Waveform modelling using locked-mode synthetic and differential seismograms: application to determination of the structure of Mexico

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SUMMARY
We have developed algorithms for modelling seismic waveforms in order to constrain regional Earth structure. The seismogram is represented as a sum of locked-mode travelling waves in a layered medium. This representation is convenient as it allows us to model structures with slowly varying heterogeneity and to construct differential seismograms. This paper describes the techniques we have implemented that enable us to compute synthetic and differential seismograms in an efficient and stable manner. The computational methods are sufficiently rapid that many modes can be included and in some cases the entire seismogram may be modelled. These algorithms are applied to model a set of seismograms of southern Mexican earthquakes recorded in northern Mexico. The frequency bandwidth of these data is centred at 0.067 Hz and we demonstrate that even at these relatively high frequencies, many features of the seismogram can be successfully modelled. Our results suggest that the structure within the recording array in northern Mexico is resolvably different from that to the south. We find that the average shear velocity of the lower lithosphere of southern Mexico is very low, approximately 4.3 km s⁻¹. If the low-velocity region is confined to the Trans Mexican Volcanic Belt, the shear velocities between 20–80 km depth are approximately 3.3 km s⁻¹. This may be correlated with partial melt and is consistent with the active volcanism and high heat flow found in the region.

Key words: Differential seismograms, Mexico, surface wave modes, velocity structure, waveform modelling

INTRODUCTION
In regions where data from densely spaced seismic stations are available, seismograms containing several surface wave modes may be used with spatial filtering techniques to constrain detailed models of regional Earth structure (e.g. Cara 1978). Unfortunately, in most places adequate data are not available to do this type of analysis. A differential seismogram algorithm may be employed when a large data set is not available. In such algorithms, the difference between a synthetic and observed seismogram is related directly to changes in modelled Earth structure. Differential seismogram algorithms have previously been used with relatively low frequency data to constrain global aspherical structure (Woodhouse & Dziewonski 1984). In this paper we describe the application of a differential seismogram technique to recover regional Earth structure in Mexico from shorter period data (dominant period of approximately 15 s). Our technique is based on the representation of the seismogram as a sum of locked-mode travelling waves. The method is computationally efficient so that many modes can be included and in some cases the entire waveform may be modelled.

The first part of this paper gives a description of the algorithms with more detail provided in the Appendices. The second half describes the application of these techniques to twelve sets of three-component seismograms of southern Mexican earthquakes recorded in northern Mexico. The objective of this study is to characterize the structural variability in northern and southern Mexico. The bandwidth of these data is essentially identical to that obtained on the long-period World Wide Standard Network so that the dominant wavelengths are 40–200 km and structural variations within the region should be resolvable. Models of the crust and upper mantle that serve as starting models for the present study are derived from analysis of phase velocity and traveltime data performed in an earlier study of northern Mexico (Gomberg et al. 1988, hereafter Paper I). A description of the instrumentation is also found in Paper I. Our results demonstrate that is is possible to model many features of the entire seismogram, even at these relatively high frequencies.
COMPUTATION OF SYNTHETIC AND DIFFERENTIAL SEISMOGRAMS

Our intent in this section is to present the theoretical background and formulae required for efficient computation of locked-mode synthetic seismograms. While many of the computational algorithms may be found elsewhere in the literature, we describe several new improvements and derive formulae for differential seismograms with respect to shear and compressional velocities, density, and source location. A brief outline of the theory behind the locked-mode approximation is presented first in order to establish our notation. The usual Fourier transform of the conservation of momentum equation in cylindrical coordinates is considered:

\[- \rho \omega^2 \mathbf{u} = \mathbf{f} + \nabla \cdot \mathbf{\sigma},\]

where \(\mathbf{u}\) is the displacement field, \(\mathbf{\sigma}\) is the stress tensor, \(\mathbf{f}\) is a body force density, \(\omega\) is frequency and \(\rho\) is density. We first consider the solution to equation (1) when \(\mathbf{f}\) is zero. \(\mathbf{u}\) is expanded in vector cylindrical harmonics (Takeuchi & Saito 1972):

\[u_\ell(z, \omega, k) = \sum_{m=-\infty}^{\infty} \frac{1}{2\pi} \int_0^\infty k \, dk [A(z, \omega, k)R^m_k + B(z, \omega, k)S^m_k + C(z, \omega, k)T^m_k],\]

where

\[R^m_k = Y^m_k,\]

\[S^m_k = -\frac{1}{k} \frac{\partial Y^m_k}{\partial r} + \frac{1}{k} \frac{\partial Y^m_k}{\partial \phi},\]

\[T^m_k = \frac{1}{k} \frac{\partial Y^m_k}{\partial r} - \frac{1}{k} \frac{\partial Y^m_k}{\partial \phi},\]

and

\[Y^m_k = J^m_k(r) e^{i m \phi}.\]

\(J^m_k(r)\) is an \(m\)th-order Bessel function and \(k\) is the wavenumber. In an isotropic, elastic medium, equation (2) can be substituted into equation (1) and three second-order ordinary differential equations are obtained for \(A(z)\), \(B(z)\) and \(C(z)\) at a fixed \(\omega\) and \(k\). The equation for \(C(z)\) is independent of the other two and governs Love wave motion. The coupled equations for \(A(z)\) and \(B(z)\) govern Rayleigh wave motion. We follow Woodhouse (1980) and define quantities proportional to the components of displacement and stress. For Love waves let

\[b_1 = \omega C, \quad b_2 = \mu \frac{dC}{dz}\]

represent these quantities for the transverse components and for Rayleigh waves we define quantities proportional to the vertical and radial components:

\[b_1 = \omega A, \quad b_3 = \sigma \frac{dA}{dz} - \lambda kB,\]

\[b_2 = \omega B, \quad b_4 = \mu \left( kA + \frac{dB}{dz} \right).\]

\(\lambda\) and \(\mu\) are the Lamé parameters and \(\sigma = \lambda + 2\mu\). The three second-order ordinary differential equations may now be written as two sets of first-order equations with a high degree of symmetry (Chapman & Woodhouse 1981):

\[\frac{db}{dz} = \omega \mathbf{A} b\]

where

\[\mathbf{A} = \begin{bmatrix} \frac{\lambda p}{\sigma} & \sigma^{-1} & 0 \\ -\rho & 0 & 0 \\ -\rho & 0 & p \end{bmatrix} \cdot \mu^{-1} \frac{-\rho \lambda}{\sigma} 0 \]

for Rayleigh waves and

\[\mathbf{A} = \begin{bmatrix} 0 & \mu^{-1} \\ p\mu^{-1} - \rho & 0 \end{bmatrix} \]

for Love waves. Note that \(p\) is the inverse phase velocity or horizontal slowness, i.e. \(p = c^{-1} = k/\omega\). We require solutions to equation (6) that satisfy the boundary conditions (i.e. that \(b \rightarrow 0\) as \(z \rightarrow \infty\), \(b_z = b_{z4} = 0\) at the free surface for Rayleigh waves and \(b_z = 0\) at the free surface for Love waves, and that \(b\) is everywhere continuous except at a fluid/solid interface where \(b_2\) may be discontinuous to accommodate slip). Efficient algorithms for integrating equation (6) exist if the structure is composed of homogeneous layers. We can then write (Woodhouse 1980)

\[\mathbf{A} = \mathbf{RAR}^{-1}.\]

