth} and vector length)

Example: present relative motion of Pacific plate past North America is
.78°/m.y. about a pole at 49°N, 78°W

RELATIVE PLATE DISPLACEMENTS are described by
"finite poles" or "Euler poles"

For each plate pair need:
(a) pole position and (b) angle of displacement
( it is NOT a vector)

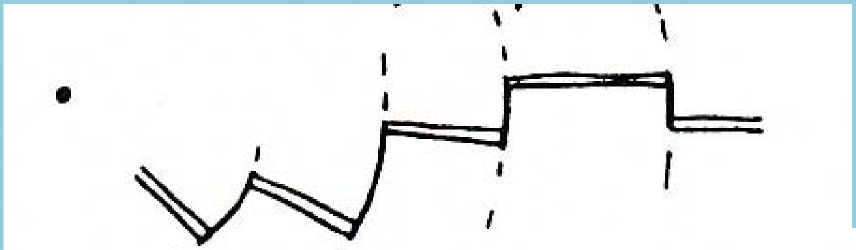
Example: to reconstruct the location of North America with respect to Europe at anomaly 24 we rotate it 13° about a pole at 68°N, 147°W

# Present day plate motions (velocities)

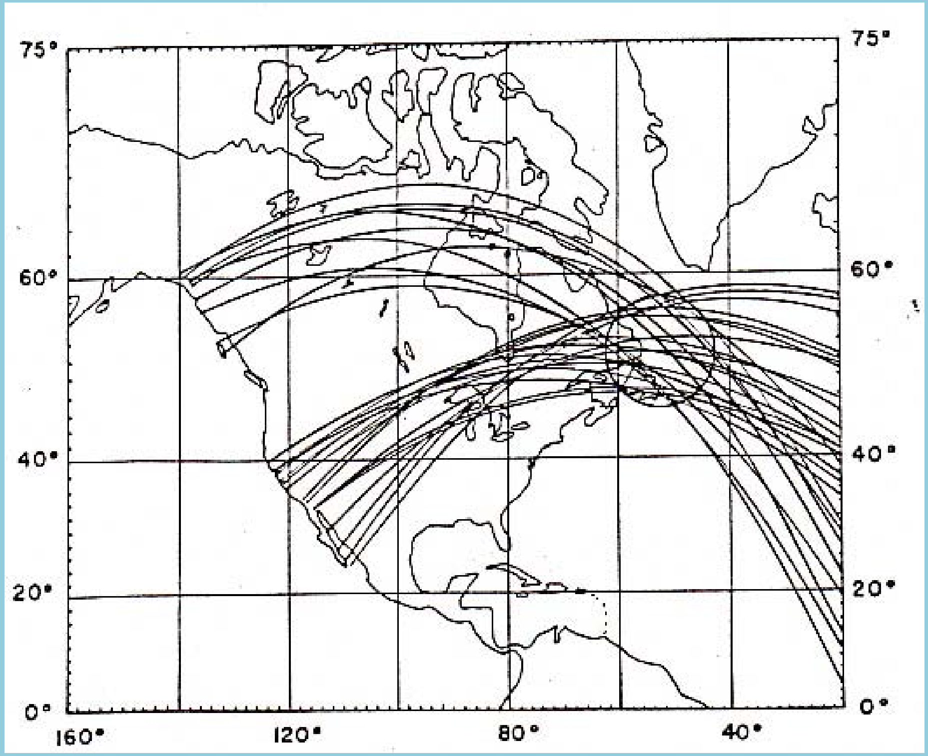
We use a) spreading rates and b) transform fault azimuths (or earthquake slip vectors) to determine Euler vectors

1) Transform faults should form small circles about the rotation pole position

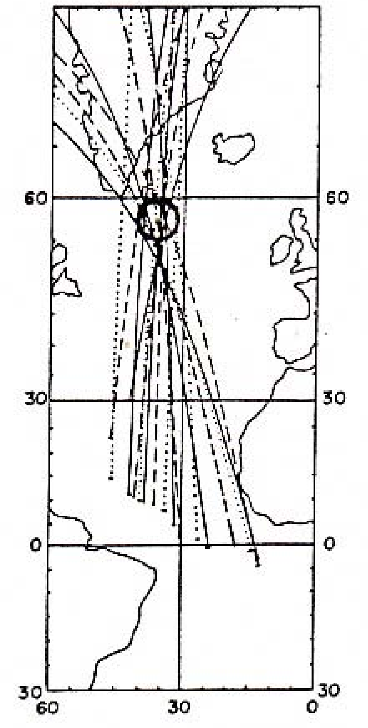
2) Perpendiculars to transform faults should all intersect at the pole



Ex. 1: Gulf of CA, San Andreas, Fairweather Faults (Pac – North Am.)

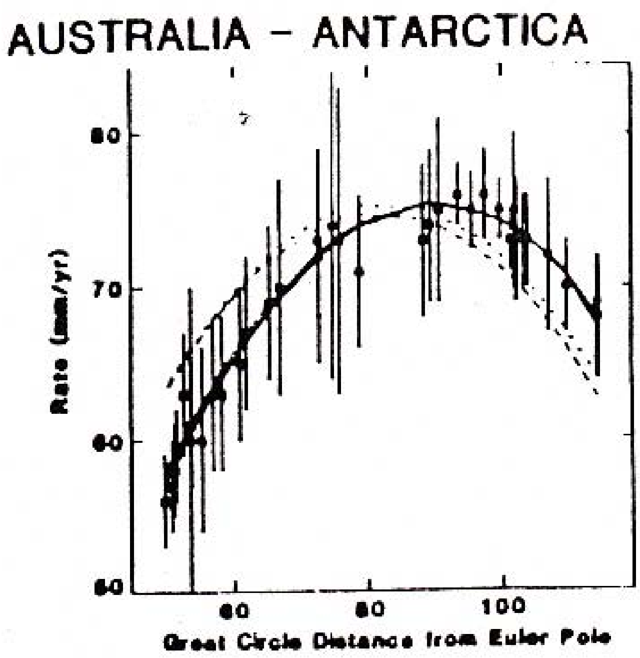


Morgan (1968)

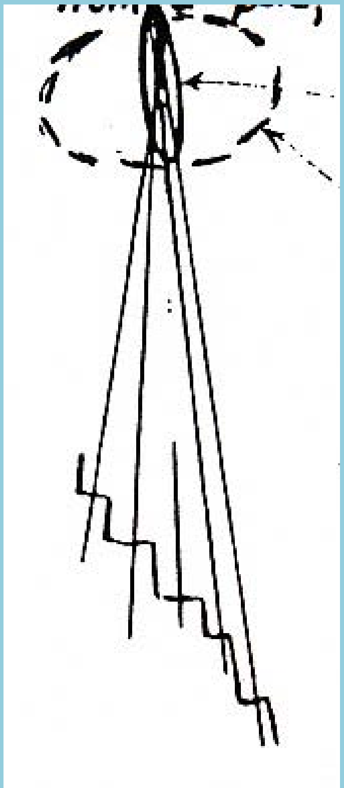


Ex. 2: South Atlantic (Africa - So. Amer.)

Rates of relative motion should vary as sine of angular distance from the pole.



Ex.: Southeast Indian Ridge



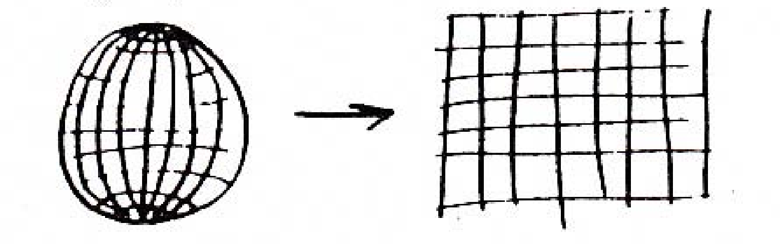
Uncertainties: Usually data are clumped in a smallish region in one general direction from a poles so that :

Transform crossing errors form a long ellipse

Rate errors form a larger, wide ellipse

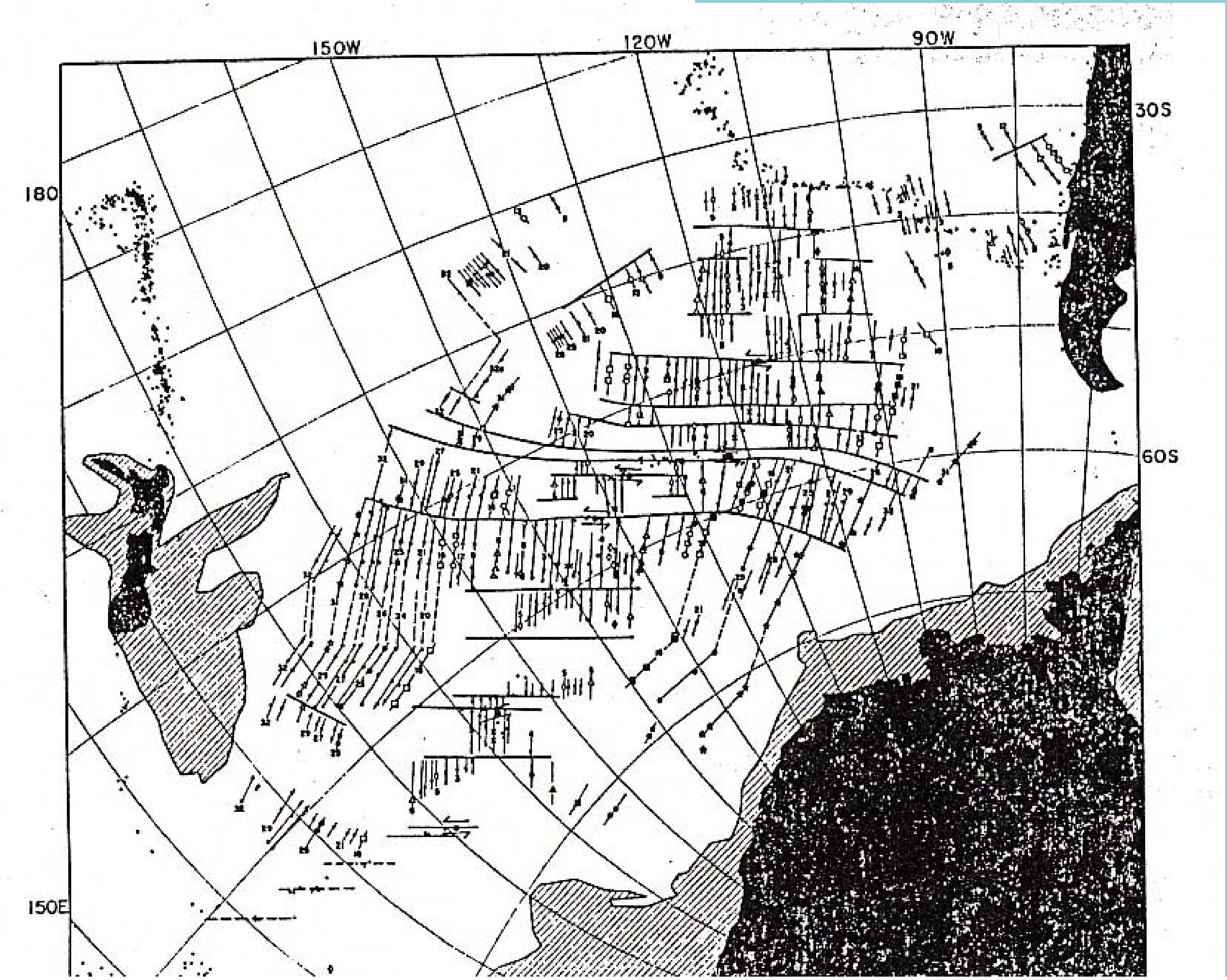
Combination actually gives a long ellipse, +/- 5 or 10°, elongated toward data region.

One way to check fit: Plot data on an “Oblique Mercator” projection using the Euler pole instead of the North pole.



