Heat-Flow Determinations in the Northwestern United States

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Eleven new heat-flow determinations in the northwestern United States, based on data from twenty-one drill holes and two mine shafts, together with previously available data, suggest a heat-flow pattern similar to that observed in the southwestern United States. In particular the high heat flow found in the Basin and Range province, about 2.0 μ cal/cm³ sec, is characteristic of the Northern Rocky Mountains province and possibly the Columbia Plateaus as well. This region of high heat flow is referred to in this paper as the Cordilleran thermal anomaly zone. Most of central Wyoming, central Montana, and western Washington may have normal heat flow. A heat-flow determination in the Black Hills is high. The Northern Rocky Mountains and Basin and Range provinces have similar regional heat flow, crustal structure, upper mantle P_n velocity, and Cenozoic geologic history. Thus, tectonic schemes that emphasize the Basin and Range province as a unique area on the North American continent must be reevaluated.

INTRODUCTION

The geophysical characteristics of the crust and upper mantle in the western United States have been the subject of active investigation in the past few years [Lee and Uyeda, 1965; Roy et al., 1968b; Sass et al., 1968; Henyey, 1968]. The heat flow is normal in the Great Plains province and high (>1.6 μ cal/cm² sec) in the Southern Rocky Mountains. The heat flow seems to be normal in at least part of the Colorado Plateaus province but is uniformly high (the average is approximately 2 μ cal/cm² sec) in the Basin and Range province. The heat flow in the Sierra Nevada Mountains and the Southern California batholith is normal to low. The Franciscan belt east of the San Andreas fault appears to have a high heat flow while the crystalline belt west of the San Andreas has a normal heat flow. The object of this study is to investigate the heat-flow pattern in the northwestern United States.

The locations of the new heat-flow determinations are shown in Figure 1. Precambrian metamorphic and igneous rocks form the basement in the Dakotas, Wyoming, and eastern and central Montana. One heat-flow value was obtained near the edge of the Beartooth Plateau, part of the

1958; Goldich et al., 1966]. The site is only about 70 km from the thermal areas in Yellowstone National Park. Two determinations, one on either side of the outlier of old basement, were made in younger Precambrian metamorphic rocks, in the Black Hills of South Dakota, and in the Little Belt Mountains in Montana. The rocks of the Black Hills and Little Belt Mountains were last metamorphosed about 1700 and 1900 m.y. ago, respectively [Goldich et al., 1966; Catanzaro and Kulp, 1964]. A determination is also reported for a site in the Absaroka Mountains of northwestern Wyoming. The Absaroka Mountains are composed of early Cenozoic volcanic and volcanic detrital units and are atypical of the Middle Rocky Mountains. The bedrock in most of northwestern Montana and northern Idaho is late Precambrian sedimentary rocks of the Belt Series; four heat-flow determinations are reported from this terrain. A value in northeastern Washington is in Paleozoic sediments that are a part of a highly deformed belt of sedimentary rocks called the Kootenay Arc [Yates et al., 1966]. The basement in central Idaho and northern Washington is composed of Mesozoic intrusives and metamorphic rocks. The only determination in these rocks is in the Colville batholith in northern Washington. The age of the Colville batholith is about 100 m.y. [Becraft and Weiss, 1963, p. 32].

large outlier of very old Precambrian basement (K-Ar ages > 2.5 b.y.) in Wyoming [Gast et al.,

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Fig. 1. Large outcrops of basement rocks in the northwestern United States. Solid pattern, Precambrian metamorphic rocks; line pattern, Precambrian sedimentary rocks; dotted pattern, Mesozoic and Cenozoic intrusive rocks.

In addition, a determination in the Late Cretaceous Boulder batholith of western Montana (Blackwell and Robertson, in preparation) will be mentioned. A single determination was made in the northern Cascades in the Snoqualmie batholith. According to K-Ar dating, the batholith was intruded only 17 m.y. ago [Lipson et al., 1961], a date which is in agreement with stratigraphic evidence. Several estimated heat flow values in the Wyoming basin, the Columbia Plateaus in Oregon, and along the Washington coast furnish evidence on the heat flow in areas for which no precise data are available.

DATA

Eleven heat-flow determinations based on measurements from 21 drill holes and 2 mine shafts are reported. Because of the large amount of data, only the briefest summary of the details pertinent to each heat-flow determination is possible. The data and a more comprehensive discussion of each determination may be found in the author's thesis [*Blackwell*, 1967].