For Rayleigh waves in a solid this is

\[\mathbf{R} = \rho^{-1/2} \begin{bmatrix} r_1 & 0 & 0 & r_2 \\ 0 & r_3 & r_1 & 0 \\ 0 & r_4 & r_2 & 0 \\ r_5 & 0 & 0 & r_4 \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} 0 & q_o & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & q_p \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad r_1 = 1, \quad r_2 = 2\mu p, \quad r_3 = p, \quad r_4 = (2\mu p^2 - \rho), \quad r_5 = 1 - r_1 r_4 = 1
\]

\[q_o = p^2 - \frac{\rho}{\sigma} q_p = p^2 - \frac{\rho}{\mu}.\]

In a fluid

\[\mathbf{R} = \begin{bmatrix} \rho^{-1/2} & 0 \\ 0 & -\rho^{-1/2} \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} 0 & q_o \\ 1 & 0 \end{bmatrix} \]

and for Love waves

\[\mathbf{R} = \begin{bmatrix} 0 & \mu^{-1/2} \\ \mu^{-1/2} & 0 \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} 0 & q_o \\ 1 & 0 \end{bmatrix} \]

To extend the validity of the solutions, we apply Earth-flattening transformations to account for the sphericity of the Earth (Biswas & Knopoff 1970; Biswas 1972) and for physical dispersion by assuming weak, frequency-independent attenuation (Liu, Anderson & Kanamori 1976). All models are calculated for a reference frequency of 0.1 Hz. Fig. 1 shows a comparison of four higher mode Rayleigh wave phase velocity curves calculated for a flattened Earth, together with the appropriate normal modes calculated for a spherical Earth. The approximation
Figure 1. Theoretical Rayleigh wave phase velocity curves for the first four higher modes are shown by the solid curves and diamonds represent normal modes. A modified form of model 1066B is used in both calculations and the Earth-flattening transformation of Biswas (1972) is applied before computing the Rayleigh wave curves. Gravitational terms are neglected and an infinite $Q$ is assumed in both. The normal mode branch at a phase velocity of $\approx 8.4$ km s$^{-1}$ corresponds to the Stonely mode at the core–mantle interface (absent in the flat Earth model).

appears to break down near 200 s period [Alterman, Jarosch & Pekeris 1961 make similar observations]. Fig. 2 illustrates that the Earth-flattening transformation produces fundamental-mode Rayleigh wave Green's functions that are almost indistinguishable from normal mode Green's functions calculated for a spherical Earth. Similar comparisons demonstrate that the flat Earth approximation produces accurate results out to distances of 80° for fundamental-mode seismograms at frequencies higher than 0.01 Hz.

We write the solution to equation (6) in terms of a propagator matrix (Gilbert & Backus 1966), i.e.

$$\mathbf{b}(z_2) = \mathbf{P}(z_2, z_1) \mathbf{b}(z_1)$$

and from equation (9)

$$\mathbf{P}(z_2, z_1) = \mathbf{R} \exp (\omega \Delta h) \mathbf{R}^{-1},$$

where

$$\exp (\omega \Delta h) = \begin{bmatrix} C_\alpha & q_\alpha S_\alpha & 0 & 0 \\ S_\alpha & C_\alpha & 0 & 0 \\ 0 & 0 & C_\beta & q_\beta S_\beta \\ 0 & 0 & S_\beta & C_\beta \end{bmatrix}$$

and

$$C_\alpha = \frac{1}{2} \left[ \exp (\omega h \sqrt{q_\alpha}) + \exp (-\omega h \sqrt{q_\alpha}) \right],$$

$$S_\alpha = \frac{1}{2 \sqrt{q_\alpha}} \left[ \exp (\omega h \sqrt{q_\alpha}) - \exp (-\omega h \sqrt{q_\alpha}) \right], \quad h = z_2 - z_1.$$

Thus, when $q_\alpha$ is positive there is exponential behaviour and when $q_\alpha$ is negative the behaviour is oscillatory. Substitution
of α with β gives the terms $C_\mu$, $S_\mu$, and $q_\mu$. For Love waves

$$\exp(i\omega \Delta t) = \begin{pmatrix} C_\mu & q_\mu S_\mu \\ S_\mu & C_\mu \end{pmatrix}.$$ 

It is well known that straightforward propagation of Rayleigh wave stress-displacement vectors can result in poorly determined phase velocities at sufficiently high frequencies (Knopoff 1964; Dunkin 1965; Schwab & Knopoff 1972; Abo-Zena 1979). The method of minors (Gilbert & Backus 1966; Abo-Zena 1979; Menke 1979) is used to remove this numerical problem. We follow the notation of Woodhouse (1980) in which the minor of two vectors $b_1$ and $b_2$, is formed by taking the six possible combinations (non-zero) of the elements of each vector:

$$m = [b_1, b_2] = (b_1 b_2 - b_2 b_1)^T$$

so that

$$m = [b_1, b_2] = \begin{pmatrix} b_1 b_2 - b_2 b_1 \\ b_1 b_2 - b_2 b_1 \\ b_1 b_2 - b_2 b_1 \\ b_1 b_2 - b_2 b_1 \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{pmatrix}.$$ 

The minor of a matrix can also be generated in a similar manner so that the propagator matrix for the minor vector is straightforward to calculate (Woodhouse 1980). An efficient form for the propagator of $m$ is given in Appendix B. Two useful identities are

$$m_1(z) + m_2(z) = 0$$

and

$$m_1(z) m_4(z) + m_3(z) m_5(z) - m_2(z) m_6(z) = 0.$$  \hspace{1cm} (10)

The first one reduces the size of the system ($m_2 = -m_1$) and the second is useful in finding the eigenvalues of the system (see below). It is easily shown that the ellipticity is

$$m_2(0)$$

and that the boundary condition that the stresses vanish at the free surface $z = 0$, is

$$m_6 = b_1 b_2^2 - b_2 b_1^2 = 0.$$  \hspace{1cm} (11)

An initial value of $m$ at the half-space depth $z = Z$, is determined by the boundary conditions for $b(Z)$ and $m$ is propagated to the surface. This is a much more stable way to solve equation (6) since it eliminates the need to difference the elements of $b^1$ and $b^2$ in equation (11) (which can result in a significant loss of accuracy). The number of computations is also reduced as a single fifth-order system is propagated instead of two fourth-order systems.

$b(z)$ is recovered from $m(z)$ using a technique suggested by results of Woodhouse (1980). We specify a continuous

\[ Figure 2. \] Fundamental-mode synthetic seismogram Green's functions calculated for a modified form of model 1066B. The vertical component is on the left, the north-south (approximately transverse) component is on the right. The source depth is 200 km and the source-receiver distance and azimuth are 2886 km and 114° respectively. Solid lines are computed as a Rayleigh surface wave in a flat earth, using the Earth-flattening transformation of Biswas (1972). The dashed lines are calculated according to a normal mode formulation in a spherical earth. An infinite Q is assumed and gravitational terms are neglected in both cases. Note the good agreement between the surface wave and normal mode Green's functions; the computation required for the latter is orders of magnitude greater.
solution \( y(z) \), that does not satisfy the boundary conditions but is a solution to
\[
\frac{dy(z)}{dz} = \omega \mathbf{A}(z) y(z)
\]
by propagating a solution downwards from the free surface with the starting solution \( y(0) = [1, 0, 0, 0]^T \). A matrix \( \mathbf{N}(z) \) is formed from the minor vector at each interface depth as
\[
\mathbf{N} = \begin{bmatrix}
-m_2 & -m_3 & 0 & m_1 \\
-m_4 & -m_5 & -m_1 & 0 \\
0 & -m_6 & -m_2 & -m_4 \\
m_6 & 0 & -m_3 & -m_5
\end{bmatrix}
\]
which has the property that
\[
\mathbf{N} \mathbf{b} = 0,
\]
where \( \mathbf{b} \) is a solution that satisfies the boundary conditions and also
\[
\mathbf{N} \mathbf{N} = 0.
\]
If we now let
\[
\mathbf{b} = \mathbf{N} \mathbf{y}
\]
it follows that \( \mathbf{b} \) is a solution which does satisfy the boundary conditions and is the required stress-displacement vector. This method of construction of \( \mathbf{b} \) is found to be numerically stable in nearly all cases (with the exception of structures that have strong internal wave guides).