1) Transform faults should be horizontal lines

2) Young magnetic anomalies should be evenly separated

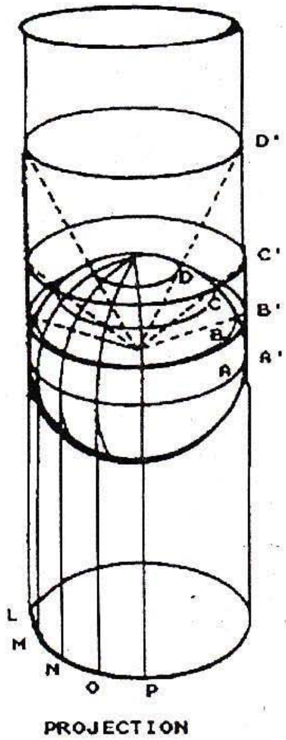


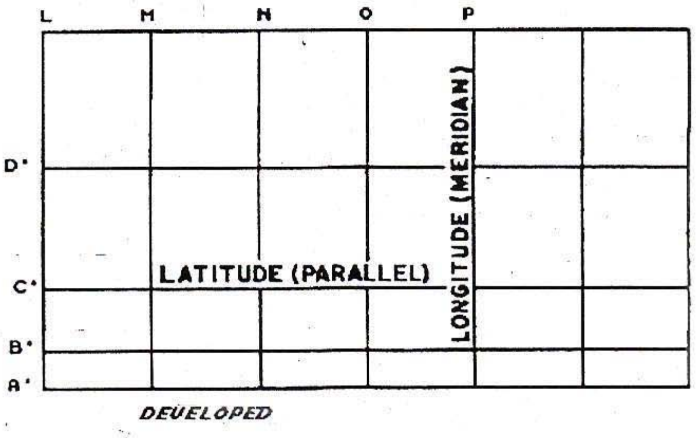
Ex.: Pacific-Antarctic ridge

Molnar et al. (1975)

Cylinder aligned with spin axis

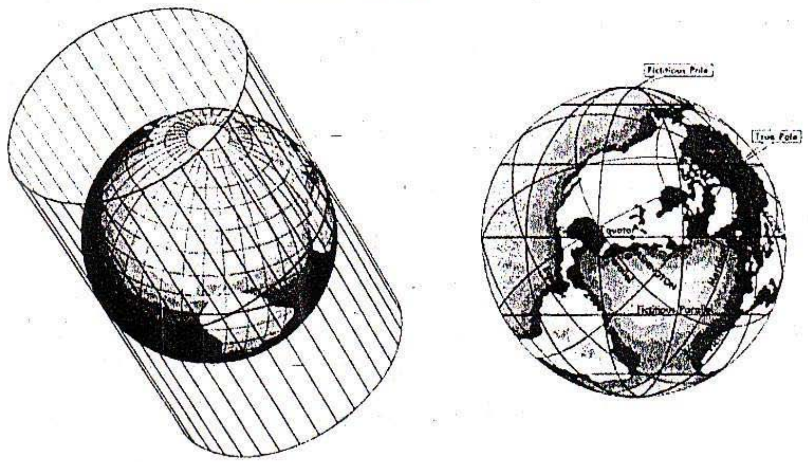
## Mercator Map Projection

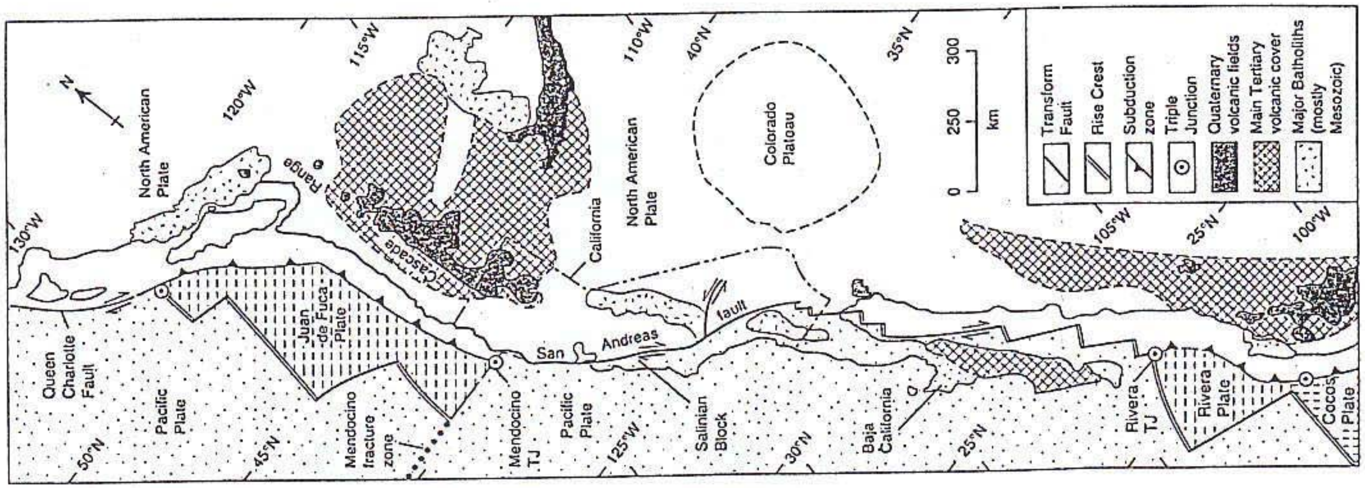




Shift cylinder to Euler pole

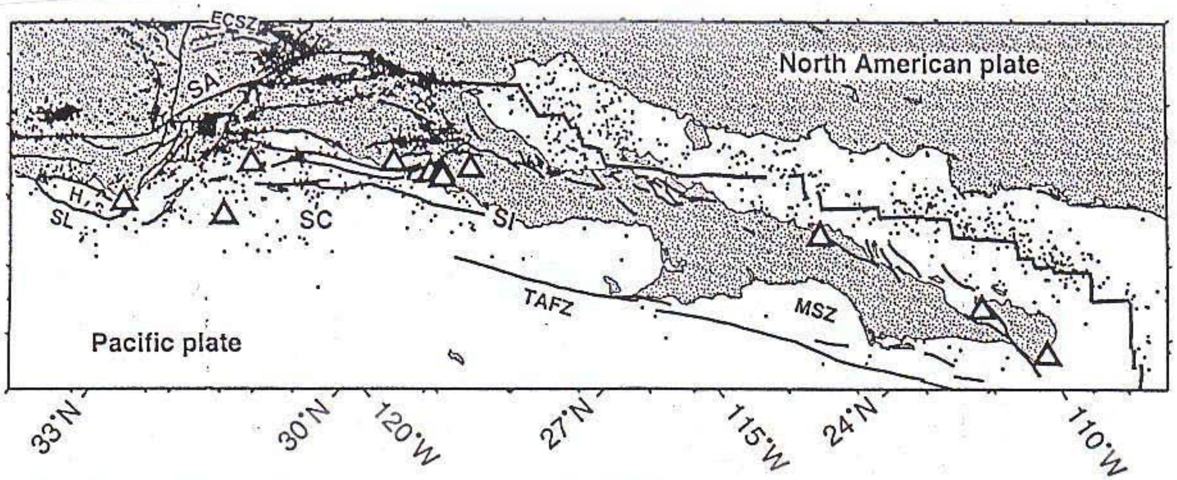
## Oblique Mercator Map Projection*





Tectonic map of Western North America shown in an Oblique Mercator projection about a pole at 53°N, 53°W (the Pacific-North America pole from Morgan, 1968).

Dickinson and Snyder (1979) as redrawn by Moores and Twiss (1995)



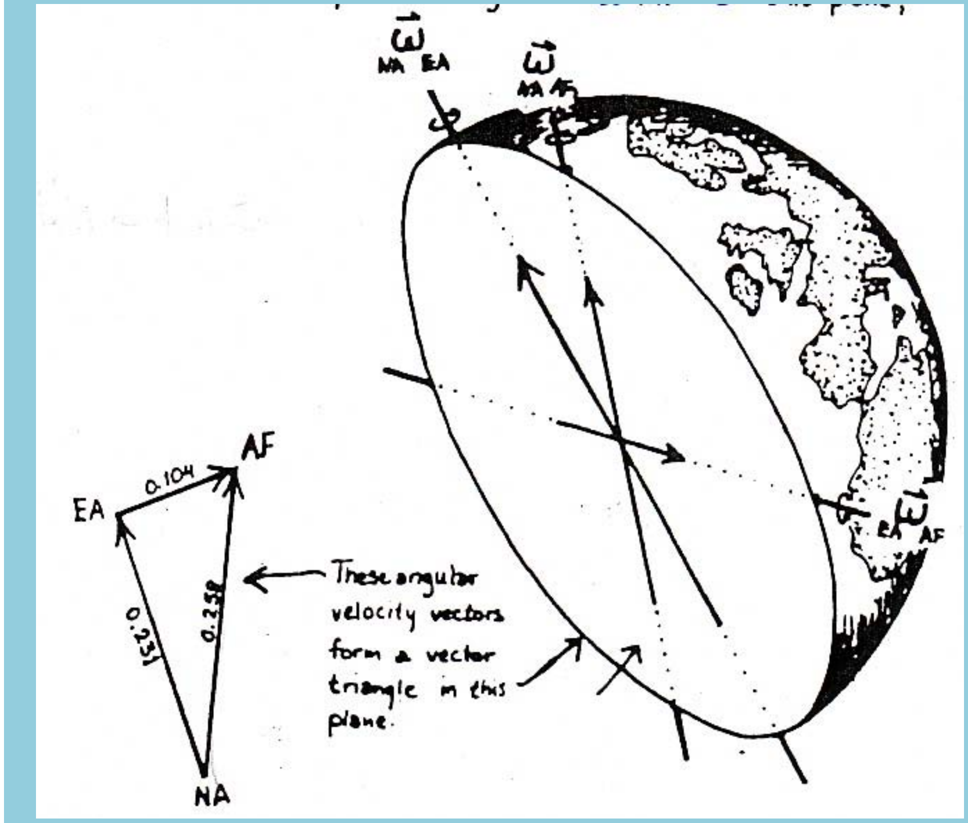
Newer Oblique Mercator pole: 50° N, 77° W (Dixon et al. , 2000)

# Euler vectors can be added (vector addition) to find others.

For example: add sea floor spreading in North and Central Atlantic to find motion across Mediterranean

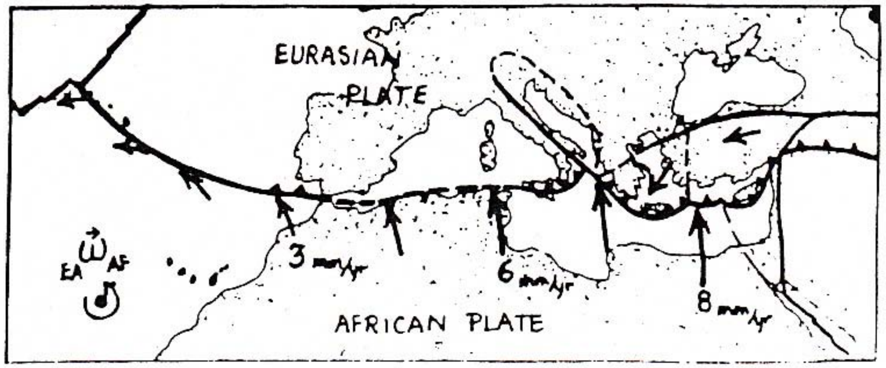
$$\vec{\omega}_{EA\,NA} + \vec{\omega}_{NA\,AF} = \vec{\omega}_{EA\,AF}$$