The mechanical details of the data acquisition

and reduction are summarized by Roy et al. [1968b]. All the heat-flow determinations discussed were made by using holes drilled for the purpose of mineral exploration. The heat-flow determinations are listed in Table 1 and in Roy et al. [1968b]. All of the heat-flow values, except as noted, were calculated by the resistance integral method. The gradients are least-squares straight lines fitted to the temperature-depth data in the given interval. The conductivity values listed are mean harmonic averages. The errors shown beneath the heatflow values are statistical and relate only to the internal consistency of the data. The error limits shown for the best value for each area are discussed more fully in the section pertaining to that particular heat-flow determination. Temperature-depth curves for most of the drill holes are shown in Figures 2–5. For clarity only points at 20-meter intervals are plotted although the heat flow values were calculated using measurements made at 10-meter intervals.

Several of the heat flow determinations presented have rather large topographic corrections.

The procedure suggested by Jeffreys [1938] and modified by Birch [1950] gives satisfactory results if corrections are no greater than 10-15%. The method is much more accurate for drill holes in valleys than on hills, but in both situations the tendency is to overcorrect. Additional error for large corrections may arise owing to departures from the assumed uniform rate of variation of surface temperature with elevation or lateral variations in thermal conductivity. The details are reserved for a later publication, but in two areas the correction calculated in the normal manner was modified to allow for the inaccuracies of the plane approximation. The resulting error of the heat-flow determinations from this source should be no more than 10-15%.

Because the northwestern United States is an area of rugged topography, the effect of topographic evolution was considered for all of the drill holes. For all but two areas the maximum reasonable corrections are not significant (<5%) and so are neglected.

HEAT-FLOW DETERMINATIONS

Lead, South Dakota. Temperature data from the Yates and No. 4 and No. 5 Shafts were supplied by the Homestake Mining Company (see Figure 2). The technique of temperature measurement is described by Noble [1948]. The formations shown in Figure 2 are described by Noble and Harder [1948]. The variation in gradient in the Yates Shaft correlates with conductivity variations between formations, so the change in gradient noted by Noble [1948] is due to a change in thermal conductivity and there is no evidence for an increase in heat flow with depth.

Meeteetse, Wyoming. A temperature-depth curve for DDH-17 is shown in Figure 3 (curve 4). The drill hole is in a tuff pipe intruded into the Wiggins Formation of late Eocene and



Fig. 2. Temperature-depth curves and rock units in Yates (open circles) and No. 4 and No. 5 (solid circles) Shafts, Homestake Mine, Lead, South Dakota.



Fig. 3. Temperature-depth curves: 1: Libby, Montana; 2, 3: Wilbur, Washington; 4: Meeteetse, Wyoming; 5, 6: North Bend, Washington.

Oligocene age [Wilson, 1964]. The thermal conductivity contrast between the tuff and the surrounding volcanics appears to be small so refraction is not important, although the tuff body may be rather small. A time-dependent terrain correction was applied to the determination. The value listed in Table 1 was calculated assuming uplift of 1.8 km and erosion of 0.6 km in the last 2-9 m.y. [see Love, 1939, p. 114].

Cooke City, Montana. The determination is based on data from two drill holes in the Cooke City mining district [Lovering, 1929]. The holes penetrate Cambrian sedimentary rocks and the underlying Precambrian granitic rocks. Only DDH-1 is represented by a temperature-depth curve (Figure 4, curve 1). The change in gradient in DDH-1 is matched by a change in thermal conductivity. The heat flow in both drill holes is undisturbed at a depth of only 50 meters.

Neihart, Montana. The heat flow is based on data from two drill holes in the Precambrian metamorphic core of the Little Belt Mountains, but only DDH-37 is illustrated (Figure 4, curve 2). Several porphyry dikes of about the same conductivity as the metamorphic rocks are cut by the drill holes. Above 170 meters in DDH-37 the temperatures are disturbed by flowing water and are not used in the calculation of the heat flow. Marysville, Montana. Results from three drill holes in contact metamorphosed Belt sediments and Tertiary dikes in the Marysville mining district [Barrell, 1907] give abnormally high heat-flow values. The gradients are abnormal (see Figure 5), not the conductivity, so the high values cannot be attributed to refraction. Because there are no hot springs in the area, the best explanation for the high values seems to be a shallow recent intrusion or an untapped reservior of hot water at depth.

Lincoln, Montana. The two drill holes (DDH-1 and DDH-29, curves 3 and 4, Figure 4) cut flat-lying Belt argillite and quartzite and altered quartz monzonite intrusives. The conductivity for DDH-1 was estimated from samples from above 66 meters and from samples in DDH-29. The uncorrected values differ by 25% but the terrain-corrected values differ by only 5%.