An important enhancement to the algorithm as described above has been made by Woodhouse (1981). This is the ability to count the number of mode branches between two phase velocities at a fixed frequency. The behaviour of Love waves is governed by a Sturm–Liouville equation which has the property that the zeroes of \( \sin \omega z \) do not satisfy the boundary conditions. The radial component Love wave may be obtained by multiplying equation (13a) by the ellipticity for each mode and \( \mathbf{e}(\mathbf{s}, \omega) \).

For vertical component Rayleigh waves
\[
S = \left[ p b_2(z) s_1 + \left( \frac{b_2(z) + p \lambda(z) b_2(z)}{\sigma(z)} \right) s_2 \right] + i \frac{b_2(z)}{\mu(z)} s_3 \times \frac{p b_1(z)}{4l_4} \sqrt{\frac{2}{nk \tau}}.
\]
where \( p = \sin 2 \phi + M_{xx} \sin 2 \phi + M_{yy} \sin 2 \phi + M_{xy} \cos 2 \phi \),
\[
s_1 = M_{xx} \sin 2 \phi + M_{yy} \sin 2 \phi + M_{xy} \cos 2 \phi
\]
\[
s_2 = M_{xx} \sin \phi - M_{yy} \cos \phi
\]

The complete expressions for \( \mathbf{u}_n(\omega) \) used in the computation of our synthetic seismograms are of the form
\[
S_n \mathbf{e}^{-\gamma_n \mathbf{e}^{-ikr(\cos \theta + \pi/2)}}.
\]
For vertical component Rayleigh waves
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S = \left[ p b_2(z) s_1 + \left( \frac{b_2(z) + p \lambda(z) b_2(z)}{\sigma(z)} \right) s_2 \right] + i \frac{b_2(z)}{\mu(z)} s_3 \times \frac{p b_1(z)}{4l_4} \sqrt{\frac{2}{nk \tau}}.
\]

The source term can also be written in terms of a moment tensor \( \mathbf{M} \). We only consider point sources so that \( \mathbf{M} \) has six independent elements (specified by a 6-vector \( \mathbf{M} \)) and the displacement spectrum becomes
\[
\mathbf{u}_n(z, t, \Phi, \omega) = \sum_{j=1}^{6} \mathbf{M}_n \mathbf{G}_{n,j}(z, t, \Phi, \omega, 0, 0, 0, \omega)
\]
where \( \mathbf{G}_{n,j}(z, t, \Phi, \omega, 0, 0, 0, \omega) \) is the \( n \)th modal Green’s function associated with the \( \mathbf{M}_j \)th element of the moment tensor (Mendiguren 1977; Aki & Richards 1980). Further simplification is achieved through the use of the asymptotic form of the Bessel functions of the surface harmonics corresponding to outgoing plane waves (chapter 9, Abramowitz and Stegun 1972).

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\]
where \( t_i \) is a time point, \( \frac{\partial u_i(t)}{\partial \ln m_k} \) is the differential seismogram for the \( i \)th difference seismogram and the \( k \)th model parameter. In addition to allowing several seismograms to be modelled simultaneously, other types of data and beliefs about the plausible range of model parameters can easily be included as constraints on the solution.

We solve the above system of equations and any additional constraints with a singular value decomposition (SVD) and stabilize the solution by using a truncated set of singular values. The solution, \( \left( \frac{\partial m_k}{\partial \ln m_k} \right)_{k=1, \ldots, K} \), is added to the starting model and the process is iterated with the new model. A per cent variance reduction, the difference and differential seismograms calculated for singular values. The solution, \( \left( \frac{\partial m_k}{\partial \ln m_k} \right)_{k=1, \ldots, K} \), is calculated as a measure of goodness of fit after each iteration and the entire process is iterated until the fit no longer improves.

The procedure for the computation of differential seismogram kernels is illustrated by referring to equation (12). Small changes in model parameters result in a small change in the modal spectrum \( \Delta u \) where, to first order,

\[
\Delta u_n = \sum_{n=1}^{N} \left[ -\Delta S_n (\delta \gamma_n + i \delta k_n) r + \delta S_n \right] e^{-\gamma r e^{-ikr e^{i(w \nu + \pi/4)}}},
\]

(15)

where \( \Delta S_n \) and \( \delta k_n \) are related to perturbations to a \( K \)-layered model by

\[
\delta S_n = \sum_{k=1}^{K} \frac{\partial \ln c_n}{\partial \ln m_k} \frac{\partial m_k}{m_k}
\]

\[
\delta k_n = -k_n \sum_{k=1}^{k} \frac{\partial \ln c_n}{\partial \ln m_k} \frac{\partial m_k}{m_k}
\]

and \( c_n \) is the phase velocity of the \( n \)th mode. These are substituted into equation (15), the order of summation is reversed and Fourier transforming each of the \( K \) modal sums produces the differential seismograms. The excitation partial derivatives, \( \frac{\partial S_n}{\partial \ln m_k} \), are calculated numerically and computation of the phase velocity derivatives, \( \frac{\partial \ln c_n}{\partial \ln m_k} \), is described in Appendix A.

The multiplication of the terms with \( (\delta \gamma + i \delta k) \) in equation (15) by the distance \( r \) makes them significantly larger than the terms in \( \Delta S \) which may then be omitted with a negligible loss of accuracy. This makes the inversion procedure much more computationally efficient. Fig. 3 shows a comparison of differential seismograms for the model parameters density, compressional velocity and shear velocity calculated with and without the excitation perturbation terms. The greatest differences are in the amplitudes. However, these differences are far smaller than amplitude differences which result from uncertainties in source mechanism, focal depth, and mutipathing.

The expressions for differential seismograms with respect to the source colatitude \( \Theta_s \), a longitude term \( \Psi_s = \Phi_s \sin \Theta_s \) where \( \Phi_s \) is the longitude and with respect to the event origin time \( t_0 \), are obtained by differentiating the seismogram spectrum with respect to \( r, \Phi \) and \( t_0 \). These are

\[
\frac{\partial u}{\partial \Theta_s} = \left( \frac{\partial u}{\partial r} \right) \left( \frac{\partial r}{\partial \Theta_s} \right) + \left( \frac{\partial u}{\partial \Phi} \right) \left( \frac{\partial \Phi}{\partial \Theta_s} \right)
\]

\[
\frac{\partial u}{\partial \Psi_s} = \left( \frac{\partial u}{\partial r} \right) \left( \frac{\partial r}{\partial \Psi_s} \right) + \left( \frac{\partial u}{\partial \Phi} \right) \left( \frac{\partial \Phi}{\partial \Psi_s} \right)
\]

\[
\frac{\partial u}{\partial t_0} = -i \omega u,
\]

where

\[
\frac{\partial u}{\partial r} = \left( \gamma - \frac{1}{2} kr \right) u
\]

\[
\frac{\partial S}{\partial \Phi} = \frac{\partial \Phi}{\partial \Phi} e^{-\gamma - i kr e^{i(\omega t_0 + \pi/4)}}.
\]

The geometric terms relating \( \Theta_s \) and \( \Psi_s \) to \( r \) and \( \Phi \) are

\[
\frac{\partial r}{\partial \Theta_s} = -a \cos \Phi
\]

\[
\frac{\partial r}{\partial \Psi_s} = -a \sin \Phi
\]

\[
\frac{\partial \Phi}{\partial \Theta_s} = \sin \Phi \cot \left( \frac{T}{a} \right)
\]

\[
\frac{\partial \Phi}{\partial \Psi_s} = -\left[ \cos \Phi \cot \left( \frac{T}{a} \right) + \cot \Theta_s \right]
\]

where \( a \) is the radius of the earth.