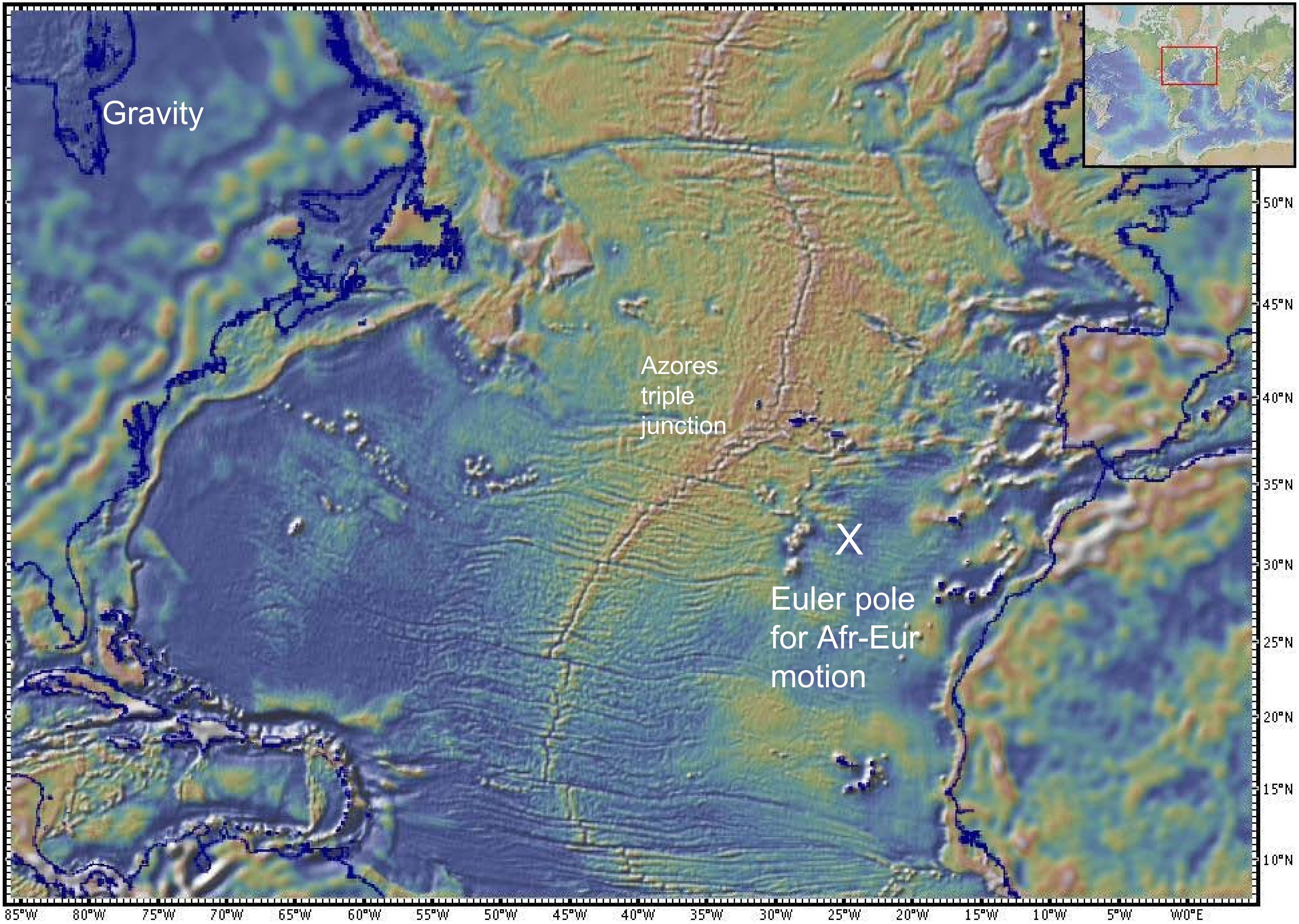
Addition of angular velocity vectors for Eurasia-North America and Africa-North-America to find Eurasia-Africa motion



The vectors, centered at the center of the earth, show locations of poles of rotation and their anti-poles. Two vectors define a plane through the earth. In this plane a vector triangle can be constructed to find the third vector

Map showing the location of $_{EA}\vec{\omega}_{AF}$ and the small circles about it. Arrows and numbers show directions and rates of motion across the Eurasian - African plate boundary described by this pole.




Gravity
Azores triple junction
X
Euler pole for Afr-Eur motion
50°N
45°N
40°N
35°N
30°N
25°N
20°N
15°N
10°N
85°W
80°W
75°W
70°W
65°W
60°W
55°W
50°W
45°W
40°W
35°W
30°W
25°W
20°W
15°W
10°W
5°W
W0°E

# GLOBAL SOLUTIONS FOR PRESENT-DAY PLATE MOTIONS

DeMets, Gordon, Argus, + Stein (1990), Geophys. J. Int., v 101, p 425
"NUVEL-1"

previous versions: "RM2" Minster + Jordon (1978) J.G.R., 83, 5331.
"PO71" Chase (1978) E.P.S.L., 37, 355

Did inversion (giant least squares fit) of global data set to find Euler vectors for major plate pairs.

a) Assume world plate model of 12 plates (ignore Philippine and Juan de Fuca plates for now) and define plate boundaries.

b) Collect and cull data set:

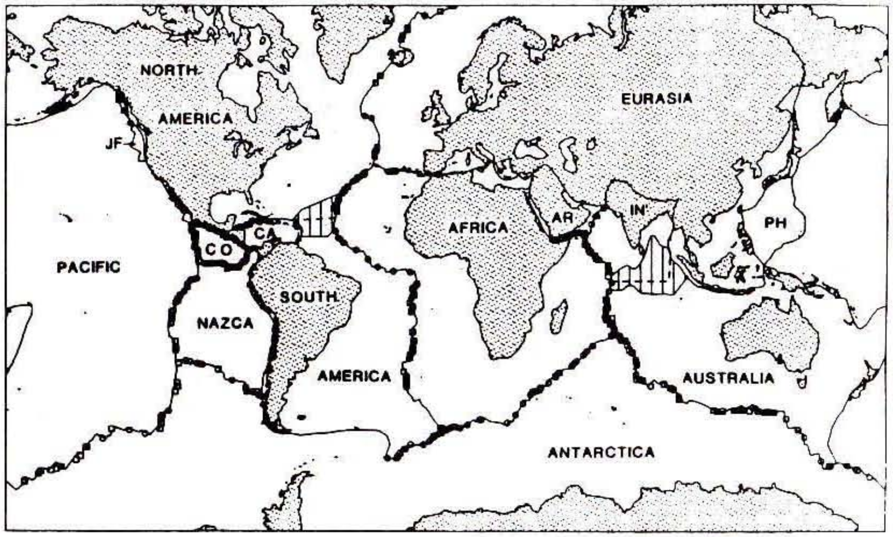
| | NUVEL-1 |
|---|---|
| Spreading rates, anom 2a ↔ 2a (3 m.y.) | 277 |
| Transform fault azimuths | 121 |
| Earthquake slip vector azimuths | 724 |
| (+ estimated uncertainty for each) | 1122 data pts |

Data not used for calculation of NUVEL-1:

oblique subduction slip vectors
short offset transforms
complex, multi-fault boundaries

Instantaneous = 3 Ma

DATA SET for NUVEL-1



1) Find individual "Best-fitting pole" for each plate pair with data on boundary. (Check internal consistency of data.)

2) Check local plate circuits for closure, e.g., around a triple junction.

3) Use all data at once to find global best fit: Euler vectors for all plate pairs
+ uncertainty ellipse for each vector
+ "importance" of each datum.
"NUVEL-1"

( k poles for global solution with individual "best-fitting" poles.

σ

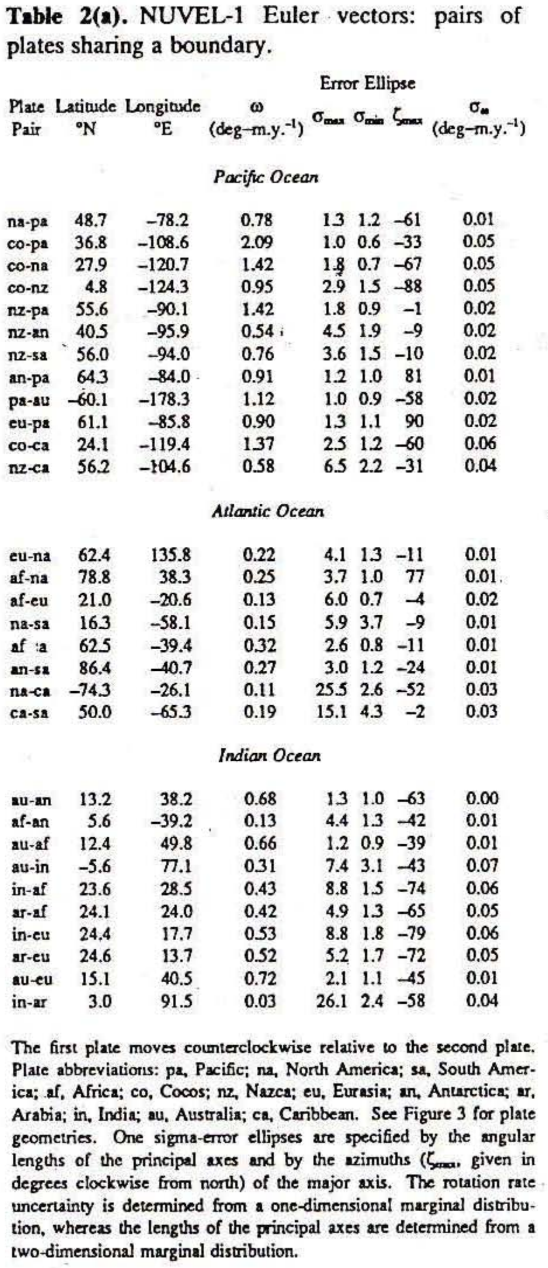
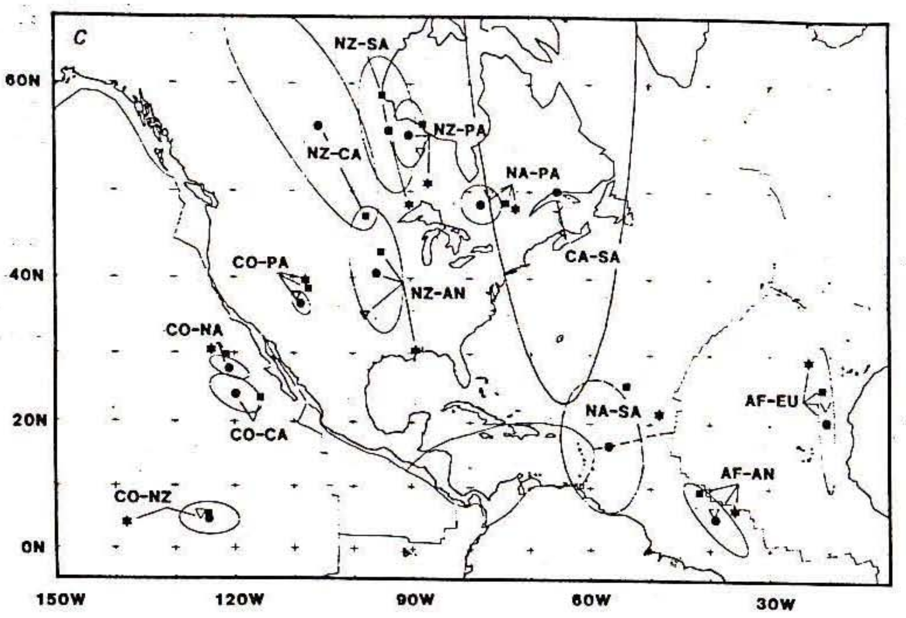


