Reply to “Comment on ‘A model of earthquake triggering probabilities and application to dynamic deformations constrained by ground motion observations’ by Ross Stein”

Joan Gomberg¹ and Karen Felzer²

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[1] We thank Stein [2011] for pointing out the errors in the caption of Figure 1 of Gomberg and Felzer [2008], and we correct the caption appropriately herein. However, as we show below, the figure itself is not in error. We were also remiss in not providing a thorough description of how Figure 1 was created, and thus, do so in this reply. We also emphasize that the foundations of the study described by Gomberg and Felzer [2008] do not depend on its Figure 1, but instead Figure 1 simply strengthens them. The primary focus of Gomberg and Felzer [2008] was to glean some physical understanding about the processes that give rise to aftershocks, particularly the aftershock characteristics revealed in the study of Felzer and Brodsky [2006]. They showed that the aftershock density follows a continuous inverse power law decay over distances of many multiples of the rupture dimension and traditionally defined aftershock zone out to at least 50–100 km, separately for main shocks within bins M2–3, 3–4 and 5–6. Indeed, while Figure 1 of Gomberg and Felzer [2008] further corroborates the remarkable linearity of aftershock log-density and its lack of dependence on main shock dimensions by combining results for multiple magnitudes, the premise of the paper derives from the original results of Felzer and Brodsky [2006], with or without this additional corroboration.

[2] Stein’s [2011] message is that the M2–3 and M5–6 aftershock density measurements have not been correctly combined to show the continuity inferred by Gomberg and Felzer [2008]. Stein has four concerns: (1) he correctly notes errors in the caption of Figure 1, (2) he takes issue with the interpretation of the overlap between the two populations, (3) inclusion of only subsets of measurements from each population, and (4) the certainty of the correction for differences in populations due to the dependence of aftershock rate on measurement duration. We address these concerns sequentially.

[3] First, Stein correctly notes that the statement “All aftershocks are M > 2 and occur in the first 5 min after their main shock” is in error, as it only applies to the M2–3 main shocks and the inequality should be M ≥ 2 (this inequality was similarly incorrectly reported by Felzer and Brodsky [2006]). In other words, we failed to note in the caption of Figure 1 that aftershocks are M ≥ 3 in the first 2 days for the M5–6 main shocks. However, these errors affect the caption only, and were accounted for in the creation of Figure 1.

[4] Stein also correctly notes a second error in the caption of Figure 1 regarding the distance range of densities plotted for the M5–6 main shocks. We reported these were plotted only to distances of r < 3 km, when in fact as can be seen in Figure 1, they are plotted to 12 km and thus overlap with the M2–3 measurements from 3 to 12 km. However, we disagree that this overlap obscures the continuity between the two data sets and note that the overlap more robustly establishes continuity than had each data set been terminated at a “3 km boundary.” In addition, the overlap makes sense physically because the transition from near to far field occurs over a finite distance range, not at a sharp boundary.

[5] Stein’s third concern pertains to the omission of measurements, that “outliers at distances greater than 10 km in Figure S5, and a change in slope of the data at 80–100 km in Figure S1, are excluded.” We remind Stein and the reader the latter is explained in the caption of Figure S1 of Felzer and Brodsky [2006], which states “At far distances there are more background earthquakes than aftershocks, and thus that the aftershock decay can no longer be observed. Since the main shock selection criteria only eliminated larger main shocks within 100 km (see Methods), there is contamination from aftershocks of larger mainshocks beyond 100 km, as well as contamination from other sources of background seismicity.” Careful inspection of Figure S5 reveals that in Figure 1 no points from the M5–6 data set render “the M5–6 portion of the Gomberg and Felzer [2008] Figure 1 irreproducible.” We note that this does not appear to be supported by the fact that Marsan and Lengliné [2010] had no difficulty duplicating the results of Felzer and Brodsky [2006]. The critical point is being able to determine accurate locations for the main shock fault planes; Marsan and Lengliné [2010] developed an automated algorithm for fault plane location and found that the aftershocks of a larger

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set of M5–6 main shocks displayed a very similar aftershock density decay with distance to that shown in our figure. We emphasize again that the point of Gomberg and Felzer [2008] was not to reconfirm or debate the validity of Felzer and Brodsky’s [2006] approach and results, but rather to interpret them.