The locked-mode, travelling wave representation of the seismogram and plane layer model parameterization can be modified so that wave propagation in laterally varying structures may be modelled. This is possible when the scale of heterogeneity is large with respect to wavelength \( \lambda \) or equivalently when the medium heterogeneity is slowly varying so that the change in structure is significant only after passage of many cycles of the wave. A local mode asymptotic representation is then appropriate (Bretherton 1968; Woodhouse 1974) and the elements of the stress-displacement vector \( b \) and the wavenumber \( k = \frac{2 \pi}{\lambda} \), may be considered as slowly varying functions of position. (We assume that the heterogeneity is only along the \( \hat{r} \) direction.) The modal spectrum, equation (12), becomes

\[
u_n = S_n(r) \exp \left[ -i \int_0^\gamma \gamma_n(r') \, dr' \right] \exp \left[ -i \int_0^k k_n(r') \, dr' \right],
\]

(16)

where the excitation \( S_n \), attenuation \( \gamma_n \), and wavenumber \( k_n \), are now slowly varying functions of position and \( r \) is the total path length. Different source and receiver structures can be accommodated by computing the terms containing the stress-displacement vector elements in equation (13) separately for the source and receiver structures as if they were locally homogeneous.

**MODELLING OF MEXICAN DATA**

We use the differential seismogram technique to model 12 sets of three-component, long-period, digital seismograms of
Figure 3. (a) Examples of vertical (centre) and north-south (approximately transverse, right) component differential seismograms with respect to the shear velocity model (left) are shown in this figure. The numbers on the left of each seismogram correspond to the appropriate model layer and the asterisk at 20 km depth indicates the source depth. All differential seismograms contain six modes, are computed for a strike of 300°, a dip of 15°, a slip of 270° (thrust) and each is normalized to the peak amplitude of all three sets of seismograms. The solid lines are calculated including excitation perturbations and the dashed lines are calculated neglecting these. The dashed lines are barely visible, illustrating the negligible contribution of the excitation perturbations. Note also that the relative amplitudes indicate the insensitivity of the seismogram to density and compressional velocity below the lower crust. (b) As in (a) but for compressional velocity. (c) As in (a) but for density.
southern Mexican earthquakes recorded in northern Mexico. We attempt to determine models of the structure along each of the propagation paths shown in Fig. 4a. The model variables include shear velocity, compressional velocity, and density. A single Q structure, appropriate for regions of the western US (Solomon 1972; Patton & Taylor 1984; Cheng & Mitchell 1981), is used in all synthetics. Our experience using different Q structures demonstrates that our results are insensitive to the details of the attenuation model. Information about source characteristics is given in Table 1. Events are hereafter identified by the number assigned to them in Table 1. Fig. 4b shows four pairs of seismograms recorded at stations DUR and MON. The higher mode amplitudes, with respect to the fundamental mode, are systematically larger at DUR which suggests to us that structural variations should be resolvable with these data.

An important assumption in our analysis technique is that...
strong heterogeneities are not present. Effects of strong heterogeneities might include mode conversion, conversion of Rayleigh to Love energy (and vice versa) when the heterogeneity is transverse to the propagation direction, and scattering (Kennett 1972, 1984; Aki & Richards 1980). We can identify two likely sources of significant heterogeneity in the study region. The first is the east-west trending Trans-Mexican Volcanic Belt (TMVB) which cuts across the Cordilleran fold-thrust belt of the Central Plateau and Sierra Madre Oriental provinces and forms the boundary between the Sierra Madre Occidental and Sierra Madre Del Sur (see Fig. 4). The TMVB is a complex structure as well as having distinctly different geological and geophysical characteristics from the regions on either side; it is characterized by volcanism, numerous geothermal fields and high heat flow (Ziagos, Blackwell & Mooser 1985; Lubimova & Prol 1979). Molnar & Oliver (1969) report that the phase $S_n$ is attenuated or stopped upon incidence at the TMVB.
Figure 4. (a) Propagation paths for the events analysed in this study. Coordinates, focal depths and source mechanisms are found in Table 1. Numbers next to each event (Julian day of the event) serve to identify each event. Recording stations CHI, MON and DUR are indicated by the squares. Tectonic regions of Mexico are also labelled according to maps of Guzman & de Cserna (1971), Nixon (1982), Atwater (1970) and Dickinson (1976). (b) Examples of vertical component seismograms recorded at stations MON and DUR. Amplitudes are in digital counts and each pair is plotted on the same time-scale. Note the larger higher mode amplitudes for paths to DUR with respect to paths to MON, illustrating that resolvable, systematic differences exist between seismograms corresponding to westerly paths to DUR and more easterly paths to MON.
The width of the TMVB is approximately 200 km and typical wavelengths are about 70 km so that the scale of the heterogeneity may be sufficiently large to consider the structure as a slowly varying medium and the local mode representation is appropriate. To reduce the number of unknowns in our modelling, we can reasonably assert that the receiver structure is known from our previous study (see Paper I). Unfortunately, little is known about the structure to the south of the TMVB. We postulate that the source and receiver structures are the same and that any structural variation is concentrated in the vicinity of the TMVB. This may be justified as the 40.5 km-thick crust found at the receiver is in agreement with the refraction study of Valdes et al. (1986) in which they determine that the crustal thickness at the southeastern edge of the TMVB is 41–49 km and thins towards the west. The distinct difference in the geophysical and geological characteristics of the TMVB with respect to those on either side and the apparent continuity in the Cordillera across the TMVB also suggest that it may disrupt an otherwise continuous structure. An alternative approach to treating the TMVB as a separate entity would be to determine homogeneous models that best represent the average structure along the entire path. However, use of the local mode representation allows us to preserve the assumed source and receiver structures which is more consistent with our assertion that the source structure is more similar to that near the receiver than to that in the volcanic zone. The modal spectra are calculated according to equation (16) as

\[ u_n = S_n e^{-\gamma_n r} e^{-ik_n\phi} e^{-\gamma_{nr} z} e^{-ik_{nr} r}, \]

where \( r_2 \) is the width of the TMVB for the particular path, \( \gamma_n \) and \( k_n \) are calculated for a TMVB structure, \( r_1 = r - r_2 \), and \( S_n, \gamma_n \) and \( k_n \) are calculated for the structure outside the TMVB.

The second source of large-scale heterogeneity is the Cocos and Rivera plates which are being subducted along the southwestern coast of Mexico. The dip of the subducted slab north of about 14° is 10–20° and is slightly steeper to the south (LeFevre & McNally 1985; Valdes et al. 1986). Therefore, its upper surface is at a depth of approximately 40 km at a distance of 110 km from the trench. For most of the events studied, the fraction of the propagation path that samples structure within 100 km from the trench is about 10 per cent. Thus, the presence of the subducted plate should not complicate the propagation of waves travelling in the crust or the source excitation occurring at shallow focal depths. We anticipate that this will limit our ability to model seismograms of deeper events which undoubtedly occur within the subducted plate.