**Table 2(a).** NUVEL-1 Euler vectors: pairs of plates sharing a boundary.

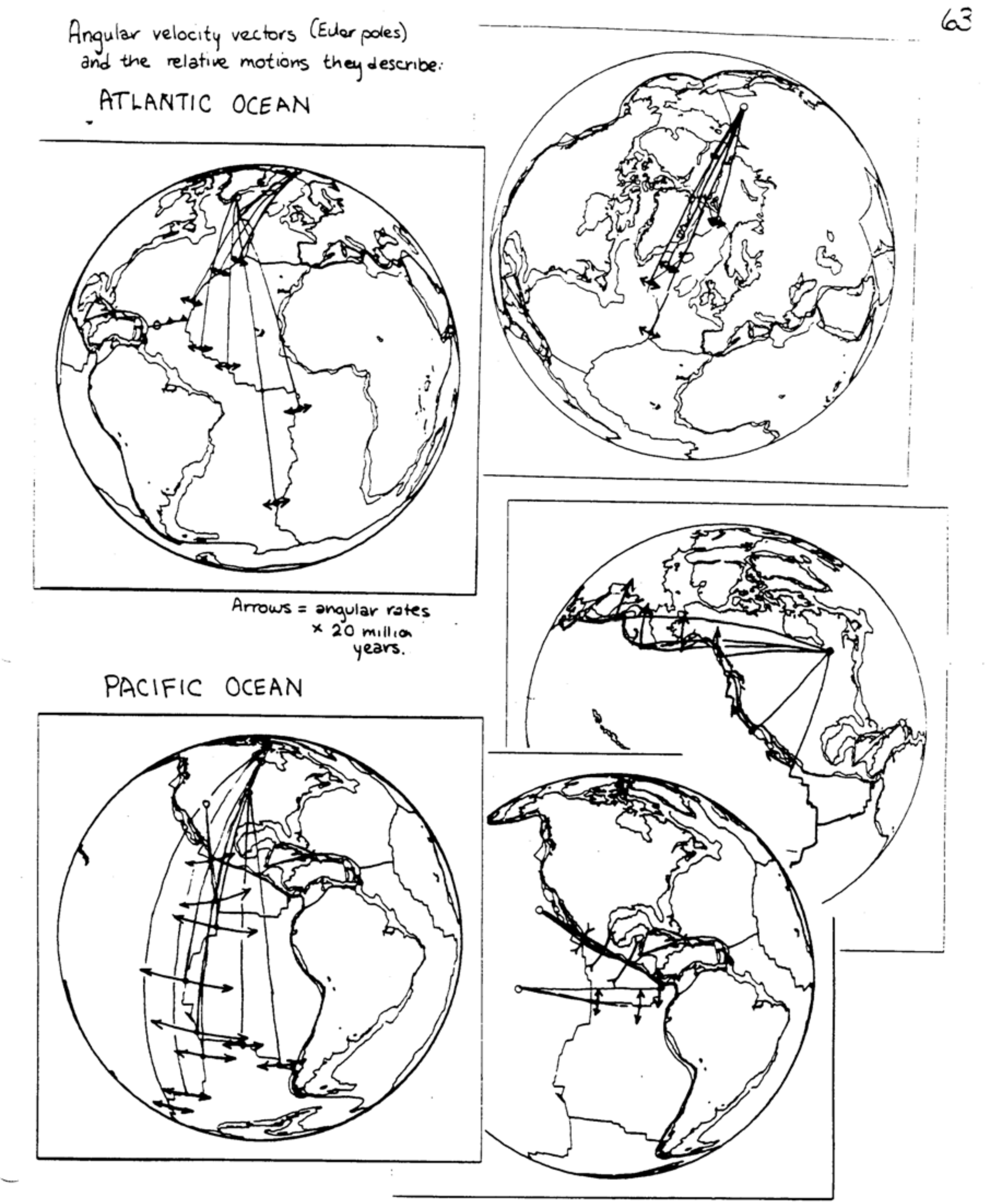
| Plate Pair | Latitude °N | Longitude °E | $\omega$ (deg–m.y.$^{-1}$) | Error Ellipse $\sigma_{max}$ | $\sigma_{min}$ | $\zeta_{max}$ | $\sigma_\omega$ (deg–m.y.$^{-1}$) |
|---|---|---|---|---|---|---|---|
| | | | *Pacific Ocean* | | | | |
| na-pa | 48.7 | –78.2 | 0.78 | 1.3 | 1.2 | –61 | 0.01 |
| co-pa | 36.8 | –108.6 | 2.09 | 1.0 | 0.6 | –33 | 0.05 |
| co-na | 27.9 | –120.7 | 1.42 | 1.8 | 0.7 | –67 | 0.05 |
| co-nz | 4.8 | –124.3 | 0.95 | 2.9 | 1.5 | –88 | 0.05 |
| nz-pa | 55.6 | –90.1 | 1.42 | 1.8 | 0.9 | –1 | 0.02 |
| nz-an | 40.5 | –95.9 | 0.54 | 4.5 | 1.9 | –9 | 0.02 |
| nz-sa | 56.0 | –94.0 | 0.76 | 3.6 | 1.5 | –10 | 0.02 |
| an-pa | 64.3 | –84.0 | 0.91 | 1.2 | 1.0 | 81 | 0.01 |
| pa-au | –60.1 | –178.3 | 1.12 | 1.0 | 0.9 | –58 | 0.02 |
| eu-pa | 61.1 | –85.8 | 0.90 | 1.3 | 1.1 | 90 | 0.02 |
| co-ca | 24.1 | –119.4 | 1.37 | 2.5 | 1.2 | –60 | 0.06 |
| nz-ca | 56.2 | –104.6 | 0.58 | 6.5 | 2.2 | –31 | 0.04 |
| | | | *Atlantic Ocean* | | | | |
| eu-na | 62.4 | 135.8 | 0.22 | 4.1 | 1.3 | –11 | 0.01 |
| af-na | 78.8 | 38.3 | 0.25 | 3.7 | 1.0 | 77 | 0.01 |
| af-eu | 21.0 | –20.6 | 0.13 | 6.0 | 0.7 | –4 | 0.02 |
| na-sa | 16.3 | –58.1 | 0.15 | 5.9 | 3.7 | –9 | 0.01 |
| af-sa | 62.5 | –39.4 | 0.32 | 2.6 | 0.8 | –11 | 0.01 |
| an-sa | 86.4 | –40.7 | 0.27 | 3.0 | 1.2 | –24 | 0.01 |
| na-ca | –74.3 | –26.1 | 0.11 | 25.5 | 2.6 | –52 | 0.03 |
| ca-sa | 50.0 | –65.3 | 0.19 | 15.1 | 4.3 | –2 | 0.03 |
| | | | *Indian Ocean* | | | | |
| au-an | 13.2 | 38.2 | 0.68 | 1.3 | 1.0 | –63 | 0.00 |
| af-an | 5.6 | –39.2 | 0.13 | 4.4 | 1.3 | –42 | 0.01 |
| au-af | 12.4 | 49.8 | 0.66 | 1.2 | 0.9 | –39 | 0.01 |
| au-in | –5.6 | 77.1 | 0.31 | 7.4 | 3.1 | –43 | 0.07 |
| in-af | 23.6 | 28.5 | 0.43 | 8.8 | 1.5 | –74 | 0.06 |
| ar-af | 24.1 | 24.0 | 0.42 | 4.9 | 1.3 | –65 | 0.05 |
| in-eu | 24.4 | 17.7 | 0.53 | 8.8 | 1.8 | –79 | 0.06 |
| ar-eu | 24.6 | 13.7 | 0.52 | 5.2 | 1.7 | –72 | 0.05 |
| au-eu | 15.1 | 40.5 | 0.72 | 2.1 | 1.1 | –45 | 0.01 |
| in-ar | 3.0 | 91.5 | 0.03 | 26.1 | 2.4 | –58 | 0.04 |

The first plate moves counterclockwise relative to the second plate. Plate abbreviations: pa, Pacific; na, North America; sa, South America; af, Africa; co, Cocos; nz, Nazca; eu, Eurasia; an, Antarctica; ar, Arabia; in, India; au, Australia; ca, Caribbean. See Figure 3 for plate geometries. One sigma-error ellipses are specified by the angular lengths of the principal axes and by the azimuths ($\zeta_{max}$, given in degrees clockwise from north) of the major axis. The rotation rate uncertainty is determined from a one-dimensional marginal distribution, whereas the lengths of the principal axes are determined from a two-dimensional marginal distribution.



De Mets et al. (1990)

Classic “highly cited” paper; everybody compares their local fault zone to this global model



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Angular velocity vectors (Euler poles) and the relative motions they describe:

ATLANTIC OCEAN

Arrows = angular rates × 20 million years.

PACIFIC OCEAN

Boundary between NoAmer and EurAsia runs across Arctic ocean, into Siberia, and beneath Euler pole

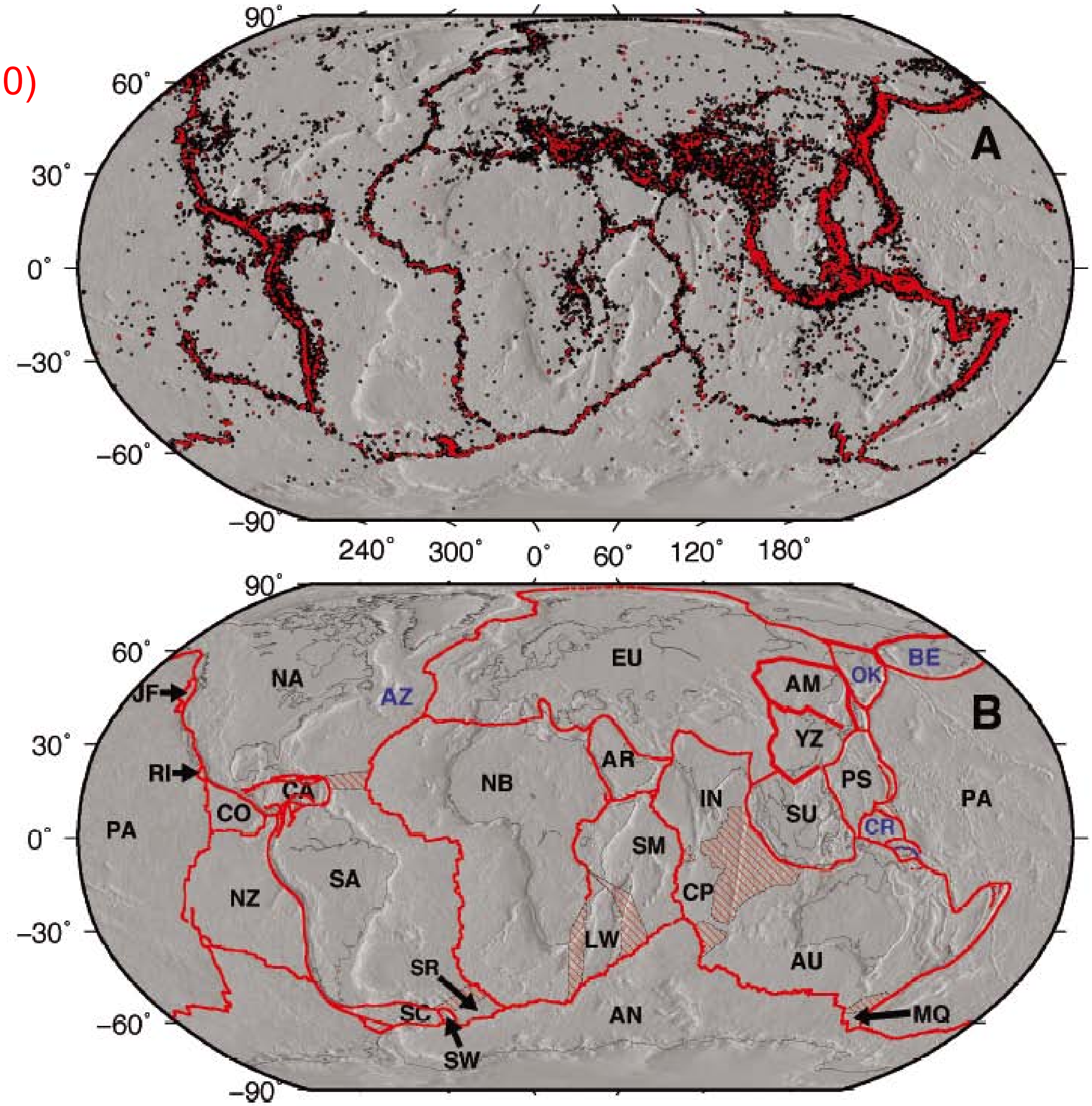
Arrows = angular rates x 20 Million years

# MORVEL

DeMets et al. (2010)