[6] Finally, we were remiss in not discussing the scaling of the two measurement populations and do so now to address Stein’s last concern. As we now show, the scaling to correct for differences in main shock and aftershock magnitudes, the numbers of main shocks, and the time periods covered in the M2–3 and M5–6 main shock/aftershock data sets is ~1. Herein we describe our derivation of the scaling factor and results of new analyses we conducted to respond more quantitatively to Stein’s concern about the confidence intervals on the scaling factor. Our scaling estimate differs from Stein’s in how we account for the different measurement periods and assess the uncertainty in the scaling (period in Table 1 of Stein). We suggest that our approach results in a similar scaling but significantly smaller uncertainties. Like Stein, we employ a Gutenberg-Richter frequency-magnitude relation, an aftershock productivity versus magnitude relation, and consideration of how aftershock rate decays with time.

[7] We concur with Stein that the aftershock “densities of the M5–6 data need to be rescaled by the magnitude difference (a factor of about 10 for a Gutenberg-Richter b value of 1), the relative number of main shocks in the two data sets (a factor of 821), and the observation that larger main shocks produce more aftershocks (a factor of about 0.001, based on the work by Felzer et al. [2002]); together these three factors come to a about 8.2.” We estimate a product of 8.4, but the difference from 8.2 appears to be only due to precision in the factors. We describe our calculations for completeness; the factor of 10 for the aftershock magnitude difference is the same, the relative number of M2–3 to M5–6 main shocks is 7396/9 = 822, and the factor accounting for the greater number of aftershocks for larger main shocks is 1/977. We derive the last factor using the magnitude of the earthquake that would produce the average number of aftershocks expected to be produced by the main shocks in each data set, $M_{\text{effective}}$, where $M_{\text{effective}} = \log_{10}(\Sigma 10^{M_i}/N)$ where M is the magnitude of each main shock and N is the total number of main shocks, and aftershock productivity is assumed to vary as $10^M$ [Felzer et al., 2004]. $M_{\text{effective}} = 5.68$ for the M2–3 and M5–6 populations, respectively. The larger main shock would have an average of $10^{5.68}$ more aftershocks, with $\Delta M \approx (5.68-2.47)$, resulting in a factor of 977.

[8] We differ from Stein in our estimate of the factor that corrects for the difference in the number of aftershocks between $t_1 = 5$ min and $t_2 = 2$ days for the M2–3 and M5–6 populations. Stein estimates a correction for the different time periods covered and its uncertainties using Omori’s law (a model of aftershock decay rate) and obtains a range that spans 2 orders of magnitude, from 0.01 to 1.0 (his Table 1 and Figure 1). The parameter range that Stein chooses for the application of Omori’s law is too large, as described below. We avoided the need to invoke a model and accompanying parameters by simply taking the ratio of the number of aftershocks that occurred in the first 2 days to those in the first 5 min after the M5–6 main shocks. This empirical method is affected by the fact that aftershock data are known to be more incomplete earlier in the sequence (see Helmstetter et al. [2005] for a complete discussion) so we also compare the mean magnitude of catalog events in the first 5 min and first 2 days, finding M3.94 and M3.35, respectively, and correct for this incompleteness by assuming that both periods are characterized by the Gutenberg-Richter magnitude frequency distribution with a b value of 1.0. In an earlier version of this paper we provided an illustrative example based on the work of Helmstetter et al. [2005] that we no longer include because it appears to have misled Stein to think that we used Helmstetter et al.’s equations. We did not, and thus our calculations required no assumption of Omori’s law.

[9] We estimate that we need to scale the aftershocks of the M2–3 main shocks by a factor of 10.9 to correct for the time difference spanned by the two populations, with a 95% confidence range of 8.3 to 14.3. This value and error bars are specifically calculated as follows. Thirty-five M5–6 main shocks and $N = 503$ total M ≥ 3 aftershocks met our selection criteria for being sufficiently isolated from larger earthquakes [see Felzer and Brodsky, 2006, Supplementary Figure 5] over the first 2 days and a distance range of 0 to 500 km (the distance in Felzer and Brodsky’s Supplementary Figure 5). Thirteen aftershocks, or ~2.5% of this total, occur in the first 5 min. Given the difference in mean magnitudes between the first 5 min and first 2 day periods we estimate that there were likely really a total of ~50 M ≥ 3 aftershocks in the first 5 min, or 9.2% of the total aftershocks that occurred over the 2 day period. Given the size of our data set, the standard error, $\sigma$, on this fraction, f, is $\sigma = \sqrt{f(1-f)}/\sqrt{N} = \sqrt{0.092(1-0.092)}/\sqrt{503} = 0.0129$. To estimate the 95% confidence intervals, we multiply this by 1.96 to get 0.0253. Thus, the 95% confidence interval on the fraction aftershocks that occurs in the first 5 mins of our 2 day period is 6.7% to 11.7%. This means that we need to multiply the aftershocks of the M2–3 main shocks by 1/0.092 = 10.86 to correct for the time difference, with a 95% confidence range of 8.3 to 14.3.