Libby, Montana. Temperatures measured in a drill hole in Belt sediments in extreme northwestern Montana are isothermal above 280 meters due to water circulation. Below that depth the gradient is quite linear (curve 1, Figure 3). The sediments in the vicinity of the drill hole are almost flat-lying, although in general the deformation is moderate to intense [Gibson, 1948]. The terrain correction is large, but because of the depth of the drill hole the probable error is small. The large estimated

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	N. Lat.	W. Long.	Elev., meters	Depth Range, meters	-	No.	$\frac{\Delta T}{\Delta z},$ °C/km	Heat Flow, µcal/cm ² sec	
Locality					K, mcal/cm sec °C			Unc.	Corr.
Idaho									
Crescent Mine	47°30′	116°05′	1341	1538-1604	12.6	21	17.6	2.20	2 22
					0.2		0.2	0.03	
Silver Summit	47°30′	116°02′	1189	1382–1435	11.7	9	18.8	2.23	2.25
Mean					0.3		0.2	0.03	99101
Montana									2.2 ± 0.1
Cooke City 1	45°30′	109°57'	2793	50-300	7.2	30	18.4	1.29	1.25
					0.2		0.2	0.01	
Cooke City 2†			2870	50-150	8.1	13	18.0	1.45	1.37
Meen					0.4		0.4	0.05	
Libby	48°14′	115°55′	1682	280-390	8 9	2A	15 5	1 29	1.3 ± 0.1
				200 000	0.4	20	1.0	0.04	1.75 ± 0.8
Lincoln 1†	47 °02′	112°23′	1597	100-250	8.8		27.5	2.4	2.1
							0.2		
Lincoln 29			1919	170-270	11.2	10	17.0	1.92	2.22
Мевр					0.4		0.2	0.03	
Marysville 2	46°43′	112°21′	2043	100-210	11 7	13	70 4	8.0	2.2 ± 0.1
	-• -•		2010	100 210	0.8	10	2.1	0.2	
Marysville 4			2001	50-270	8.4	13	72.5	6 38	
					0.4		0.5	0.07	
Marysville 6			2043	100-280	9.0	13	72.5	6.68	
Best value					0.6		1.6	0.15	
Neihart 36	46°58′	110°43′	2006	70-150	6 6	10	94 3	0.5 ± 0.2	1 60
				10 200	0.6	-0	0.6	0 02	1.00
Neihart 37			1939	170-280	6.5	12	29.1	1.82	1.72
					0.2		0.2	0.03	
Mean South Dalasta									1.7 ± 0.1
Lead No. 4 Shaft	44°21′	103°45'		1455-2048	7 0	12	93 A	1 99	1 94
		100 10		1100 2010	0.3	10	0.8	0.07	1.04
Yates Shaft			1618	584-1508	10.7	15	19.0	1.98	1.96
					0.8		1.2	0.05	
Mean Washington									1.9 ± 0.1
Leadnoint 1	48°55/	117928/	711	100 040	14.9	10		0 40	0.00
Longount 1	40 00	117 00	711	100-240	14.3	10	23.9	8.43 0.01	3 00
Leadpoint 3			820	290~340	14.4	16	22 0	3.16	2.93
					0.1		0.1	0.02	
Best value ‡									2.0 ± 0.3
North Bend 1	47 '30'	121*22'	838	80-130	9.4	9	16.2	1.52	1.18
North Bend 2			585	80-951	0.5	04	0.3	0.03	1.94
			000	00-201	0.2	44	202	0.08	1.04
Best value§								0.00	1.2 ± 0.2
Wilbur A†	48°04′	118°42′	892	90-170	9.1	2	14.5	1.33	1.77
W/IL D			1000	100 050			0.1		
WIDUR D			1036	130-250	99	11	12.0	1.17	1 75
Wilbur C			963	200-470	91	28	13.5	0.03	1 51
					0.4	20	0 5	0.03	1.51
Weighted mean									1.6 ± 0.2
Wyoming	10.050/	1000			_				
1VICCTCCTSC	43 '52'	109 17'	3010	140-440	6.97	36	30.35	2 14	1.95
Best values					0.05		0.03	0.01	18401
									X.0 = V.1

TABLE 1. Measurements of Harmonic Conductivity, Gradient and Heat Flow*

* Standard error shown immediately below mean values.

† Heat flow calculated from product of mean harmonic conductivity and least-squares gradient.

‡ Heat flow corrected for refraction.

§ Heat flow corrected for effects of topographic evolution.



Fig. 4. Temperature-depth curves: 1: Cooke City, Montana; 2: Neihart, Montana; 3, 4: Lincoln, Montana; 5, 6: Leadpoint, Washington; 7, 8, 9: Coeur d'Alene mining district, Idaho.

error for the determination, however, is due to the combined effects of complex geology, possible water circulation, and a large terrain correction.