25 plates

Blue plates not included

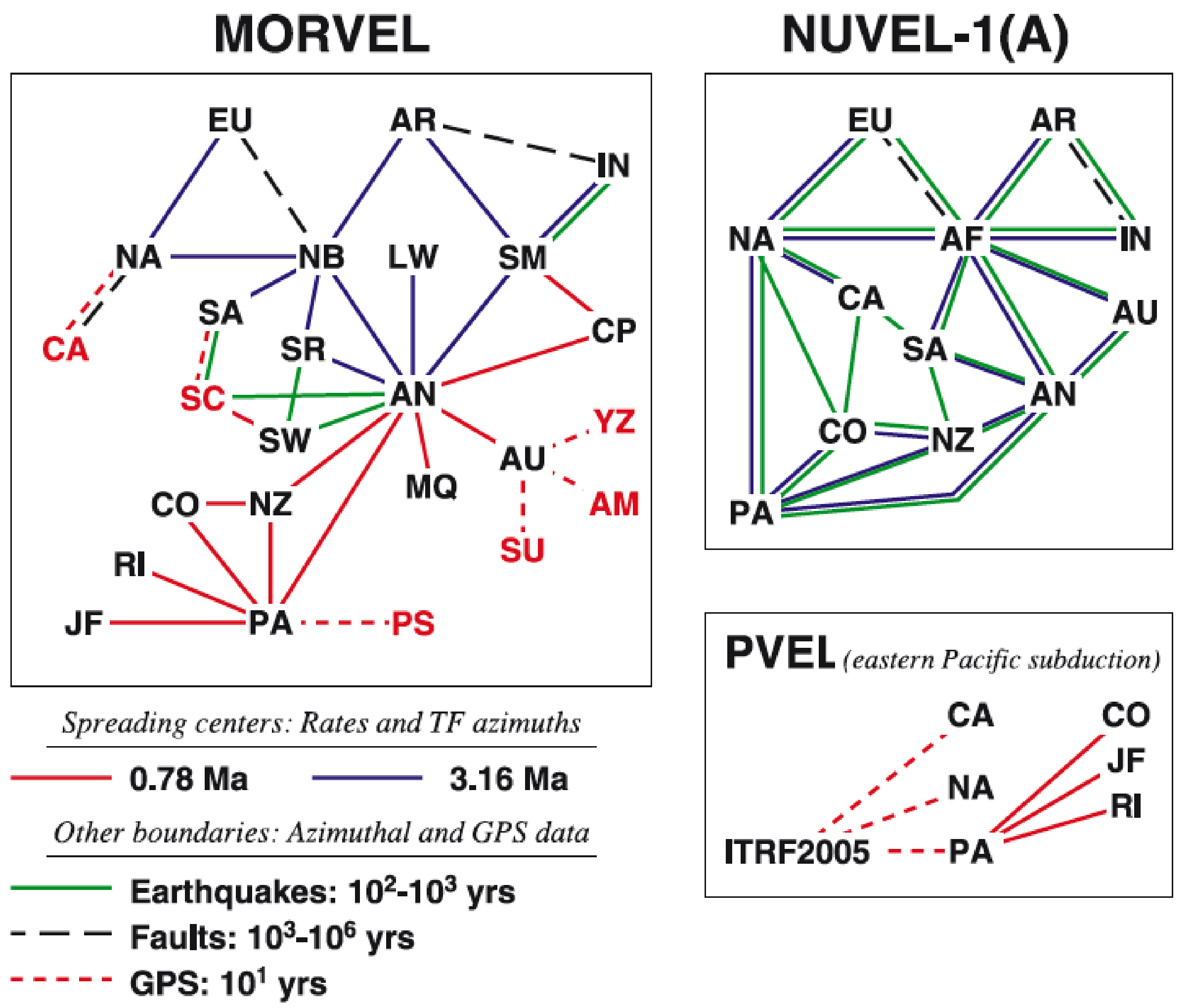


Note: several diffuse plate boundaries

earthquakes define zones of intraplate crustal deformation

Ex.: Africa = Nubia + Somalia + Lwandle plates

**Figure 1.** (a) Epicentres for earthquakes with magnitudes equal to or larger than 3.5 (black) and 5.5 (red) and depths shallower than 40 km for the period 1967–2007. Hypocentral information is from the U.S. Geological Survey National Earthquake Information Center files. (b) Plate boundaries and geometries employed for MORVEL. Plate name abbreviations are as follows: AM, Amur; AN, Antarctic; AR, Arabia; AU, Australia; AZ, Azores; BE, Bering; CA, Caribbean; CO, Cocos; CP, Capricorn; CR, Caroline; EU, Eurasia; IN, India; JF, Juan de Fuca; LW, Lwandle; MQ, Macquarie; NA, North America; NB, Nubia; NZ, Nazca; OK, Okhotsk; PA, Pacific; PS, Philippine Sea; RI, Rivera; SA, South America; SC, Scotia; SM, Somalia; SR, Sur; SU, Sundaland; SW, Sandwich; YZ, Yangtze. Blue labels indicate plates not included in MORVEL. Patterned red areas show diffuse plate boundaries.



Updated version:

MORVEL

DeMets et al. (2010)

25 plates

Instantaneous = .78 Ma
on intermediate and fast
spreading ridges

But still use Anom 2A
(3 Ma) on slow ridges

# Finite Rotation Poles (or Euler Poles)

Measure relative plate displacements

Euler Pole: Latitude, Longitude, Ω

or $E = (E_x, E_y, E_z)$ (Cartesian Coordinates)
$\Omega$ = Angle

Use matrix multiplication to rotate a point

if A is a point prior to rotation
and A' is the point after rotation

then A' = R A where R is a 3 x 3 "rotation" matrix

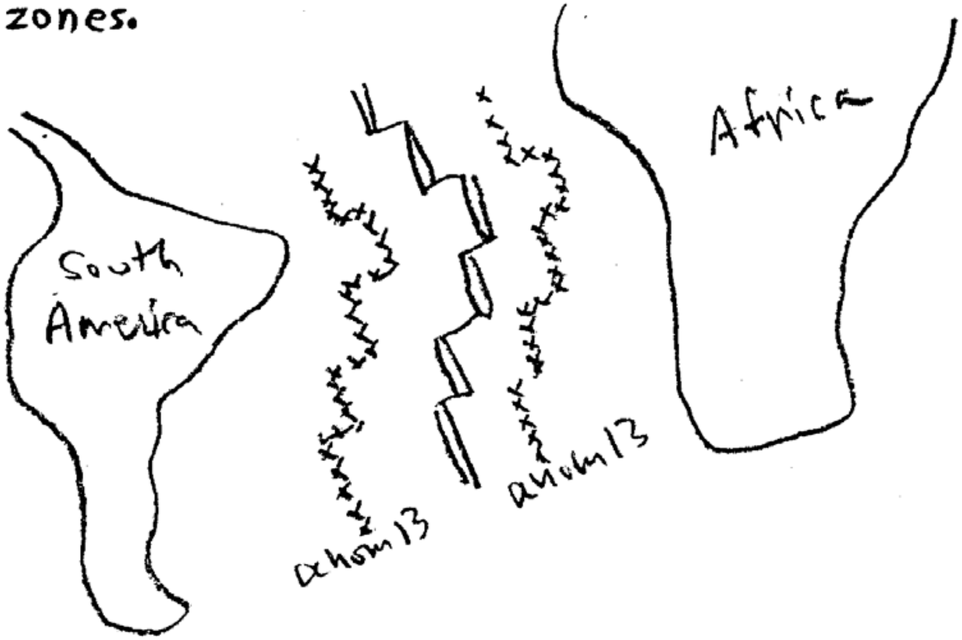
$$\begin{bmatrix} A'_x \\ A'_y \\ A'_z \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$R_{11} = E_x E_x (1 - \cos\Omega) + \cos\Omega$$
$$R_{12} = E_x E_y (1 - \cos\Omega) - E_z \cos\Omega$$
$$\vdots$$
$$R_{33} = E_z E_z (1 - \cos\Omega) + \cos\Omega$$

See Cox and Hart Box 7.3

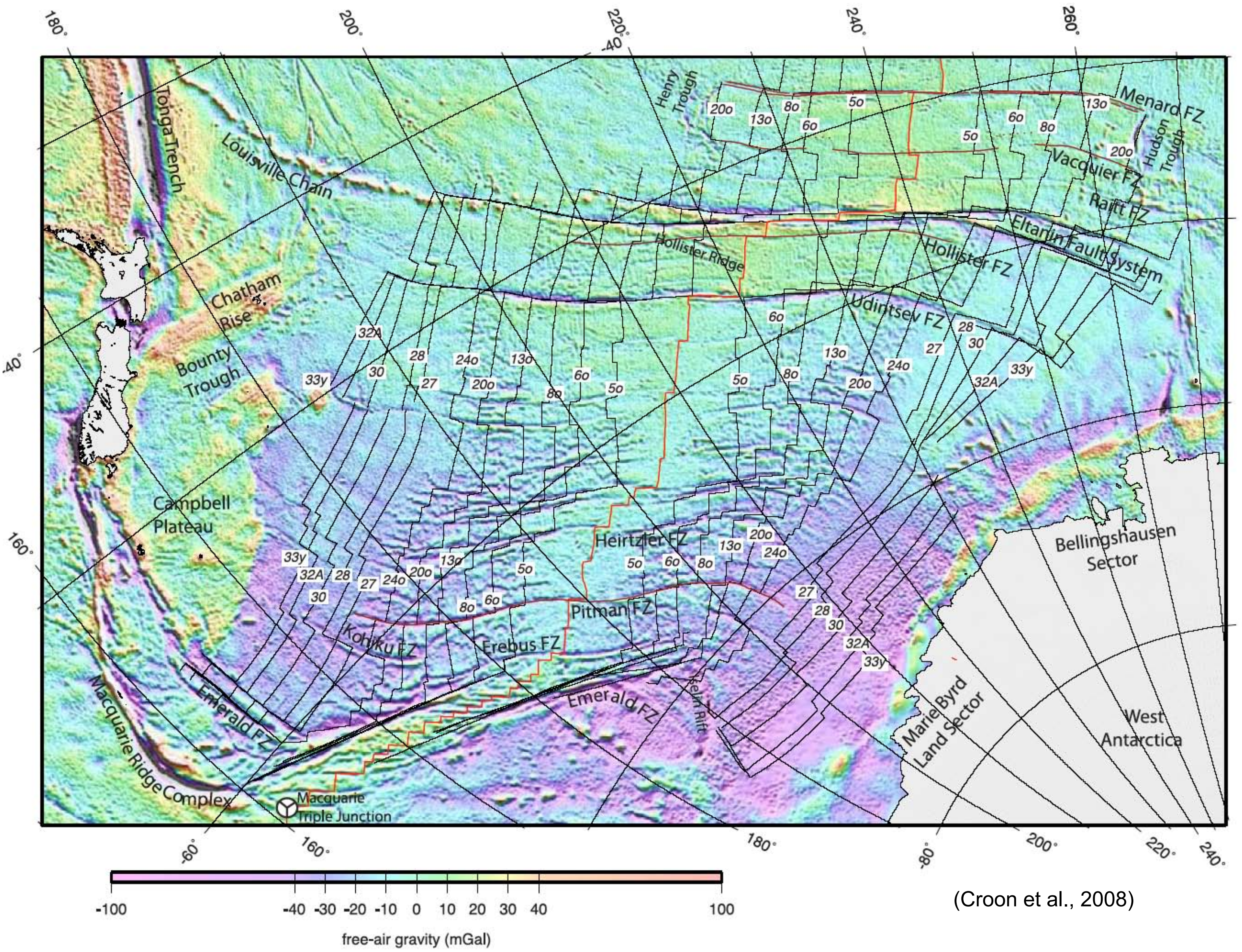
# How to determine a Finite Rotation Pole

Practically, we determine finite rotation poles by the trial-and-error fitting of magnetic anomalies (isochrons) and segments of fracture zones.



This used to be done "by eye." Now there are several different search programs that use different "best-fitting" algorithms and generate uncertainty ellipses.

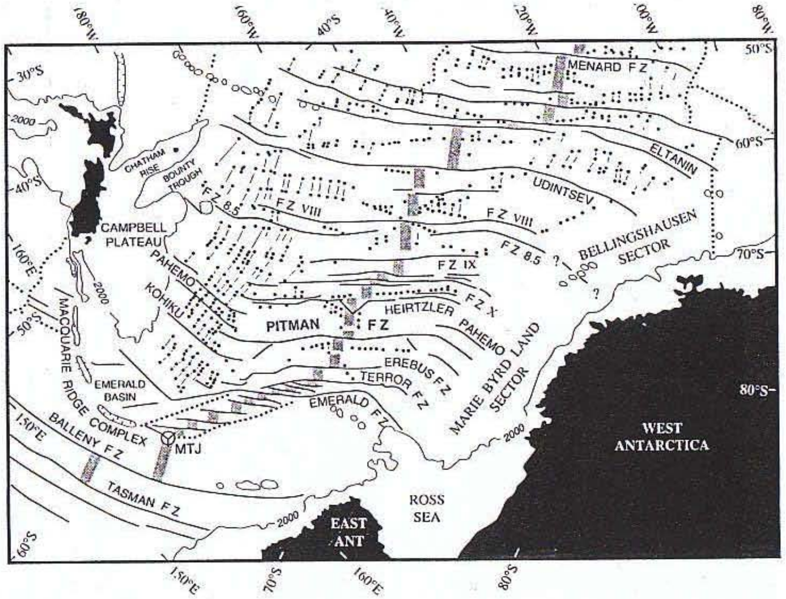
Euler poles that rotate a plate from its present position to some past position are also referred to as "total rotation poles" or "reconstruction poles."