[10] For completeness, we illustrate explicitly how Stein arrived at the uncertainties in the corrections (period values in Stein’s Table 1) for the different periods covered by the M2–3 and M5–6 data sets. Stein’s correction is described by $N(t_1,t_2)/N(0,t_2) = 1 - N(t_1,t_2)/N(0,t_2)$, in which $N(t_1,t_2)/N(0,t_2)$ is the unknown fraction of aftershocks between $t_1 = 5$ min and $t_2 = 2$ days. Stein’s approach using Omori’s law requires specification of model parameters $p$, $c$, and $K$. The equations describing this model are

$$N(t_1,t_2) = K \int_{t_1}^{t_2} (c + t)^{-p} dt$$
$$= K \left[ (c + t_2)^{-p} - (c + t_1)^{-p} \right] \quad p \neq 1$$
$$= K [\ln(c + t_2) - \ln(c + t_1)] \quad p = 1$$
and
\[
\frac{N(t_1, t_2)}{N(0, t_2)} = \frac{(c + t_2)^{1-p} - (c + t_1)^{1-p}}{(c + t_2)^{1-p} - c^{1-p}} = \frac{1 - (\frac{c+t_1}{c+t_2})^{1-p}}{1 - (\frac{c}{c+t_2})^{1-p}} p \neq 1
\]
\[
= \ln\left(\frac{c + t_2}{c + t_1}\right) - \ln\left(\frac{c + t_1}{c}\right) = \ln\left(\frac{c + t_2}{c + t_1}\right) - \frac{\ln\left(\frac{c + t_2}{c}\right)}{\ln\left(\frac{c + t_1}{c}\right)} p = 1
\] (2)

[11] Stein [2011] shows that the range of $p$ and $c$ values he assumes lead to values of $N(0, t_1)/N(0, t_2)$ that vary over 2 orders of magnitude. We note that this range is an overestimate because the Omori law parameters Stein used correspond to those measured for both direct and total aftershock sequences (e.g., the former being triggered solely by a given main shock and “total” sequences comprised of both direct aftershocks and those triggered by other aftershocks [see Felzer et al., 2002]), but only total aftershock sequence parameters appropriate to early times after the main shock should be used. Moreover, the $c$ and $p$ parameters are derived as pairs and cannot be interchanged. Stein writes in his comment that “the central contention of Felzer and Brodsky [2006] that all events within the first 5 min (or 2 days) are direct aftershocks of their main shocks, which is why Felzer and Brodsky take the triggering distance to be the main shock–aftershock separation, not the closest distance between two aftershocks.” This is not correct, nor do Felzer and Brodsky [2006] state or imply that they consider only direct aftershocks. It is true that the spatial decays of aftershock density with distance from the main shock fault plane for the full aftershock sequence (direct plus secondary shocks) and for direct aftershocks alone are very similar when many sequences are averaged together, as was done by Felzer and Brodsky [2006]. However, this similarity does not exist in the temporal domain where the decays of the total aftershock sequence and direct-only aftershocks can be very different at short times after the main shock. Of the parameters that Stein cites, appropriate values include $c = 0.014\text{ day} = 0.34\text{ h}$, $p = 1.08$ and $c = 10^{-5}\text{ day} = 0.75$ from Felzer et al. [2003] and $c = 60\text{ s} = 6.94 \times 10^{-4}\text{ days}$, $p \approx 1$ from Kagan and Houston [2005]. The cited parameters from Felzer and Kilb [2009] correspond to direct aftershocks only, and the $c = 20\text{ s}$ of Peng et al. [2007] was based on careful inspection of the coda to find additional events not listed in the standard catalog so a correction for the superior completeness would be required to use this $c$ value before it could be appropriately applied to the data in our figure. If we only use the allowed parameters, we obtain values of $N(0, t_1)/N(0, t_2) = 0.0536$ and 0.165 for the Felzer et al. [2003] estimates and 0.225 for the Kagan and Houston [2005] parameters, corresponding to a range of a factor of ~4 rather than 2 orders of magnitude. We also note that this range is only slightly larger than and encompasses our empirical estimate of $1/14.3 = 0.0699$ to $1/8.3 = 0.121$.