Coeur d'Alene mining district, Idaho Temperatures were measured in three steeply-inclined underground drill holes, two in the Crescent Mine belonging to the Bunker Hill Mining Company and one in the Silver Summit Mine belonging to the Hecla Mining Company. The geology in the district is complex, and in the vicinity of the mines, on the north limb of an anticline south of the Osburn fault [Hobbs et al., 1965, section B, plates 2 and 3], the average dip is about 70°. The two drill holes in the Crescent Mine (DDH-841, curve 8; DDH-854, curve 9, Figure 4) are only a few meters apart in a recently opened crosscut and are treated as a single value. They penetrate rocks of the Revett quartzite. The drill hole in the Silver Summit Mine (DDH-3417, Figure 4, curve 7) in the St. Regis argillite is in an old crosscut, and the ventilation effects penetrate to 80-90 meters (the calculated effect shown as a dotted curve agrees well with the observed effect). The rocks are anisotropic, and because the drill holes were inclined the vertical conductivity of the samples was obtained from measurements perpendicular to bedding (K_{\perp}) and parallel to bedding (K_{\parallel}) . The ratios of K_{\parallel} to K_{\perp} range from 1.1 to 1.5 for the quartzites and from 1.7 to 2.5 for the argillites. The average value of K_{\perp} is 7.9 (ranging from 4.4 to 10.8) for the rocks of the St. Regis formation cored by DDH-3417 and is 10.0 (ranging from 7.8 to 13.6) for rocks of the Revett quartzite cored

by DDH-841 and DDH-854. The equivalent average values of K_{\parallel} are 12.6 and 13.0, so that the more steeply inclined the rocks, the more uniform the vertical thermal conductivity. The agreement of the two heat-flow values is excellent, but the possibility of large-scale refraction cannot be ruled out.

Leadpoint, Washington. Temperature-depth curves from two drill holes in the Cambrian Metaline dolomite are shown in Figure 4 (DDH-1, curve 5; DDH-3, curve 6). The temperatures are disturbed by water movement above 100 meters in DDH-1 and above 290 meters in DDH-3. However, the heat-flow value for the interval 200-340 meters in DDH-3 (3.1 ± 0.1) does not differ from that in the



Fig. 5. Temperature-depth curves and gradients, Marysville, Montana.

most stable interval (290-340 meters). The heat flow in both drill holes is abnormally high. Geologic mapping suggests that the dolomite body is pod-shaped [Yates, 1964, section BB']. According to the formula given by Jaeger [1965] for a horizontal semi-elliptical cylinder of the appropriate size, the heat flow in the country rock at a large distance from the dolomite would be between 1.7 and 2.3 if the average thermal conductivity of the country rock lies between 6.0 and 11.0. The best value of regional heat flow is taken to be 2.0 ± 0.3 .

Wilbur, Washington. Temperatures were measured in three holes in an area of high relief and complex alteration. There is a partial conductivity-gradient correlation in DDH-C (Figure 3, curve 2). Data from DDH-B are also shown (Figure 3, curve 3). The terrain corrections were calculated by using surface temperature data from the drill holes rather than an assumed uniform lapse rate. The data will be described in more detail in a later publication. For the purposes of this paper, the best value is taken to be the average (weighting DDH-C twice) of the values.

North Bend, Washington. Data are available in two shallow drill holes in the northern Cascades. Below 70 meters in DDH-1 (Figure 3, curve 5) there is a good gradient-conductivity correlation. The temperature-depth curve in DDH-2 shows abundant evidence of water movement. Least-square lines fitted to all of the data or to the points possibly least disturbed give the same result (the line fitted to the latter data points is shown in Figure 3, curve 6). The heat-flow values calculated for the two drill holes agree in spite of the uncertainties. The terrain-corrected values were increased by 5% to allow for inaccuracies in the correction. A correction for 2 km of uplift and 1.0–1.5 km of erosion in about 6 m.y. was also applied to the data. Geological data indicates that the Snoqualmie batholith was intruded very close to the surface. If so, a correction for the initial heat of the batholith is negligible.

Published and Estimated Heat-Flow Determinations

Although no other heat-flow study has dealt with the northwestern United States, some published information on the heat flow is available (Table 2A). Garland and Lennox [1962] report heat-flow values of about 1.5 in two oil wells in the Alberta plains at 54°N. In addition, some data on gradients in holes drilled for hydrocarbon exploration in the northwestern United States and Canada are available. Although the gradients alone cannot be used for precise heatflow determinations, they will in some cases indicate whether the heat flow is high or normal. Anglin and Beck [1965] report gradients from two localities in southwestern Alberta, but they do not identify the section involved. Van Ostrand [1951] published gradients from many wells in the northwestern United States. Recently Spicer [1964] published a tabulation of all of the temperature measurements made during the early 1900's by Van Ostrand and others of the U.S.