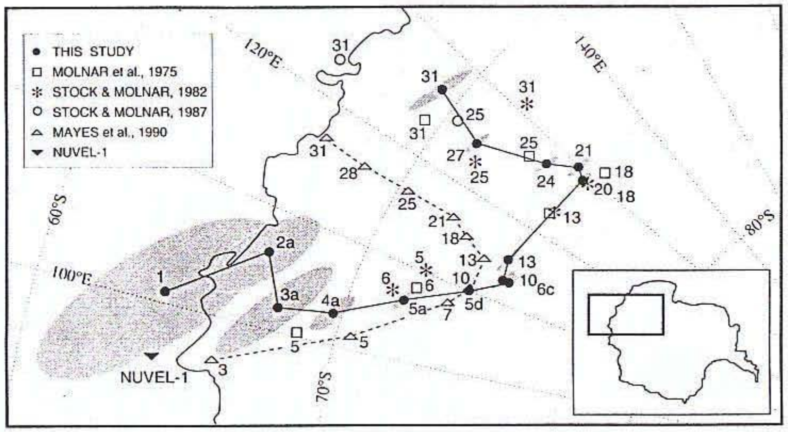
(Croon et al., 2008)

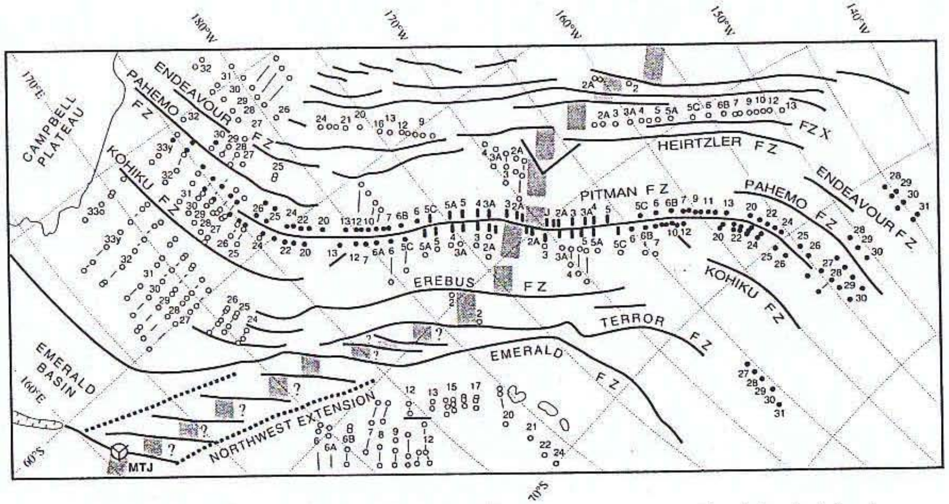
# Example: Pacific-Antarctic Ridge

Magnetic anomaly and fracture zone data shown on an Oblique Mercator projection using Euler pole for anomaly 3A



Finite rotation poles for Pacific-Antarctic plates; gray ellipses show 95% confidence zone





Zoom on data from Pitman Fracture Zone; new magnetic data in black

**Table 1.** Finite rotations of the Pacific relative to Antarctica plates. Counterclockwise rotations are positive. Ages are from (52). An., anomaly.

| Age (Ma)? | An. | Lat. (°N) | Long. (°E) | Angle |
|---|---|---|---|---|
| 0.78 | 1 | 64.25 | −79.06 | 0.68 |
| 2.58 | 2a | 67.03 | −73.72 | 2.42 |
| 5.89 | 3a | 67.91 | −77.93 | 5.42 |
| 8.86 | 4a | 69.68 | −77.06 | 7.95 |
| 12.29 | 5a | 71.75 | −73.77 | 10.92 |
| 17.47 | 5d | 73.68 | −69.85 | 15.17 |
| 24.06 | 6c | 74.72 | −67.28 | 19.55 |
| 28.28 | 10 | 74.55 | −67.38 | 22.95 |
| 33.54 | 13 | 74.38 | −64.74 | 27.34 |
| 42.54 | 20 | 74.90 | −51.31 | 34.54 |
| 47.91 | 21 | 74.52 | −50.19 | 37.64 |
| 53.35 | 24 | 73.62 | −52.50 | 40.03 |
| 61.10 | 27 | 71.38 | −55.57 | 44.90 |
| 67.67 | 31 | 69.33 | −53.44 | 51.05 |

# Addition of Finite Rotation Poles

Consider the plate circuit:

$$\underset{\text{Fixed}}{{}_{EU}}ROT_{AF} = \underset{\text{Fixed}}{{}_{NA}}ROT_{AF} + \underset{\text{Fixed}}{{}_{EU}}ROT_{NA}$$

Use matrix multiplication to sum two or more rotations

If $A' = R\ A$ (1st rotation)

And $A'' = R'\ A'$ (2nd rotation)

Then $A'' = T A$ where $T = R'\ R$

$$T = \begin{bmatrix} T_{11} & \cdots & T_{13} \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & T_{33} \end{bmatrix} = \begin{bmatrix} R'_{11} & \cdots & R'_{13} \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & R'_{33} \end{bmatrix} \begin{bmatrix} R_{11} & \cdots & R_{13} \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & R_{33} \end{bmatrix}$$

where $T_{11} = R'_{11}R_{11} + R'_{12}R_{21} + R'_{13}R_{31}$ etc.

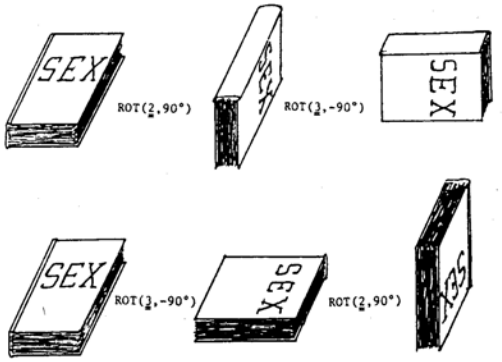
or $T_{ij} = \sum_{k} R'_{ik} R_{kj}$

See Cox and Hart Box 7.5

# Adding finite rotations:

Finite rotations can be added but, unlike instantaneous poles, the addition is not commutative.

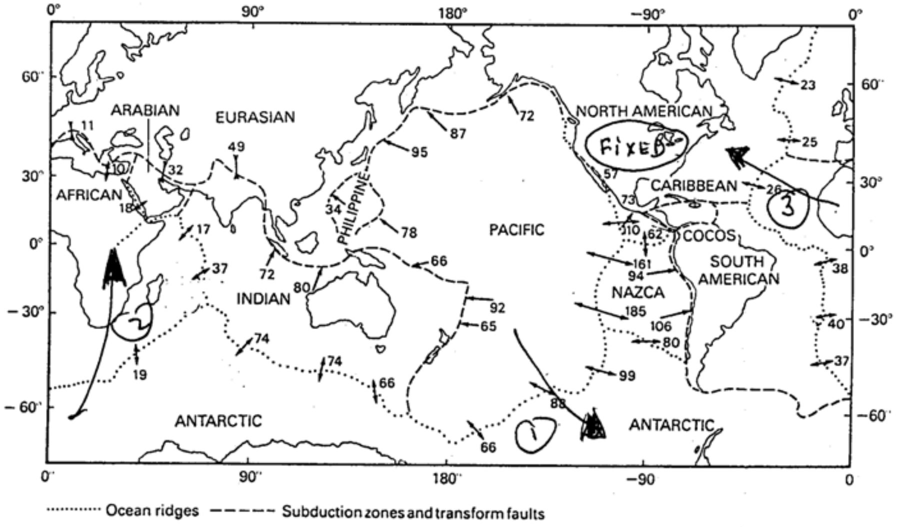
$$\mathrm{ROT}_A + \mathrm{ROT}_B \neq \mathrm{ROT}_B + \mathrm{ROT}_A$$



COX & HART (1986)

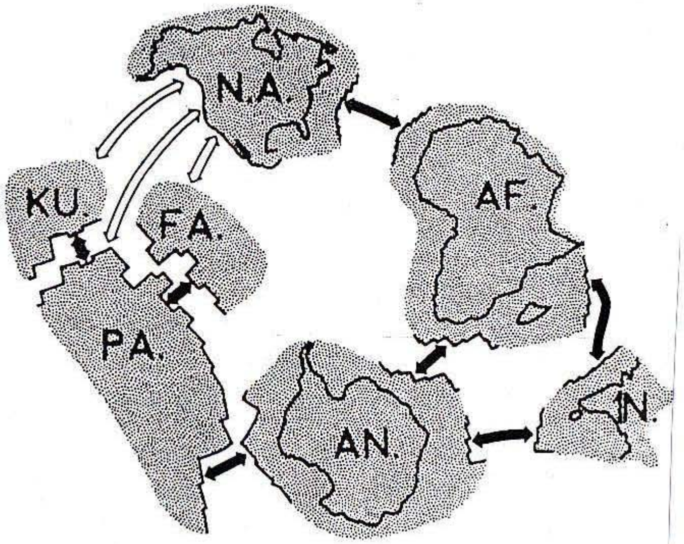
When summing poles around a plate circuit, you have to define a "fixed" plate and sum them in the right "direction." (Towards the fixed plate).

$${}_{NAM}\mathrm{ROT}_{PAC} = {}_{ANT}\mathrm{ROT}_{PAC} + {}_{AFR}\mathrm{ROT}_{ANT} + {}_{NAM}\mathrm{ROT}_{AFR}$$
(fixed)



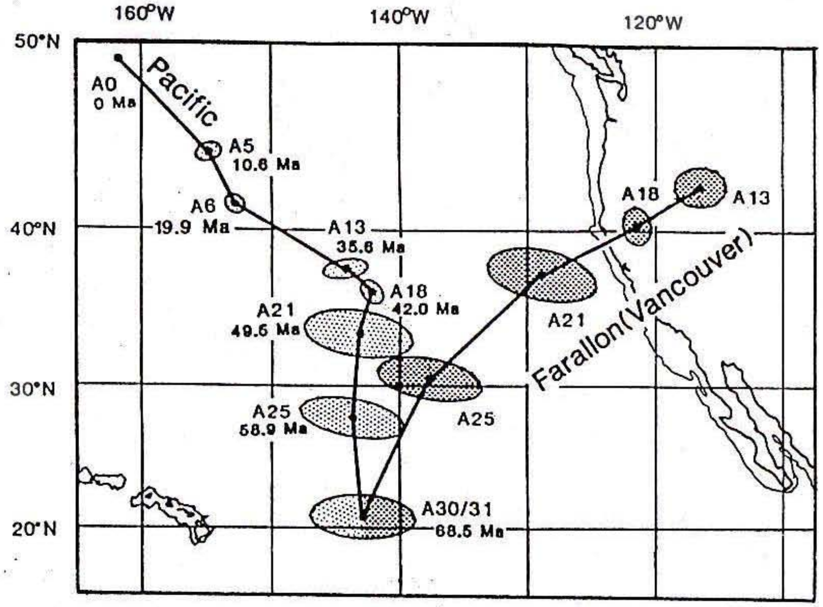
# “The global plate circuit”

Plate Circuit Reconstructions



Atwater (1989)

Circuit used to determine motion of Pacific and Farallon plates relative to North America



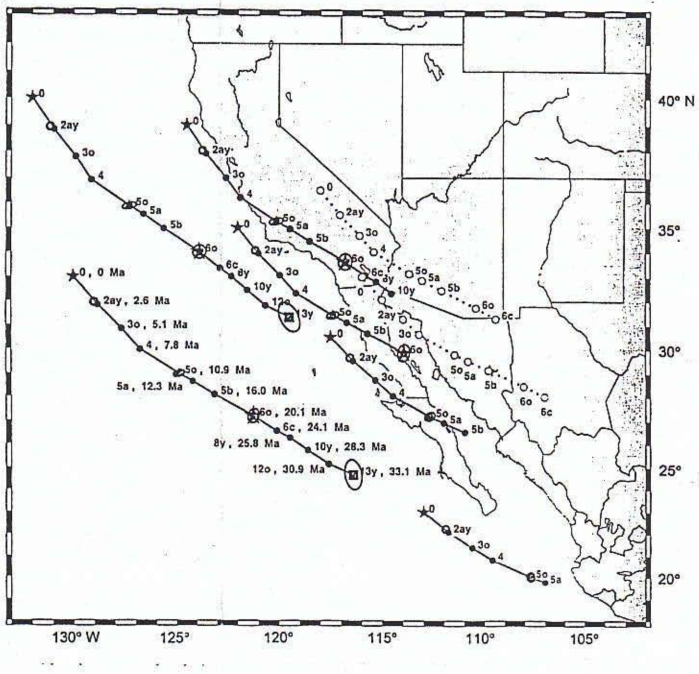
Motion of an arbitrary point relative to North America since anomaly 30 assuming it moved with the Pacific plate (light ellipses) or Farallon plate (dark ellipses).