Uncertainties in source mechanism, focal depth, and epicentre must also be considered. Experimentation with synthetic seismograms demonstrates that the seismogram phase is quite stable with respect to source mechanism and focal depth errors, even when the amplitudes are poorly predicted. We employ an iterative procedure to recover the moment tensor elements from observations of body waves recorded globally on the Global Digital Seismic Network (GDSN). The dominant frequencies in these data are sufficiently high that focal depths are not a significant source of uncertainty. This is illustrated in Fig. 5 which shows a comparison of observed body wave waveforms (a subset of those used for this event) together with theoretical waveforms corresponding to a solution determined with our algorithm and the Harvard centroid moment tensor solution (CMT) (derived from data in a lower frequency band; Dziewonski, Chou & Woodhouse 1981; centre and right columns). The source depth is the same in both of these solutions and they result in equal variance reductions. However, an improved variance reduction is achieved for a third set of waveforms which are calculated for a shallower depth (left column in Fig. 5).

The source retrieval procedure is summarized in the following steps. (i) Individual body wave arrivals are windowed out of the seismograms. (ii) Green's functions are initially calculated for the focal depth published in the National Earthquake Information Service (NEIS) monthly bulletin. Although spherically symmetric earth models are used to compare WKBJ Green's functions (Chapman 1978), the algorithm accounts for local perturbations in Earth structure by permitting interactive alignment of synthetic and observed waveforms when the main source of misfit is corrected by a simple time shift. [Ward (1980) and Doornbos (1985) give similar algorithms.] These pointed-source Green's functions are convolved with the instrumental impulse response and attenuation is included using an absorption band operator (Doornbos 1985). (iii) The least-squares solution is found using an SVD and an average

**Table 1.** Epicentral parameters

<table>
<thead>
<tr>
<th>Location</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Date (Day)</th>
<th>Depth (km)</th>
<th>( m_s ), ( M_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Guerrero</td>
<td>16.783N</td>
<td>98.420W</td>
<td>7/2/84 (184)</td>
<td>20</td>
<td>5.8, 6.1</td>
</tr>
<tr>
<td>2) Guerrero</td>
<td>17.215N</td>
<td>99.993W</td>
<td>7/14/84 (196)</td>
<td>45</td>
<td>5.3</td>
</tr>
<tr>
<td>3) Offshore</td>
<td>14.386N</td>
<td>92.224W*</td>
<td>8/3/84 (216)</td>
<td>20</td>
<td>5.3, 5.5</td>
</tr>
<tr>
<td>4) Oaxaca</td>
<td>15.868N</td>
<td>95.246W</td>
<td>8/26/84 (239)</td>
<td>20</td>
<td>5.4</td>
</tr>
<tr>
<td>5) Chiapas</td>
<td>16.080N</td>
<td>93.286W</td>
<td>8/31/84 (244)</td>
<td>110</td>
<td>5.3</td>
</tr>
<tr>
<td>6) Offshore</td>
<td>15.024N</td>
<td>94.304W*</td>
<td>10/13/84 (287)</td>
<td>12</td>
<td>6.1, 5.7</td>
</tr>
<tr>
<td>7) Guerrero</td>
<td>16.276N</td>
<td>98.352W</td>
<td>11/30/84 (335)</td>
<td>20</td>
<td>5.4, 5.4</td>
</tr>
<tr>
<td>8) Oaxaca</td>
<td>16.324N</td>
<td>96.525W</td>
<td>12/13/84 (348)</td>
<td>20</td>
<td>5.4, 5.0</td>
</tr>
<tr>
<td>9) Oaxaca</td>
<td>18.020N</td>
<td>97.199W</td>
<td>9/15/85 (258)</td>
<td>65</td>
<td>5.9</td>
</tr>
<tr>
<td>10) Michoacan</td>
<td>18.058N</td>
<td>103.097W</td>
<td>9/25/85 (268)</td>
<td>20</td>
<td>5.3, 6.1</td>
</tr>
<tr>
<td>11) Guerrero</td>
<td>17.344N</td>
<td>97.199W</td>
<td>9/28/85 (271)</td>
<td>33</td>
<td>5.0, 5.0</td>
</tr>
<tr>
<td>12) Michoacan</td>
<td>18.138N</td>
<td>102.555W</td>
<td>10/29/85 (302)</td>
<td>20</td>
<td>5.5</td>
</tr>
</tbody>
</table>

* indicates NEIS location, all others are our relocations.
variance reduction between synthetic data calculated for the
moment tensor solution and the observations is calculated.
(iv) The process is iterative as the resulting solutions depend
on the time shifts. We determine the source depths by
finding solutions for various depths until a minimum
variance reduction is achieved. All the source mechanisms
determined are given in Table 2. CMT solutions or
mechanisms consistent with the regional tectonics are
assumed for the smaller events that are not adequately
recorded on the GDSN network.

Epicentral uncertainties are a major limiting factor in
what can be learned from these data. Locations determined

Figure 5. Examples of observed waveforms (solid lines) recorded on the GDSN network together with theoretical waveforms (dashed lines) corresponding to different focal mechanisms and depths. The mechanism and depth used for the synthetics on the right is the Harvard CMT solution and the others are best-fitting solutions determined in this study. (Only a subset of the total data set for this event is shown.) The depths, fault parameters, scalar moments (dyne-cm) and variance reductions (calculated for the entire dataset) corresponding to the theoretical waveforms in each column are listed at the top. Station codes, components and phase types are indicated below each set of waveforms. The nominal conversions from counts to ground displacements are 5000 counts μm⁻¹ for SNZO, 10000 counts μm⁻¹ for KONO, and 7150 counts μm⁻¹ for RSNT. The narrower pulse widths and larger variance reductions for a focal depth of 20 km illustrate the improved depth resolution obtained with this method with respect to that of the CMT solution.
Table 2. Source mechanisms

<table>
<thead>
<tr>
<th>Location (Day)</th>
<th>Strike</th>
<th>Dip</th>
<th>Slip</th>
<th>$m_o \times 10^{24}$ (dyne-cm)</th>
<th>Type/Data Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Guerrero (184)</td>
<td>307°</td>
<td>20°</td>
<td>276°</td>
<td>24.5</td>
<td>Thrust, GDSN</td>
</tr>
<tr>
<td>2) Guerrero (196)</td>
<td>316°</td>
<td>30°</td>
<td>104°</td>
<td>1.9</td>
<td>normal, GDSN</td>
</tr>
<tr>
<td>3) Offshore (216)</td>
<td>281°</td>
<td>15°</td>
<td>290°</td>
<td>4.9</td>
<td>thrust, GDSN</td>
</tr>
<tr>
<td>Chiapas</td>
<td>4) Oaxaca (239)</td>
<td>331°</td>
<td>36°</td>
<td>107°</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td>5) Chiapas (244)</td>
<td>313°</td>
<td>84°</td>
<td>103°</td>
<td>28.0</td>
</tr>
<tr>
<td></td>
<td>6) Offshore (287)</td>
<td>289°</td>
<td>23°</td>
<td>276°</td>
<td>8.8</td>
</tr>
<tr>
<td></td>
<td>7) Guerrero (335)</td>
<td>293°</td>
<td>34°</td>
<td>292°</td>
<td>2.9</td>
</tr>
<tr>
<td></td>
<td>8) Oaxaca (348)</td>
<td>300°</td>
<td>15°</td>
<td>270°</td>
<td>3.0</td>
</tr>
<tr>
<td></td>
<td>9) Oaxaca (288)</td>
<td>315°</td>
<td>40°</td>
<td>82°</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>10) Michoacan (268)</td>
<td>300°</td>
<td>15°</td>
<td>270°</td>
<td>5.5</td>
</tr>
<tr>
<td></td>
<td>12) Michoacan (302)</td>
<td>300°</td>
<td>15°</td>
<td>270°</td>
<td>25.0</td>
</tr>
</tbody>
</table>

The two locations of event 268 are determined independently by investigators at the University of California at Santa Cruz (UCSC, Christian Sholte, private communication) and by investigators at the National University of Mexico City (UNAM, Krishna Singh, personal communication). The

![Earthquake Locations](image)

**Figure 6.** Locations of the earthquakes recorded on the northern Mexican array determined by NEIS, RESMAC, in the CMT solutions and two relocations of event 268 based on local data collected by investigators at UCSC and UNAM. Also shown are the relocations determined in this study. The crossed lines in the lower left corner each represent approximately 100 km distance.
Figure 7. Vertical component seismograms (solid lines) recorded at DUR, MON and CHI plotted with synthetic seismograms (dashed lines) calculated for the reference Plateau model. Two modes are included since the fundamental and first higher modes make the most significant contribution to these seismograms. Synthetics on the left are computed assuming the epicentre determined by UNAM and those on the right are for a relocation determined with these data and differential seismograms computed for the Plateau model. The relocated epicentre is shown in Fig. 6 by the diamond to the west of the UNAM location. The seismograms are all normalized to have the same peak amplitudes in the plot and the per cent variance reduction is given in the lower left corner of each seismogram/synthetic pair (as in subsequent figures).

