Time-dependent earthquake probabilities

J. Gomberg,[1] M. E. Belardinelli,[2] M. Cocco,[3] and P. Reasenberg[4]

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[1] We have attempted to provide a careful examination of a class of approaches for estimating the conditional probability of failure of a single large earthquake, particularly approaches that account for static stress perturbations to tectonic loading as in the approaches of Stein et al. (1997) and Hardebeck (2004). We have developed a general framework based on a simple, generalized rate change formulation and applied it to these two approaches to show how they relate to one another. We also have attempted to show the connection between models of seismicity rate changes applied to (1) populations of independent faults as in background and aftershock seismicity and (2) changes in estimates of the conditional probability of failure of a single fault. In the first application, the notion of failure rate corresponds to successive failures of different members of a population of faults. The latter application requires specification of some probability distribution (density function or PDF) that describes some population of potential recurrence times. This PDF may reflect our imperfect knowledge of when past earthquakes have occurred on a fault (epistemic uncertainty), the true natural variability in failure times, or some combination of both. We suggest two end-member conceptual single-fault models that may explain natural variability in recurrence times and suggest how they might be distinguished observationally. When viewed deterministically, these single-fault patch models differ significantly in their physical attributes, and when faults are immature, they differ in their responses to stress perturbations. Estimates of conditional failure probabilities effectively integrate over a range of possible deterministic fault models, usually with ranges that correspond to mature faults. Thus conditional failure probability estimates usually should not differ significantly for these models.

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1. Introduction

[2] It is widely held that time of occurrence of an earthquake on a fault undergoing tectonic loading is controlled both by the stress and frictional properties on that fault and by earthquakes on other faults nearby [*Stein*, 1999]. The effects of a nearby earthquake are commonly associated with the static and dynamic stress changes it produces, but they may also be related to processes set in motion by those stress changes, such as crustal fluid flow and plastic deformation. Those endeavoring to estimate the probability of an earthquake on a single fault or fault segment must take into account both basic frictional processes, which are associated with quasiperiodic sequences of earthquakes, and interactive effects, which can alter the rhythm of a sequence. One approach to do this was first suggested by *Stein et al.* [1997] and has since been applied in several places (e.g., in Turkey by *Stein et al.* [1997], *Parsons et al.* [2000], and *Parsons* [2004]; in Japan by *Toda et al.* [1998]; in California by *Toda and Stein* [2002]; to global data by *Parsons* [2002]). Most recently, *Hardebeck* [2004] suggested another approach, which shares features of *Stein et al.*'s [1997] approach. Applications of these approaches may affect public policy decisions about earthquake preparedness and business policy decisions, which in turn impact future losses of property and human life. Because of the importance of such applications, we believe that they need to be thoroughly understood by both those applying them and interpreting their results. In this paper we attempt to facilitate that understanding.

[3] A sequence of earthquake recurrences with a mean rate (which may be estimated from geological, geodetic, seismic and other data) is most simply described by a Poisson process, in which the recurrence times are completely random [e.g., *Cornell and Winterstein*, 1988]. For a Poisson model, the probability of recurrence within some time interval Δt is independent of the absolute time. Thus a Poisson model of earthquake recurrence contains no intrinsic time dependence that might be otherwise expected to arise in instances of repeated material failure as a result of continuous tectonic loading. An alternative (but often debated) to the Poisson assumption is to construct a probability

[1]U.S. Geological Survey, Memphis, Tennessee, USA.
[2]Settore di Geofisica, Dipartimento di Fisica, Universita' di Bologna, Bologna, Italy.
[3]Istituto Nazionale di Geofisica e Vulcanologia, Rome, Italy.
[4]U.S. Geological Survey, Menlo Park, California, USA.