Power of global plate circuit:

Calculate Pac-Nam motion back to 20 Ma using global circuit

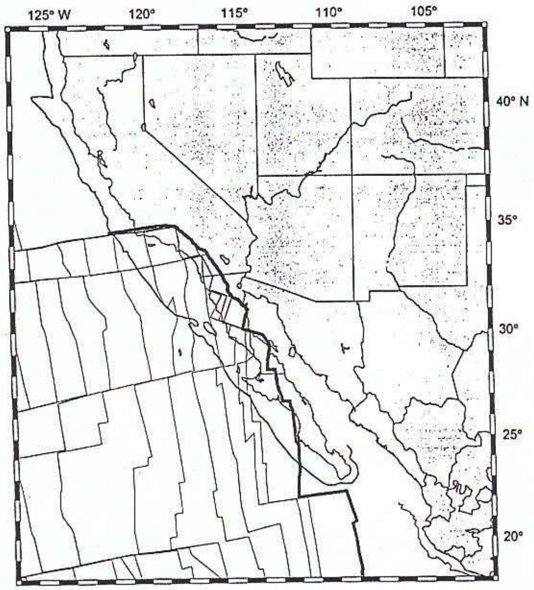
Find overlap of “reconstructed” oceanic crust onto continental Southern California

Compare to Atwater 1970



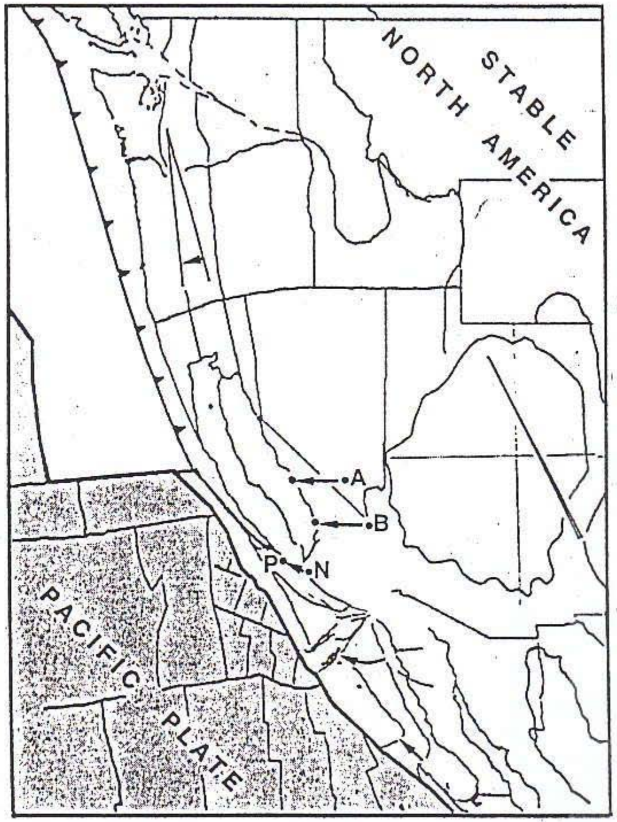
Motion of several points on the Pacific plate relative to North America. Note that prior to anomaly 4 the motion was oblique to the margin.

Reconstruction of Pacific ocean crust relative to North America at anomaly 6 (20 Ma).



Atwater and Stock (1998)

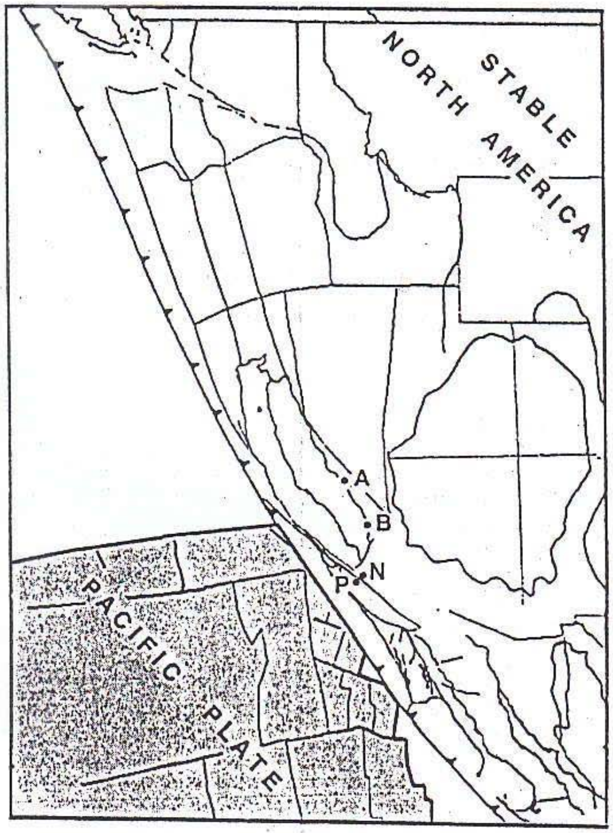
Push (collapse) North America back to east to make room for oceanic crust



Chron 5o, 11 Ma

Atwater and Stock (1998)

Reconstructions of North America taking into account the translation and rotations of various pieces. Note, for example, the 90° cw rotation of the western Transverse Ranges since chron 6 (20 Ma) and the opening of Baja since chron 3A (6 Ma).



Chron 6o, 20 Ma

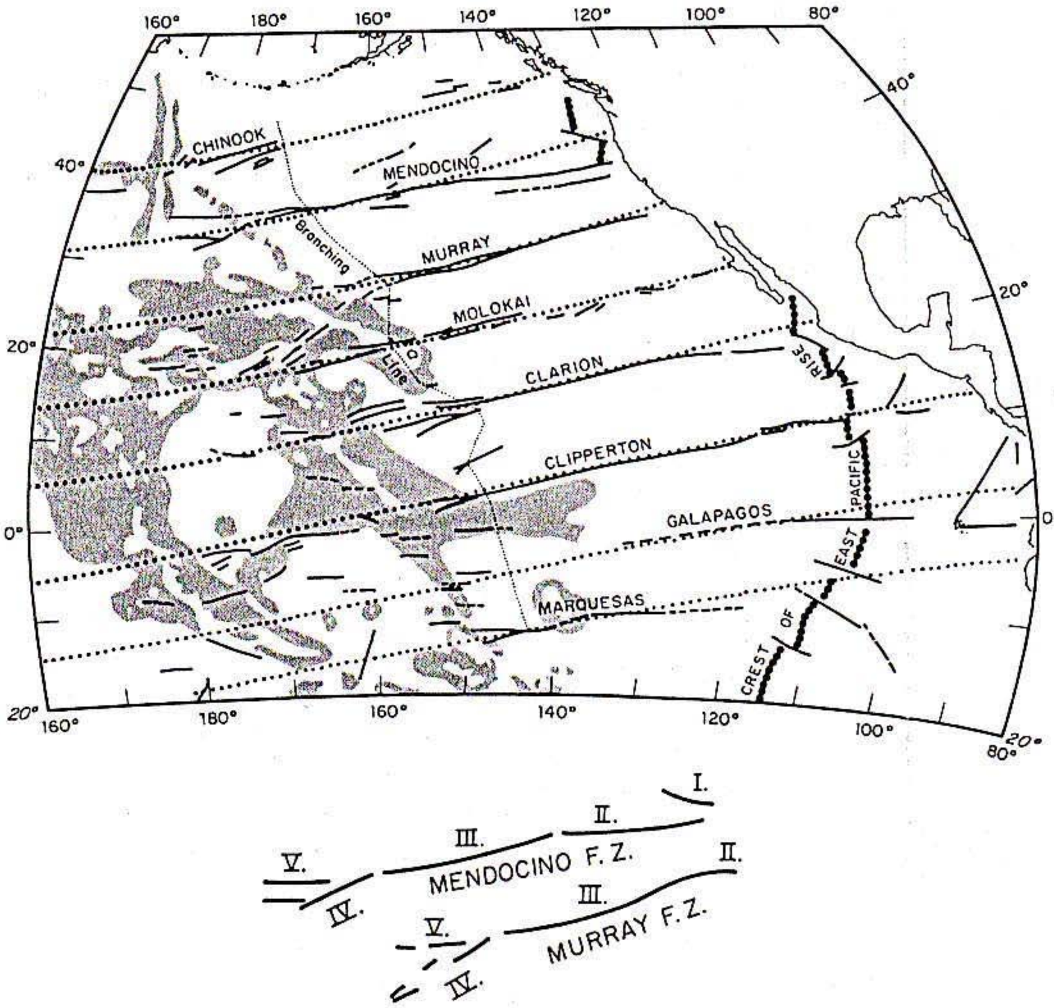
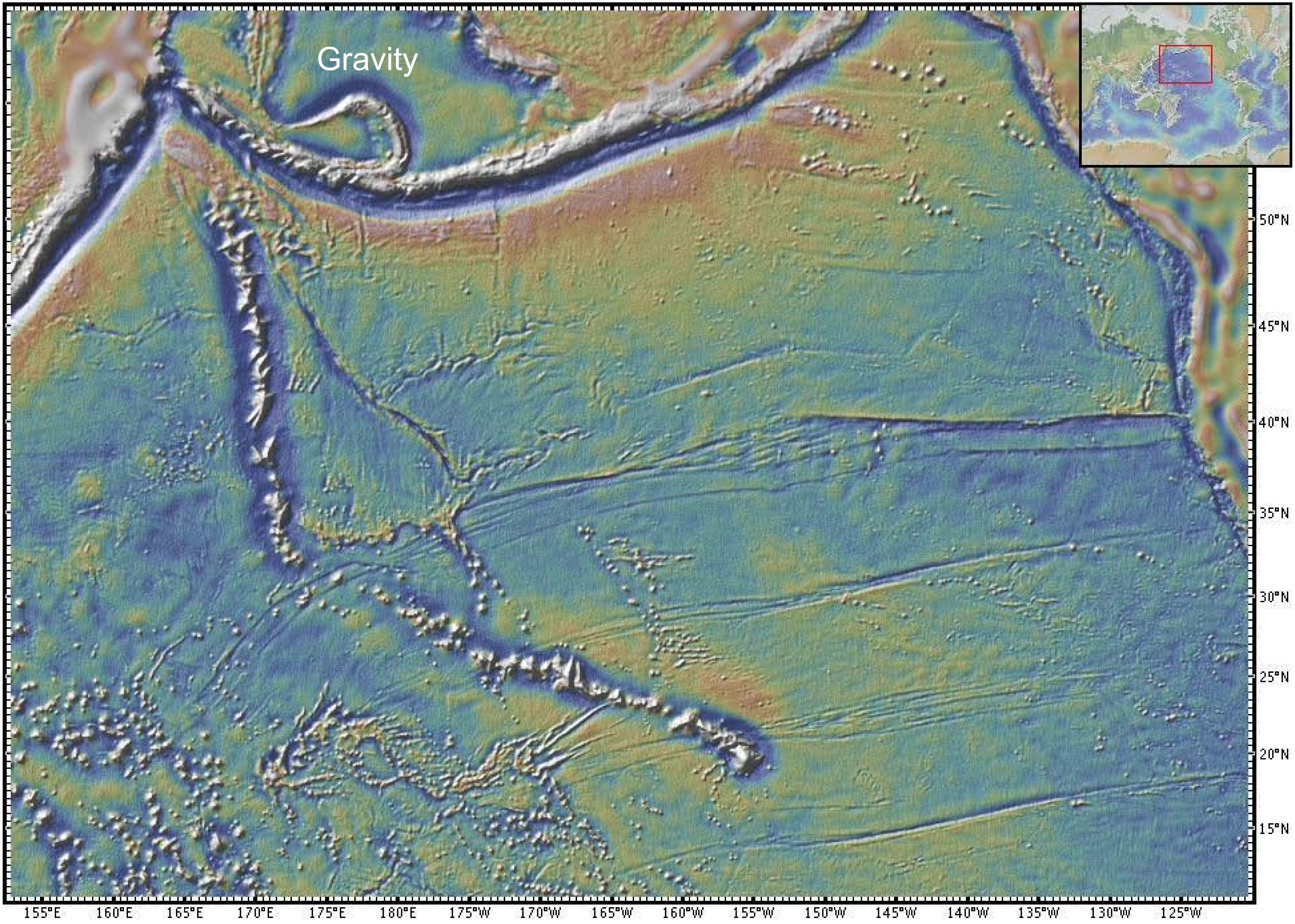
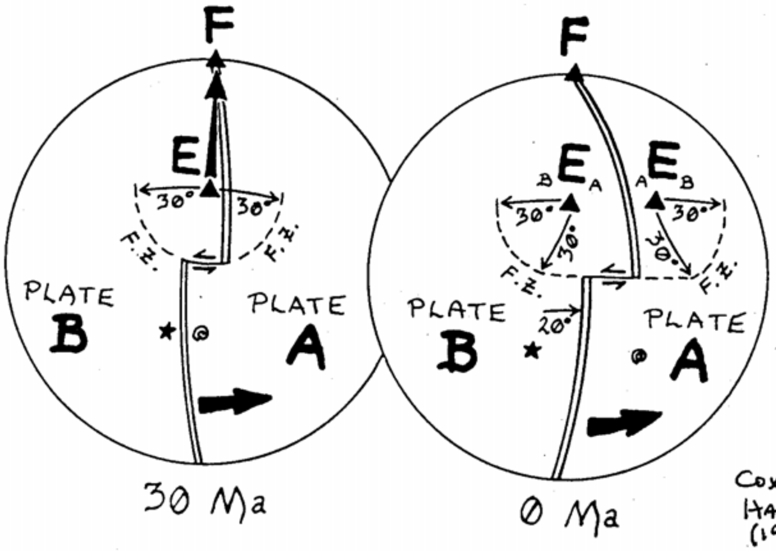


**Figure 33-2**
Fracture zones in the north-eastern Pacific showing trends corresponding to five possible spreading episodes. Dotted lines are small circles about the pole at 79°N., 111°E. suggested by Morgan (1968b). It is the pole of rotation for episode III


Gravity
50°N
45°N
40°N
35°N
30°N
25°N
20°N
15°N
155°E
160°E
165°E
170°E
175°E
180°E
175°W
170°W
165°W
160°W
155°W
150°W
145°W
140°W
135°W
130°W
125°W

# Changes in Plate Motion



Cox & Hart (1986)

Example:

Before 30 Ma, plates A and B rotated about pole E.