The change in the relocated epicentre of only 10 km with respect to the UNAM location significantly improves the average variance reduction (right column of Fig. 7). This clearly illustrates the ambiguity in attributing misfit to incorrect structure or to source location error.

Faced with this trade-off and a lack of independent constraints on the structure of southern Mexico, the following modelling strategy is employed. As much misfit as is reasonable, judged according to the results of Lermo & Singh (1985) and Eissler & McNally (1983), is first absorbed...
Homogeneous Plateau Model Synthetics – Event 268

![Seismograms of Event 268](image)

**Figure 8.** Vertical and horizontal component seismograms (solid lines) of event 268 recorded at all three stations. A homogeneous model (the Plateau model) and relocation described in Fig. 7 are used for the synthetics (dashed lines, two modes). Note the misfit of the lower frequency energy arriving just prior to the Airy phases. (The north–south component at CHI was not working at the time of the event.)

by relocating the event assuming the reference Plateau structure is appropriate for the entire region between source and receiver. The remaining misfit after relocation is then attributed to structural variations within the TMVB region.

The data for the eight components that recorded event 268 are shown in Fig. 8 together with synthetics computed with the relocated epicentre. Since the relocation procedure is most sensitive to the portion of the signal with the greatest amplitude, the fundamental mode Airy phases are reasonably well fit. However the earlier arriving, low-frequency energy is systematically misfit. A similar misfit is also observed when the same procedure is applied to seismograms of other events. The synthetic pairs shown in the centre of Fig. 9 are determined assuming this misfit is due to structural heterogeneity that is confined to a zone of width 161 km which is the width of the TMVB as it is expressed at the surface according to the map of Nixon (1982). (The widths may be varied for each propagation path and only coincidentally are all the same for this event.)

The Plateau velocity model is shown in Fig. 10 by the solid line with the shorter dashed line indicating the minimum difference necessary to fit the data assuming a TMVB width of 161 km. An alternative hypothesis is that the heterogeneity spans a greater width at depth (twice the surficial width is chosen for purposes of illustration). The longer dashed line in Fig. 10 is the minimum change in the Plateau model necessary under this assumption. The corresponding synthetic/data pairs are shown on the right of Fig. 9. Though it is not possible to distinguish between these hypotheses from these data alone, both models of the TMVB have very low shear velocities (3.3–4.3 km s\(^{-1}\)) in the lower lithosphere. This suggests to us that the existence of low shear velocities in the lower lithosphere is a real feature of the structure of southern Mexico.

An alternative hypothesis is that the misfit in the low frequencies is due entirely to source location error. This hypothesis is tested by performing a relocation with the same data after low-pass filtering the seismograms. Though the data can be fitted to approximately the same precision, the required change in location of 25 km is greater than the likely uncertainty in the UNAM or UCSC locations. The resulting model has very high surface velocities which is contrary to what one would intuitively expect in a volcanic zone. Similar results are obtained for other seismograms, further supporting the hypothesis that the low-frequency misfit is due to structural variations in the lower lithosphere.
Figure 9. Seismograms (solid lines) of event 268 recorded at DUR on three components. The synthetics in the left column are computed for the same parameters as in Fig. 8 and the arrows indicate the misfit in the low frequencies. Propagation through the TMVB structure is accounted for in the synthetics on the right and centre. The model corresponding to the centre synthetics is derived assuming a TMVB width of 161 km and that on the right is derived assuming it is 322 km wide at depth. Note the improvement in the fit of the low frequencies.

The shapes of the fundamental mode wavetrains provide structural constraints that are independent of epicentral uncertainties. The wavetrain duration of the seismograms of event 184 recorded at DUR (the only station which recorded it, Fig. 11) are longer than those of seismograms of events 268 and 302, suggesting that the low-velocity surface layers are slower and/or thicker along this propagation path. The centre set of synthetic/data pairs are computed for the TMVB model determined from seismograms of events 268 and 302 under the assumption that the heterogeneity spans twice the width of the TMVB measured on the surface (the 'western model' in Fig. 12). The low-frequency misfit in the vertical component is reduced with respect to synthetics computed for the homogeneous Plateau model, but the duration of the fundamental mode wavepacket is too short. The seismograms on the right are computed for a new TMVB model (the 'central model' in Fig. 12) which differs from the western model only in the layers of the upper crust. The wavetrain durations of these synthetics are in better agreement with the data. Though they are still shorter than the observed wavetrains, it is possible that the later arriving energy is scattered so that it is not necessarily meaningful to attempt to model it. These and other waveforms suggest that the sedimentary surface layer is thinner in the west and that a single model of the upper crust cannot fit all the data.

Nine sets of seismograms out of the 12 recorded (Fig. 13) are reasonably well fitted with synthetics computed for two TMVB models, the western and central models. (The western model is used for events 268 and 302, and the central model for the others.) Forty modes are included in the synthetics of Fig. 13 so that all the modes theoretically contributing to S wave arrivals are present. (This is determined by the number of modes at the highest frequency that have phase velocities less than or equal to the shear velocity at the depth of the turning point of an S-wave recorded at the greatest distance.) All relocated epicentres are well within the likely uncertainties of the NEIS locations (see Fig. 6). Though individually these seismograms could be better fitted, not much would be learned considering the large number of degrees of freedom available and that the model parameterization may not be entirely adequate.

Agreement between the synthetics and data within several seconds in the fundamental mode wavetrains suggests that
Figure 10. Compressional velocity, shear velocity and density models of the TMVB used in the computation of the synthetic seismograms shown in Fig. 9. The shorter dashed line is the minimum required difference in the Plateau model attributed to the TMVB assuming it is 161 km wide and the longer dashed line is the difference assuming it spans 322 km.

the lithosphere is adequately represented by our models, particularly as mismatches of several seconds are possible due to errors in source parameters. The fundamental mode amplitudes are much more sensitive than the phase to errors in source parameters, attenuation structure, and lateral heterogeneities (Lerner–Lam 1982) so that the agreement in amplitude to within 30–50 per cent is considered reasonable. P, waves observed on seismograms of the shallow events 268 and 184 (Fig. 13a, c) are predicted almost perfectly, suggesting that our crustal models are correct on average. The small discrepancies in horizontal components may be attributed to their greater sensitivity to uncertainties in source parameters and errors introduced in rotation. The phases of seismograms of the deeper event 258 (Fig. 13d) are also well fitted and the amplitude mismatch of approximately 40 per cent is not surprising since it most likely occurred within the subducted slab which is absent in our models.