Locality	N. Lat.	W. Long.	Rock Type	Heat Flow, µcal/cm ² sec	Reference
Idaho					
Wallace			Precambrian sediments	>2 0	Sass et al. [1968]
Montana					
Butte	46°03′	112°33′	Boulder batholith	22	Blackwell and Robertson, in prep.
Washington					,
Metaline	48°55′	117°20'	Cambrian dolomite	23	Roy et al. [1968b]
Central Wash.			Tertiary basalt	'normal'	Sass et al. [1968]
Western Wash.			Tertiary	'normal'	Sass et al. [1968]
(3 values)			sediments		• • • • •
Wyoming					
Wind River Mts.			Precambrian granite	1.3	Sass et al. [1968]

TABLE 2A. Published Heat-Flow Determinations in the Northwestern United States

Locality	N. Lat.	W. Long.	Average Depth, m (no.)*	Rock Type	Average Gradient, °C/km†	Estimated K , $\frac{\text{millical}}{\text{cm sec }^{\circ}C};$	Heat Flow, µcal/cm² sec
Montana							
Conrad	48°20′	111°55′	690(2)	Shale	20.5	4.0 ± 0.5	08 ± 01
			(1)	Dolomite	10 0	$10\ 0\ \pm\ 0\ 5$	1.0 ± 0.1
Kevin-Sunburst	48°45'	111°50′	400(6)	Shale	23 4	4.0 ± 0.5	0.9 ± 0.1
			(2)	Limestone	15 3	6.5 ± 1.0	10 ± 02
North Dakota			• •			-	
Lone Tree	48°18′	101°40′	1143(1)	Shale and Limestone	39.8	>3.5	>1 4
Oregon							
Astoria	46°09′	123 <i>°53′</i>	1152(1)	Sandstone and shale	30.9		
Burns	43°27′	118°6′	1140(1)	Basalt	40 0-45.0	4.5 ± 0.5	2.0 ± 0.3
Klamath County	42°12′	121°50′	400(4)	Lava and tuffs	48.0-98.0		>1 6
Vale	43°46′	117°22′	395(1)	7	77.7		>1 6
Washington							
Benton City	46 °25′	119°34′	650(1)	Basalt	38.3	4.5 ± 0.5	1.7 ± 0.2
Moclips	47°12′	124°6′	1030(1)	Shale	27.1	4.0 ± 0.5	1.1 ± 0.1
Seattle	47°36′	122°20'	762(1)	7	17.4	•••	••
Wyoming							
Big Muddy field	42°51′	106°58'	895(4)	Shale	35.5	4.0 ± 0.5	1.4 ± 0.2
Ferris field	42°10′	107°8′	871(2)	Shale	34.8	4.0 ± 0.5	1.4 ± 0.2
Gebo field	43°48′	108°14′	429(1)	Shale	39.5	4.0 ± 0.5	1.6 ± 0.2
Lance Creek field	43°4′	104°38′	964(3)	Limestone and shale	50 5	$4\ 0\ \pm\ 0.5$	2.0 ± 0.2
Little Sand							
Draw field	44°22′	109°0'	914(1)	Shale	31 9	4.0 ± 0.5	1.3 ± 0.2
Lost Soldier field	42°14′	107 °34′	730(3)	Shale, sand, and lime	39.3-86.3‡	•••	
Oregon Basin field	44°22'	108°56'	1295(1)	Shale	30.5	40 ± 05	13 ± 0.2
Rock River field	41°40′	106°7′	884(3)	Shale	28 9	40 ± 05	1.2 ± 0.2
Salt Creek field	43° 35′	106°15′	640(25)	Shale	45§	4.0 ± 05	$1 \ 8 \pm 0 \ 2$

TABLE 2B. Estimated Heat-Flow Determinations in the Northwestern United States

* Number of wells at each location.

† Temperature data from Spicer [1964].

‡ Circulating hot water along faults [Irwin, 1929] probably causes the high gradients.

§ Gradient at flank of dome [see Van Ostrand, 1934].

Geological Survey; these temperatures were used to calculate least-squares gradients for all suitable wells in the Northwest. The localities, gradients, and, where possible, estimated conductivities and heat-flow values are shown in Table 2B.

Most of the drill holes in Wyoming and Montana penetrate sections composed predominantly of relatively unconsolidated shale with only a small proportion of sand. For such a lithology a reasonable conductivity is 4.0 ± 0.5 mcal/cm sec °C [Benfield, 1947; Garland and Lennox, 1962]. The wells in the Columbia Plateaus province usually show evidence of water disturbances [see Van Ostrand, 1938], and the gradients in several of the wells were estimated from temperature points near the top and bottom of the drill hole. The conductivity of basalt was assumed to be 4.5 ± 0.5 mcal/cm sec °C, the average of several samples of the Columbia River basalt from near Yakima, Washington. The only value to which numerical significance is attached is the heat flow estimated in the Kevin-Sunburst and Conrad fields in northcentral Montana. The gradients and estimated conductivities in three distinct lithologies give consistent results of 1.0 ± 0.2 . The assumed dolomite conductivity was based on measurements of six samples of the same formation from about 60 km north of the well site.