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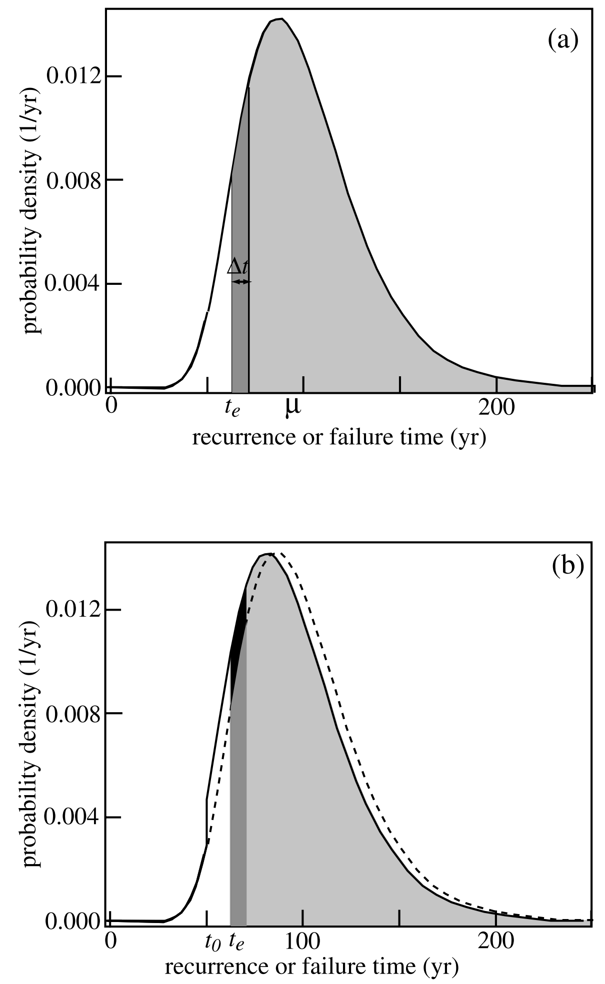

Figure 1. (a) Lognormal probability density distribution (equation (2)) of earthquake recurrence with mean recurrence $m = 100$ years and standard deviation $\sigma = 31.6$ years. Under tectonic loading alone, the conditional probability of an earthquake occurring between t_e and $t_e + \Delta t$ equals the ratio of the darker area over the entire shaded area beneath the curve (equation (1)). (b) The same PDF of Figure 1a (dashed curve) with its mean recurrence time shifted by $-\Delta\tau/\dot{\tau}$ (solid curve) to account for a permanent increase in shear stress imposed at $t_0 = 50$ years [*Working Group on California Earthquake Probabilities*, 1990; *Stein et al.*, 1997; *Toda et al.*, 1998]. The shift increases the area between t_e and $t_e + \Delta t$ (black area) and decreases the total (shaded) area under the PDF (see equation (4)), giving rise to an increase in conditional probability. The stressing rate is $\dot{\tau} = 0.1$ MPa/yr and the change in shear stress is $\Delta\tau = 0.5$ MPa.

model that embodies the historical and geophysical evidence of earthquake recurrence [e.g., see *Matthews et al.*, 2002, and references therein] characterized in terms of a mean rate, an elapse time or equivalently the time since the previous occurrence, the amount of slip in the last earthquake, loading rate, etc. One such approach is the conditional probability model of *Hagiwara* [1974]. This kind of model may additionally incorporate information about interactive processes such as stress transfer from a nearby earthquake.

[4] Specifically, we examine the assumptions and implications of approaches to estimating earthquake occurrence probability based on a simplified rate- and state-dependent fault strength model [*Dieterich*, 1992, 1994], which specifies the sensitivity of failure time to stress change. Two such approaches include the *Stein et al.* [1997] and *Hardebeck* [2004] time-dependent probability models. We provide some new equations that illustrate the similarity between these two probability models and simplify their implementation. We do not calculate earthquake probability in this study, nor do we assess the differences between the models (for such discussion, see *Hardebeck* [2004]). Instead, we attempt to understand the implications of these approaches, particularly with respect to calculating stress-induced time-dependent changes in earthquake rate and probabilities. Toward this end we suggest several conceptual physical models. We also attempt to connect models of seismicity rate change as applied to populations of faults (e.g., as in aftershocks), and to the recurrence of large earthquakes on a single fault. We uses both continuous (analytical) and discrete (numerical) representations of earthquake rate but emphasize that we use discrete populations (e.g., of faults or of nucleation sites or "patches") because they provide a conceptually easier way of understanding the behavior of a continuous distribution. This study builds on ideas presented in the companion paper by *Gomberg et al.* [2005].