The general agreement between the amplitudes and group velocity windows of the synthetic higher mode wavetrains and the data also indicates that our lithosphere structure is reasonable. However, a clear systematic increase in higher mode amplitudes (relative to the fundamental mode) for propagation paths to DUR with respect to those for paths to MON is observed but is not reproduced in our synthetics (see Fig. 1b). It is improbable that seismograms recorded at MON are consistently nodal as the source–receiver azimuths to DUR and MON vary. (This is also verified through forward modelling.) Therefore, it is likely that the observed amplitude differences are due to structural variations. The distribution of shear energy density shows that the energy of both the first higher mode and the fundamental mode is distributed within the lower crust and lower lithosphere. Thus, a change in the attenuation structure alters both modal amplitudes by similar amounts, resulting in little relative change. Structural gradients in the crust or lower lithosphere also have little effect on the relative amplitudes as the vertical wavelengths of both modes are at least several tens of kilometres in magnitude. Tests with synthetics for various Q models and structural gradients corroborate these hypotheses. While these tests are not exhaustive, it seems most probable that the variation in higher mode amplitudes is due primarily to the heterogeneity along each propagation path. Observed amplitude changes of Ls wavepackets have been correlated with structural heterogeneity in other studies (Gregersen, 1984; Kennett & Mykkeltveit 1984; Ruzaikin et al. 1977; Knopoff et al. 1974). Several theoretical studies demonstrate that conversion of Rayleigh to Love wave energy can be attributed to changes in the thickness of the crust and sediments and that scattering of surface waves can be significant in the bandwidth of our data (Gregersen & Alsop 1976; Kennett 1972, 1984; Kennett & Mykkeltveit 1984; Schneider 1986a, b, c). These studies serve to illustrate that local phase perturbations, mode conversion and scattering are theoretically feasible mechanisms and have been reasonably suggested as explanations for similar observations in other regions. We conclude that our modelling technique is inadequate to determine the precise cause of these higher mode variations.
The importance of structural heterogeneity near the source and receiver is apparent in the body waves of these synthetics. As anticipated, the inability to model the subducted slab has the most severe effect for the deeper events. This is clear in the seismograms of event 244 which occurred at a depth of approximately 110 km. The poor fit to the body wave amplitudes is most probably because the source in the synthetics is located in the model shear wave low-velocity zone when in actuality, the earthquake undoubtedly occurred in the high-velocity, subducted slab. However, the predicted amplitudes of the P-waves of the shallow events 268 and 184 (Fig. 13a, c) are in much better agreement. We are unable to model three events (287, 216, 271) which all occurred offshore. It is quite plausible that the oceanic structure at the source and along the propagation paths significantly alters these waveforms.

The body wave traveltimes are predicted within the errors that arise from focal depth uncertainties, indicating that the structure averaged over the entire ray paths is well modelled. In particular, the observed S-wave shadow zone (out to about 13°) is accurately modelled by these synthetics and is indicative of the presence of a shear wave low-velocity zone (lvz) that extends to about 300 km depth. It is also indicative of smooth gradients in the transition region below the lvz and of a lack of a well-formed 400 km discontinuity. The shadow zone cannot be a consequence of a highly attenuating mantle since clear S-waves are observed at distances greater than 13°.

**DISCUSSION**

We conclude that the structure within the recording array in northern Mexico is resolvable from that to the south and that, not surprisingly, the crustal structure is not uniform over all of southern Mexico. Fundamental mode data suggest that the uppermost sedimentary layer thins towards the west. Systematic differences in crustal higher modes also indicate that the structure is variable within the study region and we conclude that the differences are due to processes that we cannot model with the assumed parameterization. Large-amplitude codas following fundamental mode packets in many of the seismograms are also...
indicative of the presence of strong heterogeneities in the crust of southern Mexico. Schneider (1986a, b, c) demonstrates theoretically that significant scattering of fundamental mode surfaces waves at periods of 20 s or less can be caused by structural change such as that inferred for the TMVB or topographic relief of the magnitude found in Mexico. However, despite these limitations, we are able to successfully model many features of the data. This suggests that a more complicated approach is not warranted in order to model the main characteristics of regional seismograms.

Very low shear velocities are found to exist in the lower lithosphere in the region of the TMVB, though precise definition of these velocities is not possible. We also find that epicentral uncertainties result in significant trade-offs between source location and structure. None the less, we can conclude that the shear velocity of the lower lithosphere, averaged over the entire region studied, is approximately 4.3 km s\(^{-1}\). If the low-velocity region is confined to the TMVB then the shear velocities in the lower crust and lithosphere may be as low as 3.3 km s\(^{-1}\). The latter is consistent with the active volcanism and high heat flow (2.19 ± 63 hfu) in the TMVB (Ziagos et al. 1985). We also find that an S-wave shadow zone is indicative of the presence of a mantle shear wave low-velocity zone that extends to about 300 km depth and of smooth gradients at the base of the low-velocity zone and where the 400 km discontinuity usually exists. Similar structures are found in other volcanic arcs and have also been correlated with high heat flow and partial melting in the lithosphere and upper mantle (Jacob & Hamada 1972; Langston 1977, 1981; Blackwell et al. 1982).

CONCLUSIONS

We have described a technique for modelling of multi-mode surface wave seismograms recorded at regional distances and have applied this technique to seismograms with propagation paths in northern and southern Mexico. Many features of these seismograms are well fitted by our synthetics suggesting that our models are plausible representations of the true velocity structure. Results of modelling experiments also suggest that mode conversion, scattering, and a more accurate characterization of the source structure need to be included in the computation of synthetic seismograms if these data are to be modelled in greater detail. Perhaps the most restrictive limitation in what may be learned from these data is the need for source locations that are accurate to within a few kilometres. However, this type of analysis should be a successful method of obtaining detailed models of regional structure when used with seismograms of well-located events.

ACKNOWLEDGMENTS

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Figure 13. (a-g) Seismograms (solid lines) and synthetics (dashed lines) of nine of the 12 events recorded. All seismograms include 40 modes and the Plateau structure is used outside the TMVB (assumed to be twice as wide at depth than its surficial width). The central TMVB model is used for events 268 and 302 and the western model for the remaining events. All events have been relocated assuming these structures. Event numbers correspond to those in Table 1 and source mechanisms are given in Table 2. Source depths and source-receiver distances are listed above the seismograms from each station. Amplitudes are in digital counts and any filters applied to the seismograms and synthetics are listed at the bottom of each figure.
Waveform modelling: the structure of Mexico

Event 302 - depth = 20 km
Distance: 690 km

Event 184 - depth = 20 km
Distance: 1224 km

Event 239 - depth = 20 km
Distance: 1337 km

Figure 13. (continued)
Event 335 - depth=20 km
Distance: 1084 km

Event 348 - depth=20 km
Distance: 1108 km

Figure 13. (continued)
REFERENCES


Waveform modelling: the structure of Mexico


**APPENDIX A: DERIVATIVES AND THE EVALUATION OF LAYER INTEGRALS**

It is well known that a variational principle can be used to compute group velocity, derivatives of phase velocity with respect to model parameters, etc. (e.g. Takeuchi & Saito 1972). Our models consist of homogeneous layers so it is convenient to have analytic formulae for the integrals of the derivatives over a layer. Furthermore, integrals over the whole structure are required to compute the excitation factors of the modes. Simple formulae for these quantities are presented in this Appendix assuming that the stress-displacement vector \( \mathbf{b} \) is available at layer interfaces. Harkrider & Anderson (1966) and Harkrider (1970) have presented some similar formulae but we believe the present results are easier to implement.