DISCUSSION

Summary of heat-flow determinations. The heat-flow data available in the northwestern United States are shown in Figure 6. All values in the Northern Rocky Mountain province



Fig. 6. Heat-flow determinations in the northwestern United States. Solid circles, determinations discussed in this paper; open circles, published or estimated values >1.6; triangles, published or estimated values <1.6; dotted circles, estimated values which could be either above or below 1.6.

are high and, excluding the heat flow of 6.5 at Marysville, Montana, average 2.0 ± 0.1 . Scanty data in the Columbia Plateaus province suggest that the regional heat flow may be comparable to that in the Basin and Range and Northern Rocky Mountain provinces. The heat flow in central Montana, central Wyoming, and western Washington is generally normal, while the Black Hills and part of eastern Wyoming appear to have high heat flow.

Presently available heat-flow data in the western United States are summarized in Figure 7. The estimated values from Table 2 are not included, however. All determinations in the Basin and Range, Northern Rocky Mountains, and Southern Rocky Mountains physiographic provinces are above 1.5, while all values in the Colorado Plateaus, Middle Rocky Mountains (except the Absaroka Mountains), Wyoming basin, and Great Plains (except the Black Hills) physiographic provinces are below 1.5. Also all determinations in the Northern Cascades, Washington coast, Sierra Nevada Mountains, and Lower California provinces are below 1.5. Thus the regional zonation of heat flow in the northwestern and southwestern United States is similar.

Birch et al. [1968] and Roy et al. [1968a] show that surface heat production is an important parameter in the interpretation of heat-

flow data. Only three of the heat-flow determinations discussed here are from what might classify as large plutonic bodies. The data on radioactivity from suitable sites are listed in Table 3. The data from the Boulder batholith fall on the line relating heat flow and heat production appropriate for the Basin and Range province while the points from Cooke City and North Bend would plot midway between that line and the line for the eastern United States.

Lateral variation in the electrical conductivity structure in the upper mantle also should supply information on regional heat-flow variation. Schmucker [1964] and Swift and Madden [1967] infer an upwarping of isothermal surfaces in the southern part of the Basin and Range province based on geomagnetic field variation and magnetotelluric investigations. The boundaries of this temperature anomaly are in excellent agreement with the boundaries of the zone of high surface heat flow [Warren et al., 1968]. Caner et al. [1967] present data from geomagnetic field variations in southwestern Canada and the southwestern United States, but the correlation of their results with the distribution of regional heat flow shown in Figure 7 is not close. The reason for the lack of agreement is not known.

The best documented feature of the heatflow pattern in the Northwest, as in the South-



Fig. 7. Heat flow and physiographic provinces in the western United States (from Roy and Blackwell, in preparation). The abbreviations SRM, MRM, NRM, and SN stand for Southern, Middle, and Northern Rocky Mountains and Sierra Nevada Mountains, respectively. Published heat-flow data from Roy et al. [1968b]; Lachenbruch et al. [1966]; Sass et al. [1968]; Warren et al. [1968]; Henyey [1968]; Costain and Wright [1968]; Burns and Grim [1967]; Vacquier et al. [1967]; and Lee and Uyeda [1965]. Pluses, heat flow 0-0.99 μ cal/cm²sec; open circles, 1.0-1.49; dotted circles, 1.5-1.99; solid circles, 2.0-2.49; solid triangles, 2.5-2.99; solid rectangles, >3.0.

west, is the broad central zone of above average heat flow in the Basin and Range, Northern Rocky Mountain, and Columbia Plateau provinces. As the zone follows the axis of the Cordilleran mountain chain [see King, 1966], from the Mexican to the Canadian borders, for convenience of reference it will be referred to as the Cordilleran thermal anomaly zone (CTAZ) in the remainder of this paper.

Cenozoic normal faulting and volcanism. The identifying physical features of the Basin and Range province are alternating ranges and valleys commonly aligned north-south. Cenozoic volcanic rocks, principally silicic flows or ignimbrites, are abundant in both the ranges and valleys. The controlling factor of the topography seems to be a set of large-scale Cenozoic dip-slip faults; the valleys are either grabens or downtilted blocks and the ranges are either horsts or uptilted blocks.