[12] To conclude, we combine the magnitude and temporal scalings to obtain the ratio that the aftershocks of the M2–3 earthquakes should be multiplied by to match with the aftershocks of the M5–6 main shocks, yielding a factor of $10.86/8.4 = 1.3$, with a range of $8.3/8.4 = 1.0$ to $14.3/8.4 = 1.7$. This range includes the factor of 1 used for Figure 1 of Gomberg and Felzer [2008], with the difference having no impact on the continuity in decay between the aftershock densities for the two main shock data sets. Finally, the range of scaling factors we obtain is encompassed by the range obtained when using Omori’s law, whether Stein’s large range of Omori law parameters are used or the smaller range that we suggest is employed. Thus, given this compatibility, we are puzzled by Stein’s assertion that the data sets we combine are “too incompatible” to be described as continuous. Our interpretation that the data sets combined to create Figure 1 are very likely continuous remains justified.

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References

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Comment on “A model of earthquake triggering probabilities and application to dynamic deformations constrained by ground motion observations” by Joan Gomberg and Karen Felzer

Ross S. Stein

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[1] Felzer and Brodsky [2006] argued that a uniform power law distance-decay of aftershock density to 50 km from small main shocks provides evidence that aftershocks are triggered by dynamic stress, since static stress would be negligible more than a few kilometers from the main shocks. This study was succeeded by Gomberg and Felzer [2008], which extended the analysis. Figure 1 of Gomberg and Felzer [2008], reproduced here as Figure 1a, is presented to show that the decay of seismicity density from main shocks “is a constant over distances spanning a fraction of a main shock fault length to hundreds of main shock fault lengths, at least out to absolute distances of 50–100 km” (paragraph 6). Gomberg and Felzer’s [2008] Figure 1 caption states that “All aftershocks are M > 2 and occur in the first 5 min after their main shock; the short time window separates aftershocks from unrelated background earthquakes.” But inspection of the Felzer and Brodsky [2006] source panels for Figure 1 indicates that the circles are actually the first 2 days of M ≥ 3 aftershocks. Apart from the mislabeling, rescaling the aftershock density from 2 days to 5 min is too uncertain to make the compatibility argument advanced, and the restricted ranges of the two data sets give an appearance of continuity that is inconsistent with the full data.

[2] Gomberg and Felzer’s [2008] Figure 1 is shown with the line, shaded rectangles, and labels removed in Figure 1b. The caption to their Figure 1 states that “[t]riangles and circles are for M2–3 and M5–6 main shocks, respectively. Note that we plot only a subset of aftershocks in each magnitude range, retaining only aftershocks of M5–6 main shocks at r < 3 km and of M2–3 main shocks at larger distances. This highlights the continuity in densities across the transition from near-field to far-field, which occurs at about r ≈ D ≈ 3 km for the M5–6 main shocks.” This statement is also incorrect; the circles are aftershocks of M5–6 main shocks out to 12 km, and thus the data overlap over 3–12 km, and so cannot highlight the absence of an offset across a 3 km boundary.

[3] As Gomberg and Felzer [2008] state, the data in Figure 1 come from Felzer and Brodsky [2006]. It appears that Figure 1 combines data from Figure S5 of Felzer and Brodsky [2006] (shown here as Figure 1c) with data from Figure S1a from Felzer and Brodsky [2006] (shown here as Figure 1d). The points just outside of the selected ranges are inconsistent with the claimed continuity; the data are windowed so that outliers at distances greater than 10 km in Figure S5, and a change in slope of the data between 80 and 100 km in Figure S1, are excluded from Gomberg and Felzer’s [2008] Figure 1.

[4] For use in Figure 1 of Gomberg and Felzer [2008], the seismicity densities of the M5–6 data need to be rescaled by the aftershock magnitude difference (a factor of about 10 for a Gutenberg-Richter b value of 1), the relative number of main shocks in the two data sets (a factor of 821), and the observation that larger main shocks produce more aftershocks (a factor of about 0.001, based on the work by Felzer et al. [2004]); together these three factors come to about 8.2. But since aftershock frequency, and thus density, decays rapidly with time, rescaling Figure 1c from 2 days to 5 min also depends on the Omori c delay and p decay exponent, in which earthquake frequency is proportional to (c + t)p, where t is time. Table 3 of Felzer et al. [2003] lists a range of observed c delays (1 s to 2 h) and p exponents (0.75–1.37). The net scaling could thus range over 0.08 to 8.21 (see Table 1 and Figure 1b). Felzer and Kilb [2009] use p = 1.34 and c = 2 h in their simulations for a southern California M = 5.2 main shock, which would result in a factor of 0.16, much smaller than the value of 1.0 used in their Figure 1. Kagan and Houston [2005] find c = 60 s, and Peng et al. [2007] find a c = 20 s, the shortest estimates yet obtained; even for this narrower range, the factor could span 0.66–6.6, too uncertain to argue for continuity in seismicity density decay with distance.