**Rayleigh Waves**

Define the following four-layer integrals

\[
J_{1k} = \int_{z_k}^{z_{k+1}} \rho(b_1^2 + b_2^2) dz \\
J_{2k} = \int_{z_k}^{z_{k+1}} \left[ b_3^2 \left( 1 + \frac{\mu}{\rho} \right) - 4\mu p \left( \frac{1}{\sigma} - \frac{\mu}{\rho} \right) b_2^2 \right] dz \\
J_{3k} = \int_{z_k}^{z_{k+1}} b_1 b_4 + 4\mu p \left( 1 - \frac{\mu}{\rho} \right) b_2^2 - \left( 1 - 2\frac{\mu}{\rho} \right) b_2 b_3 \right] dz \\
J_{4k} = \int_{z_k}^{z_{k+1}} \frac{1}{\sigma} \left[ 4p^2 p^2 b_2^2 + 4\mu p b_2 b_3 + b_3^2 \right] dz
\]

where \( z_k \) and \( z_{k+1} \) are the layer interfaces. These integrals can be easily evaluated for the half space by using the known exponential decay form of \( \mathbf{b} \) in that layer. Evaluation in the layers above the half space is a little more complicated. Define the energy integrals as

\[
I_1 = \sum_{k=1}^{K} J_{1k} \quad I_2 = \sum_{k=1}^{K} J_{2k} \quad I_3 = \sum_{k=1}^{K} J_{3k}
\]

where there are \( K - 1 \) layers above the half space. It then follows from the variational principle that

\[
I_1 = I_2
\]

and

\[
U(\omega) = \frac{I_3}{I_1}
\]

where \( U(\omega) \) is the group velocity. The perturbation in phase velocity due to a perturbation in a layer property is given by

\[
\frac{\delta \ln c}{\delta \ln \rho_{k}} = \frac{1}{2\rho I_3} J_{2k} - J_{1k}
\]

Thus, we can compute all the properties of interest by evaluating the four \( J \) integrals. These are most easily evaluated by casting the problem in terms of the vector \( \mathbf{e} \) where

\[
\mathbf{e} = \mathbf{R}^{-1} \mathbf{b}.
\]
equation (A.5) so that the evaluation of equations (A.6) and (A.7) is straightforward.

The equations simplify considerably for Rayleigh waves in a fluid where we explicitly only propagate \( b_1 \) and \( b_2 \). We set

\[
C_1 = h\left[ \frac{\rho b_1^2(z_k) - \frac{g_\alpha}{\mu} b_2^2(z_k)}{\rho} \right]
\]

\[
C_3 = \frac{1}{\omega} [b_1(z_{k+1}) b_3(z_{k+1}) - b_1(z_k) b_3(z_k)]
\]

and all other \( C \)'s in equation (A.6) are set to zero. Equation (A.7) then remains valid though we find that

\[
J_2 = J_4
\]

and

\[
J_1 = J_4 + C_4
\]

in this special case.

**Love Waves**

We refer the reader to the text for the definition of the two-element \( b \) vector for Love waves. If we now define

\[
J_1 = \int_{z_k}^{z_{k+1}} \rho b_1^2 \, dz
\]

\[
J_2 = \int_{z_k}^{z_{k+1}} \left( \frac{b_2^2}{\mu} + \frac{\mu b_1^2}{\mu} \right) \, dz \quad \text{(A.8)}
\]

\[
J_3 = \int_{z_k}^{z_{k+1}} \mu b_1^2 \, dz
\]

we find that equations (A.2–4) still hold if \( J_4 \) is set to zero. We now define

\[
C_1 = h\left[ \frac{\mu q b_2^2(z_k) - \frac{1}{\mu} b_2^2(z_k)}{\rho} \right]
\]

\[
C_2 = \frac{1}{\omega} [b_1(z_{k+1})b_3(z_{k+1}) - b_1(z_k)b_3(z_k)]
\]

then

\[
J_1 = \frac{\rho}{2q_\mu} (C_1 + C_2)
\]

\[
J_2 = \frac{1}{2} (C_2 - C_1) + \frac{p^2}{2q_\mu} (C_1 + C_2) \quad \text{(A.10)}
\]

\[
J_3 = \frac{p}{2q_\mu} (C_1 + C_2).
\]

This completes the calculation of the layer integrals.

**Discontinuity derivatives**

To the best of our knowledge the correct expression for a change in phase velocity when a layer interface is moved has not appeared in the literature though Schneider (1986a) gives the special case for a perturbation in the height of the free surface. The correct expressions are most easily obtained from the equations of Woodhouse (1976) for the spherical Earth case by making use of the limiting process described by Gilbert (1976). In terms of our \( b \) vectors we find

\[
\frac{\partial \ln c}{\partial h_k} = \frac{1}{2pI_3} \left[ (\rho - \mu p^2) b_1^2 + \frac{b_2^2}{\mu} \right]
\]

for Love waves and for Rayleigh waves

\[
\frac{\partial \ln c}{\partial h_k} = \frac{1}{2pI_3} \left[ \rho (b_1^2 + b_2^2) + \frac{b_2^2}{\mu} + \frac{b_4^2}{\mu} \right]
\]

\[
+ 2pb_2 b_3 \left( 1 - \frac{2\mu}{\sigma} \right) - 2pb_1 b_4 - 4\mu p^2 b_2^2 \left( 1 - \frac{\mu}{\sigma} \right)
\]

where \( \delta h_k \) is the perturbation in the depth of the \( k \)th interface measured positive downwards. \([ \cdot ]^+\) means that the expression in brackets should be evaluated at both sides of the interface and the result found by subtracting the value on the shallower side from the value on the deeper side.

**APPENDIX B: THE MINOR VECTOR PROPAGATOR**

The numerical heart of the code to compute locked-mode seismograms is the propagation of the minor vector across a homogeneous layer. The methods of Woodhouse (1980) can be used to construct the propagator but the derivation is tedious so we give the result here for reference. Given a minor vector \( m \), at the bottom of a homogeneous layer the following algorithm results in the minor vector at the top of the layer. Note that we do not propagate \( m_5 \) as \( m_2 = -m_5 \).

\[
x_1 = 2\mu p m_1 - m_2
\]

\[
x_2 = \rho^{-1}[2\mu p(m_2 - x_1) + m_6]
\]

\[
x_3 = px_2 + x_1
\]

\[
x_3 = \mu m_3 - p(x_3 + x_1)
\]

\[
e_1 = C_{\rho} x_4 - q_{\alpha} S_{\alpha} m_3
\]

\[
e_2 = S_{\alpha} x_4 - C_{\rho} m_4
\]

\[
e_3 = C_{\rho} m_4 - q_{\alpha} S_{\alpha} x_2
\]

\[
e_4 = S_{\alpha} m_4 - C_{\rho} x_2
\]

\[
x_5 = S_{\alpha} e_2 + C_{\alpha} e_4
\]

\[
x_6 = S_{\alpha} e_2 - C_{\alpha} e_4
\]

\[
x_7 = S_{\alpha} e_2 + C_{\alpha} e_4
\]

\[
x_8 = S_{\alpha} e_2 - C_{\alpha} e_4
\]

\[
x_9 = C_{\alpha} e_1 + q_{\alpha} S_{\alpha} e_3
\]

\[
m_1 = \rho^{-1}[x_4 + p(x_2 + x_1)]
\]

\[
m_2 = 2\mu p m_1 - x_1
\]

\[
m_3 = -C_{\alpha} e_2 - q_{\alpha} S_{\alpha} e_4
\]

\[
m_4 = S_{\alpha} e_1 + C_{\alpha} e_4
\]

\[
m_5 = \rho x_2 - 2\mu p(m_2 - x_1).
\]

The algorithm is competitive with those given by Woodhouse (1980) and Schwab & Knopoff (1972) but the explicit calculation of the minor vector allows the mode counting algorithm described in the text to be implemented. It is this feature which allows the reliable and rapid calculation of all roots within a particular phase velocity window at a fixed frequency.