The Northern Rocky Mountains, southern Idaho, southeastern Oregon, southwestern Montana, and parts of extreme western Wyoming are commonly considered to exhibit Basin and Range structure [see *Gilluly*, 1965]. *Pardee* [1950] points out that the whole western third of Montana is characterized by Basin and Range structure and that the faulting occurred chiefly in the Pliocene. *Yates et al.* [1966] summarize the geology of northwestern Montana, northern Idaho, and northeastern Washington and emphasize the role of normal faulting and volcanism in the Cenozoic history of that region.

A region near the eastern boundary of both the Northern Rocky Mountain and Basin and Range provinces is called the 'zone of great

TABLE 3. Radioactive Heat Production.

	No. of	eU,*	K ,†	A 10 ⁻¹³ cal‡	
Locality	Samples	ppm	%	cm ³ sec	
Butte,				0.0	
Montana§				8.6	
Montana	12	4.6	2.1	3.5	
North Bend, Washington	10	4.4	1.6	3.3	

* Determined by the method of α -counting.

† Determined by the X-ray fluorescence method. ‡ Heat production determined by using conver-

sion factors given by Roy et al. [1968a].

§ Data from Boulder batholith (Blackwell and Robertson, in preparation).

trenches' by Eardley [1962, p. 506] because of the presence of a number of large displacement, late Cenozoic normal faults. However, the zone is merely the eastern boundary of the region of normal faulting (Figure 8). A band of seismic activity, the Rocky Mountain earthquake zone [Ryall et al., 1966], coincides with this boundary. As shown in Figure 8, normal heat-flow values are confined to the area east of the normal faulting and the earthquake zone. Thus the position of the Rocky Mountain earthquake zone supports the equating of the Northern Rocky Mountains and the Basin and Range province and suggests that the earthquakes are somehow related to the transition in heat flow along which they appear to occur.

Crustal and upper mantle structure. The crustal structure of the southwestern United States has been extensively investigated by the technique of seismic refraction profiling. The resulting picture of crustal structure is shown in Figure 9 as a cross section at about 39°N. The section is similar to one published by Pakiser [1963] with modifications made to include the results of more recent work [*Eaton et al.*, 1965; Roller, 1965; Jackson and Pakiser, 1965]. The crust is 10-20 km thinner beneath the Basin and Range province than beneath the Sierra Nevada, Colorado Plateaus, or Southern Rocky Mountains, although the average elevation differences are small. Also the P_n velocity beneath the Basin and Range province is low (7.8-7.9 km/sec.). The transition from the crustal structure of the Basin and Range to that of the Sierra Nevada and Colorado Plateaus appears to be sharp with dips of 15° or more on the M discontinuity [*Eaton*, 1966; *Julian et al.*, 1968]. In the one place where the heat-flow transition, from high values in the Great Basin to normal values in the Sierra Nevada, is investigated in detail the seismic and heat-flow boundaries are coincident and about 50 km east of the structural and physiographic boundary [*Roy and Blackwell*, 1966].

Also shown in Figure 9 is a crustal cross section at about 50°N [White and Savage, 1965]. The similarity between the two crustal sections is obvious. White and Savage [1965] found a decrease in crustal thickness east of the Coast Range and a low P_n velocity beneath the thin crust. An increase in crustal thickness was inferred to occur somewhere slightly to the west of the eastern edge of the Rocky Mountains. Both transition zones would be near the regions



Fig. 8. Heat flow, normal faults, and earthquakes (1869-1954) in the vicinity of the Rocky Mountain earthquake zone. The small dots are earthquake epicenters [*Ross and Nelson*, 1964]. The solid lines are normal faults of large displacement [*Gilluly*, 1963, Figure 18; Yates et al., 1966]. Heat-flow determinations >1.6 are shown as large solid dots and those <1.6 are shown as open circles.



Fig. 9. Crustal cross sections, western North America. See text for references.

where a heat-flow transition is suggested by data from this study.

Data on crustal thickness in the northwestern United States are scanty, but several refraction profiles in Montana exhibit some interesting features. Interpretation of two reversed and several unreversed profiles [Steinhart and Meyer, 1961; McCamy and Meyer, 1964] shows that the crust in eastern Montana is thick (45-55 km), the average crustal velocity is high, and the upper mantle velocity is normal to high (8.0-8.4 km/sec.). In western Montana a northsouth reversed profile indicates that the crust is thinner than in the plains (by 5-20 km) and that the P_n velocity is lower (7.9 km/sec). Two recently published east-west profiles in northwestern Montana [Asada and Aldrich, 1966], though not interpreted in such a fashion, are consistent with an abrupt decrease in crustal thickness (M-discontinuity dipping at 10°-15°) of 10-15 km from east to west between 112°W and 113°W and a concomitant decrease in P_n velocity from above 8.0 to 7.9 km/sec.