2. General Conditional Probability Model

[5] In the absence of a perturbing earthquake, the calculation of conditional probability, $P_c(t_e < T < t_e + \Delta t | t_e < T)$, of an earthquake occurring on a single fault between elapse times t_e and $t_e + \Delta t$ assumes that recurrence times, T, for repeated failure on a single fault are independent, identically distributed random variables having some density function $f(T; \mu, \sigma)$ whose mean and standard deviation are μ and σ, respectively (Figure 1). The specific form of $f(T)$ may be tailored to represent how the fault evolves toward failure so that, for example, probability increases with time to represent stress on a fault increasing toward some failure threshold. In section 4.2 we offer some conceptual models of how these PDFs might arise in terms of measurement uncertainty, and/or a fault's physical properties (i.e., explain why recurrence times may be variable about some mean), and what they imply with respect to changes in earthquake rates and probabilities.

[6] The probability of an earthquake recurring on a fault at some time T after the last event on the same fault in the interval t_e to $t_e + \Delta t$, conditioned on the fact that it has not occurred prior to t_e, is

$$P_c(t_e < T < t_e + \Delta t | t_e < T) = \int_{t_e}^{t_e+\Delta t} f(T)dT \Bigg/ \int_{t_e}^{\infty} f(T)dT \quad (1)$$

(Figure 1a). To be consistent with *Stein et al.* [1997] and *Hardebeck* [2004], in Figure 1 we assume a lognormal distribution

$$f(T) = \frac{1}{\sigma T \sqrt{2\pi}} \exp\left\{-[\ln(T) - \mu]^2 / 2\sigma^2\right\} \quad (2)$$

We emphasize that while other distributions may be employed [*Matthews et al.*, 2002], the specific choice of distribution does not matter for the purposes of this study (i.e., we chose one only to be able to compute an illustrative example).

[7] A method for estimating the permanent perturbing effect on P_c of a step (or static) stress increase, $\Delta\tau$, generated by a nearby earthquake at time t_0, where $t_0 \leq t_e$, has been employed long before studies that consider transient responses to stress perturbations. This uses a Coulomb clock advance, $t_{\mathrm{Coulomb}} = \Delta\tau/\dot{\tau}$, which equals time required to accumulate $\Delta\tau$ at the tectonic stressing rate, $\dot{\tau}$. The clock advance may correspond to an equivalent advance in the elapse time t_e to t_e' [*Dieterich*, 1988; *Working Group on California Earthquake Probabilities*, 1999] or, alternatively, to a reduction in the mean recurrence time μ to μ' [*Working Group on California Earthquake Probabilities*, 1990]:

$$t_e' = t_e + t_{\mathrm{Coulomb}} \quad \text{or} \quad \mu' = \mu - t_{\mathrm{Coulomb}} \tag{3}$$

In either case, the positive step in stress increases the conditional probability of an earthquake (Figure 1b). The conditional probability thus becomes

$$P_c(t_e < T < t_e + \Delta t | t_e < T) = \int_{t_e}^{t_e+\Delta t} f(T' + t_{\mathrm{Coulomb}})dT' \Bigg/ \int_{t_e}^{\infty} f(T' + t_{\mathrm{Coulomb}})dT' \tag{4}$$

in which $T' = T - t_{\mathrm{Coulomb}} > t_0$ is the perturbed recurrence time.

3. Seismicity Rate Models in Earthquake Recurrence Probabilities

[8] *Stein et al.* [1997] and *Hardebeck* [2004] present approaches to estimating earthquake probabilities that account for the effect of stress transfer on earthquake probability. The approach of *Stein et al.* [1997] is largely analytic and relies on the rate change model of *Dieterich* [1994]. *Hardebeck*'s [2004] approach is more general and numerical, but she illustrates it using the same rate change model [see also *Dieterich*, 1992]. We wish to consider earthquake rate more generally, allowing for other failure relations and assumptions. We employ a description of stress induced changes in earthquake rate developed by *Gomberg et al.* [2000] and *Beeler and Lockner* [2003], of which the formulation of *Dieterich* [1994] is a specific case. Although most of the analyses presented by *Gomberg et al.* [2000] and *Beeler and Lockner* [2003] also employ the *Dieterich* [1992, 1994] failure relations, *Gomberg* [2001] illustrates its use with alternative failure criteria. This formulation has also been used previously to consider changes in earthquake rate due to dynamic stress change [*Gomberg et al.*, 2000; *Gomberg*, 2001].