[5] Gomberg and Felzer [2011] reveal that only 9 of the 35 M5–6 main shocks that met their stated time and distance selection criteria are shown in Figure 1 (and so this is also true for Felzer and Brodsky [2006, Figures 3 and S5]) because the others “did not have well-located and well-defined fault planes.” The exclusion of 80% of the aftershocks (399 out of 503), combined with the absence of identification of main shocks that were selected, makes the M5–6 portion of Gomberg and Felzer’s [2008] Figure 1 irreproducible.

[6] Gomberg and Felzer [2011] state that unlike the approach in this paper, “the decay rate can be measured directly from the data,” but this is not what they do. First, they use the 399 aftershocks that they rejected from Figure 1, and

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Figure 1. (a) Figure 1 from Gomberg and Felzer [2008]; (b) the rescaling uncertainty of the M5–6 data from Table 1. (c and d) from Felzer and Brodsky [2006], screened and annotated to show which portions of the data are used in Figure 1 of Gomberg and Felzer [2008].

From the Figure 1 caption:
“All aftershocks are M≥2 and occur in the first 5 min after their main shock. The short time window separates aftershocks from unrelated background earthquakes. Triangles and circles are for M2–3 and M5–6 main shocks, respectively... This highlights the continuity in densities across the transition from near-field to far-field, which occurs at about r=0–3 km for the M5–6 main shocks.”

Interpretation of caption:
- ▲ First 5 min of M>2 aftershocks of M2–3 main shocks
- ○ First 5 min of M>2 aftershocks of M5–6 main shocks
Table 1. Required Rescaling for Figure 1 of Gomberg and Felzer [2008]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Data Used</th>
<th>Converted to</th>
<th>Conversion Factor</th>
</tr>
</thead>
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<tr>
<td>Aftershock magnitude</td>
<td>M ≥ 3</td>
<td>M ≥ 2</td>
<td>~10</td>
</tr>
<tr>
<td>Number of main shocks</td>
<td>9</td>
<td>7,396</td>
<td>821</td>
</tr>
<tr>
<td>Main shock magnitude</td>
<td>“M5–6”</td>
<td>“M2–3”</td>
<td>~0.001</td>
</tr>
<tr>
<td>Period (depends on Omori (c) and (p))</td>
<td>2 day</td>
<td>5 min</td>
<td>~0.01–1.0</td>
</tr>
<tr>
<td>Seismicity density rescaling*</td>
<td></td>
<td></td>
<td>0.08–8.2</td>
</tr>
</tbody>
</table>

*Seismicity density rescaling equals the conversion factor for aftershock magnitude times number of main shocks times main shock magnitude times period.

find that out of the 503 recorded in the first 2 days, 13 struck in the first 5 min. Then, they multiply 13 by 3.8 because of early aftershock incompleteness, and thus claim “that there were likely really a total of ~50 M ≥ 3 aftershocks in the first 5 min.” While they use an estimate for the magnitude of completeness as a function of time since the main shock from Helmstetter et al. [2005] to estimate the number of missed aftershocks, this must be coupled with Omori \(p\) and \(c\) parameters to arrive at a multiplier, which according to Felzer et al. [2003] are highly uncertain. They do not state what Omori parameters they used, and their 95% confidence intervals on the scaling factor do not consider any contribution of the multiplier, rendering it a severe underestimate.

[7] Finally, in the last three paragraphs of Gomberg and Felzer [2011], they claim to reproduce the calculations made for period in Table 1, but they use a narrower range of Omori parameter uncertainty than is found by Felzer et al. [2003]. They advocate for a 1.3 (~0.3 / +0.4) scaling factor but do not correct their Figure 1. They also claim that “only total aftershock sequence parameters appropriate to early times after the main shock should be used.” This contradicts the central contention of Felzer and Brodsky [2006] that all events within the first 5 min (or 2 days) are direct aftershocks of their main shocks, which is why Felzer and Brodsky take the triggering distance to be the main shock–aftershock separation, not the closest distance between two aftershocks. If, instead, the published uncertainties are honored, the 5 min and 2 day data sets are too incompatible to be described by Gomberg and Felzer [2008] as continuous.


References


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