Thus on the basis of present knowledge, the crust beneath the CTAZ is abnormally thin and the upper mantle seismic velocities are low. The boundaries of the region of anomalous crustal structure (characterized by a 5 to 20-km change in crustal thickness in 25-50 km) appear to coincide with the heat-flow transitions. The data, then, strongly imply a genetic relationship between thin crust, low P_n velocities, and high heat flow.

Origin and extent of the high heat flow. The data discussed in the previous sections establish the similarity of the Cenozoic geological history, crustal structure, and regional heat flow of the Northern Rocky Mountains and the Basin and Range province. No other areas for which data are presently available in the western United States have all the same characteristics. Although the Southern Rocky Mountains have a high regional heat flow, the crust is thick and the P_n velocity is normal. So the relation of that zone of high heat flow to the CTAZ is unknown. It is clear that the CTAZ is a dominant feature of the western Cordillera at the present time and the understanding of its development is one of the key geological and geophysical problems in the western United States.

The main questions raised by the occurrence of the broad zone of high heat flow in the western Cordillera, its origin and the ultimate source of the excess heat flow, remain unanswered. The data from the northwestern United States indicate that the average heat flow there is about the same as in the Southwest. The distance between the heat-flow determination at Cooke City, Montana, and the thermal areas in Yellowstone National Park, which presumably are in the zone of high heat flow, suggests a sharp border to the zone. Because little else is known about the shape of the anomaly in the Northwest, no detailed interpretations of the data are attempted. The model most favored to explain the regional anomaly in the Great Basin involves recurrent, province-wide solid or liquid intrusions in the upper mantle [Roy and Blackwell, 1966]. The model is a convective one, but does not involve a current system. Penetrative convection in the crust could explain the heat-flow values above about 3.0, such as the 6.5 at Marysville, Montana. Large continentwide convection models which have ascending movement beneath the CTAZ and descending movement beyond the borders of the zone will not fit the heat-flow data in the western United States, but a model with a small convection cell completely confined to the CTAZ might fit the data.

The extension of the CTAZ beyond the United States is a possible topic of speculation. Seismic refraction data indicate that the zone extends to at least 51°N. A structural feature near the eastern boundary of the zone, the Rocky Mountain trench extends to about 60°N. Directly along the strike of the Rocky Mountain trench, the Tintina trench extends from the Yukon into eastern Alaska in a broad arc. A high heat-flow value was found in the Canadian arctic $\lceil Gar$ land and Lennox, 1962] not far east of these fault systems. At the western boundary of the CTAZ the Frazer fault system extends to about 51°N and a flood basalt field to 54°N [White, 1959]. Between about 60°N and its northern end in extreme northwestern Alaska, the Denali fault parallels the Tintina trench and another fault along the strike of the Tintina trench. The separation of the two parallel fault zones is about 350 km [see King, 1966]. Several large exposures of Cenozoic volcanics occur between the eastern and western fault zones in northwestern Canada. Although many of the faults mentioned are not normal faults, it may be significant that the western boundary of the CTAZ in southern California appears to be the San Andreas system of faults [*Henyey*, 1968]. In Mexico typical Basin and Range structure, volcanism, and topography extend for several hundred kilometers south of the border.

East Pacific rise and the western United States. There are many hypotheses about the relation of the East Pacific rise to the geological features of the western United States. It is not the purpose of this section to give a synthesis of all the evidence pertaining to the problem or to give references to the voluminous literature. It will merely be pointed out that the identical nature of the present geophysical characteristics and Cenozoic history of the Basin and Range and Northern Rocky Mountain provinces casts doubt on tectonic schemes which give a unique position to the Basin and Range province [Wise, 1963; Talwani et al., 1965; many others]. Also hypotheses which single out part of the CTAZ, such as the 'zone of great trenches' as the extension of the rise [Heezen, 1960; Cook, 1962] do not appear to be justified by the data: the rise must be a regional feature if the CTAZ and the East Pacific rise are considered equivalent. The possibility that the Colorado Plateaus mark the axis of the extension of the rise [Menard, 1960] appears to be discounted by the heat-flow data. Furthermore it is now clear that there are several separate regions of high heat flow in the western United States and northeastern Pacific (Figure 7). Thus, whatever the relation of the East Pacific rise and the western United States, it is obviously not as simple as was previously supposed.

The possibility must be considered that the CTAZ is due to other causes than the continental extension of an oceanic rise system. The zone follows the axis of the Cordilleran mountain chain in the western United States so it might be related to the development of the mountain belt rather than superimposed on it. Furthermore, similar zones of high heat flow around the Pacific basin in Australia and Japan do not appear to be the extension of rise systems.

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