At 30 Ma, pole jumped to F, where it has stayed.

At 0 Ma, (after 30 Ma of opening about pole F), the position of E is not the same for plates A and B

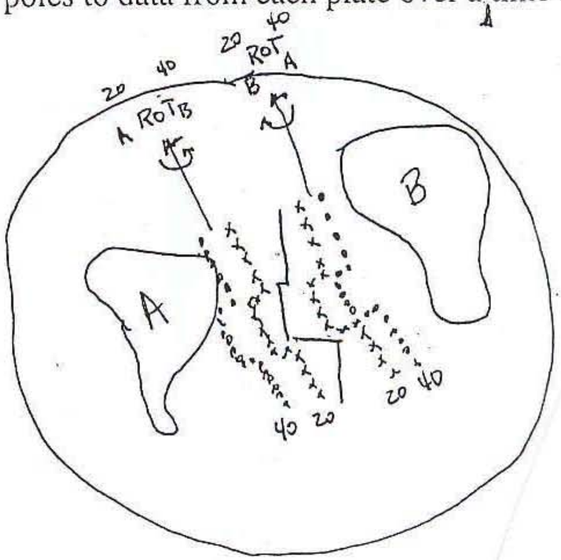
that is: $$\underset{\text{fixed}}{{}_{B}E_{A}} \neq \underset{\text{fixed}}{{}_{A}E_{B}}$$

These intermediate Euler poles are called <u>stage poles</u>

Stage poles best match actual plate motions (e.g.fracture zone trends) over a short time interval and are at the heart of tectonic studies

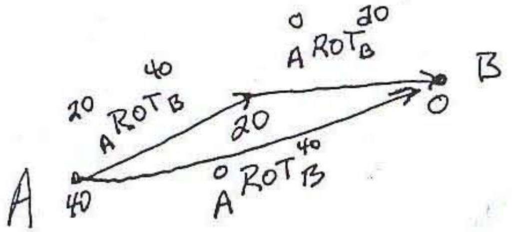
# STAGE POLES

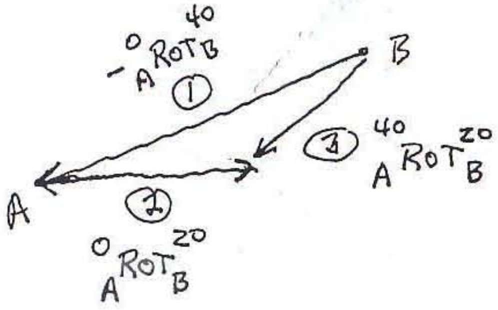
1) Can fit Euler poles to data from each plate over a specified time interval



However, this is not a very accurate method.

2) Or, can subtract (total) finite rotation poles (much better method)

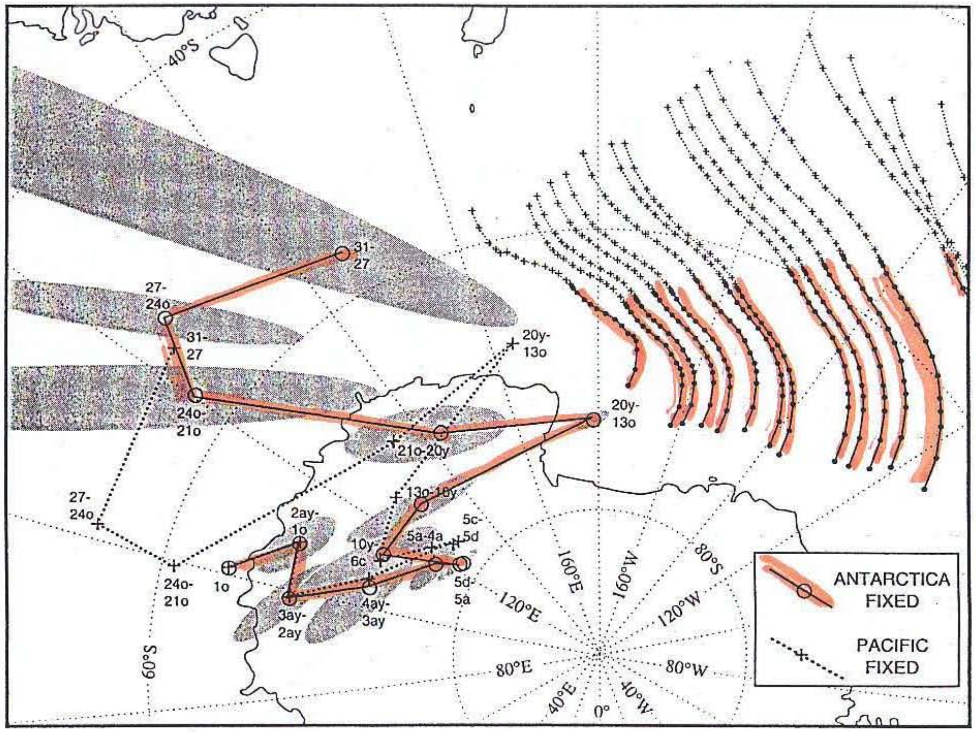




① + ② = ③

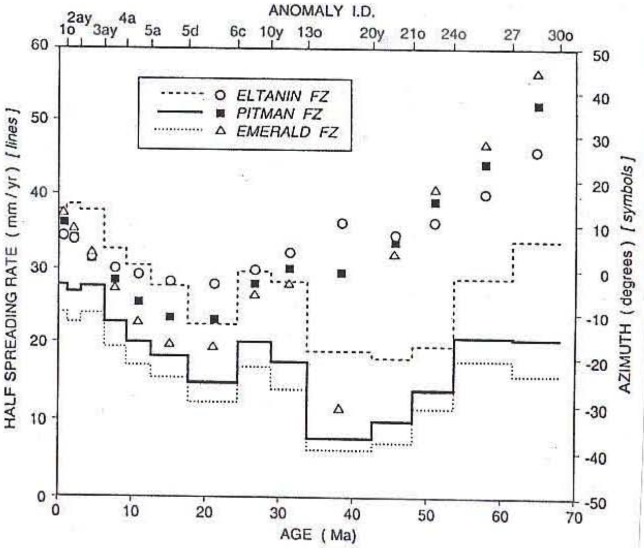
① and ② are total rotation poles
③ is the stage pole

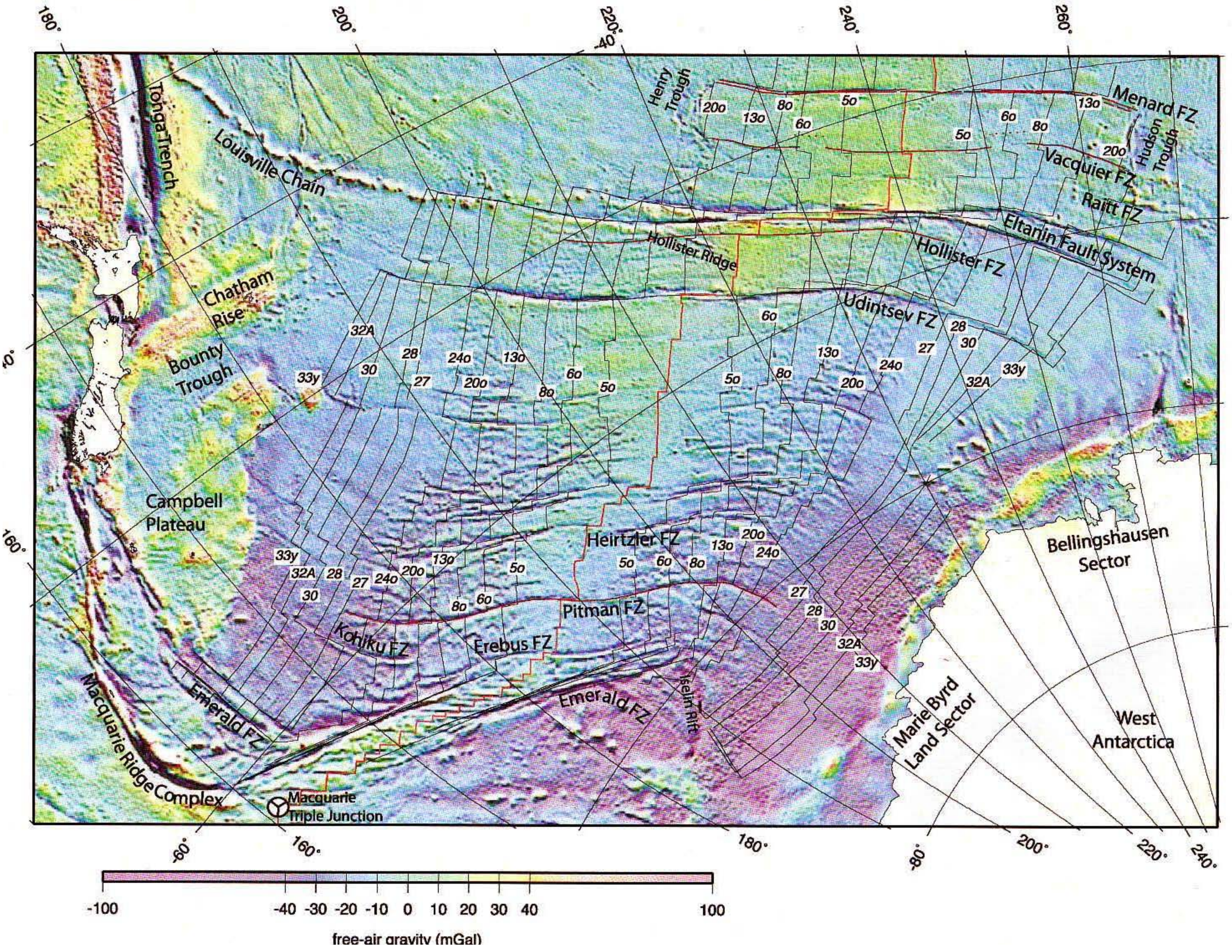
# There are two sets of stage poles: one for each plate



There are two sets of stage poles, one relative to Pacific plate, the other relative to the Antarctic plate

Stage poles are used to reconstruct fracture zones and to calculate spreading rates (lines) and azimuth of spreading (symbols) for discrete time intervals





180°
200°
220°
240°
260°
-40°
-60°
-80°
160°
180°
200°
220°
240°
Tonga Trench
Louisville Chain
Henry Trough
Menard FZ
Hudson Trough
Vacquier FZ
Raitt FZ
Eltanin Fault System
Hollister Ridge
Hollister FZ
Udintsev FZ
Chatham Rise
Bounty Trough
Campbell Plateau
Heirtzler FZ
Pitman FZ
Kohiku FZ
Erebus FZ
Emerald FZ
Emerald FZ
Iselin Rift
Macquarie Ridge Complex
Macquarie Triple Junction
Bellingshausen Sector
Marie Byrd Land Sector
West Antarctica
20o
13o
8o
6o
5o
24o
27
28
30
32A
33y
-100
-40
-30
-20
-10
0
10
20
30
40
100
free-air gravity (mGal)