3.1. A General Seismicity Rate Change Equation

[9] We consider how the failure rate, r, or equivalently the time between successive failures, changes due to a stress change. In our companion paper [*Gomberg et al.*, 2005] we examine in some detail a particular rate change model that describes the successive failures of different members of a population of faults, as in background and aftershock seismicity. Herein our task is to associate this rate change model with the change in probability of failure of a single fault in a given time interval. We begin by clarifying some similarities and differences between the background/aftershock seismicity rate change and single fault failure probability applications. In both the recurrence time, T, corresponds to the time between successive failures of the same fault. In the single fault failure probability application recurrence time or interval and failure time, measured from the time of the previous earthquake, are synonymous. The meaning of rate differs for the two applications, corresponding to the inverse of the time between successive failures of different faults in the first case and between potential failure times of the same fault in the second. Most importantly, in the latter the concepts of a rate and a PDF imply that recurrence is described by some population and distribution. These are described in detail with respect to background/aftershock seismicity in the companion paper. When considering a single fault, these may describe natural variability due to the heterogeneity and complexity of fault properties and failure processes, measurement (epistemic) uncertainty represented by a range of potential recurrence times, or some combination of both.

[10] The recurrence time altered by a stress change at t_0 may be written as $T' = T - t_c$, where t_c is the change in failure time, often called the clock advance (positive values indicate that failure time is advanced [e.g., *Gomberg et al.*, 1998]). The change in the interval between successive failures thus becomes $\Delta T' = \Delta T - \Delta t_c$ or

$$\Delta T' = \Delta T \left[1 - \frac{\Delta t_c}{\Delta T}\right] \tag{5}$$

The inverse of this is just the instantaneous rate, and if the relationship between clock advance and recurrence (failure) time is continuous, equation (5) can be written as

$$r(T') = \frac{r(T)}{\left[1 - \frac{dt_c}{dT}(T)\right]} \tag{6a}$$

or in terms of a rate change as

$$\Re(T') = \frac{r(T')}{r(T' + t_c)} = \frac{1}{\left[1 - \frac{dt_c}{dT}(T)\right]} \tag{6b}$$

For a constant unperturbed rate, equation (6) leads to the analytic rate change formula derived by *Dieterich* [1994], which we distinguish from the more generalized equation (6) by denoting it as $\Re_D$. The derivation of $\Re_D$ and its properties are discussed in some detail in the companion paper. Here we simply state the result, i.e.,

$$\Re_D(T') = \frac{r(T')}{r} = \frac{1}{1 - \left[\frac{dt_c}{dT}\right]} \tag{7a}$$

The derivative dt_c/dT is simplest if written in terms of the perturbed failure time T', resulting in

$$\frac{dt_c}{dT} = \left[1 - e^{-\Delta\tau/A\sigma}\right] e^{-[(T'-t_0)/t_a]} \qquad t_a = \frac{A\sigma}{\dot{\tau}}$$

$$\Re_D(T') = \frac{1}{1 - \left[1 - e^{-\Delta\tau/A\sigma}\right] e^{-[(T'-t_0)/t_a]}} \tag{7b}$$

A is a frictional parameter, $\dot{\tau}$ is the stressing rate, and σ is normal stress.

[11] In the companion paper we show that for the case of a population of many independent faults, an equivalent rate change expression may be derived, and that the most significant rate increase is due to the change in failure times of the most mature faults. In other words, those faults for which t_0 is close to their failure times contribute most to the rate change. For the single fault recurrence model the requirement for significant rate increase, and increase in failure probability, is a finite potential that the fault is near failure. More specifically, for some distribution of possible recurrence times (i.e., a range of potential failure times) and a stress perturbation applied at time t_0, for values of T corresponding to a fault far from failure, or $T \gg t_0$, $dt_c/dT \sim 0$, and the rate change is negligible ($\Re \sim 1$). Stated in words, the potential failure (or recurrence) times are all perturbed, or clock advanced, similarly and the time between potential failures does not change. Since the latter determines the rate, it also does not change. When the values of $T \sim t_0$, this corresponds to faults close to failure (more mature) at t_0, dt_c/dT becomes finite, and the rate change is significant ($\Re \gg 1$). A key feature of $\Re_D$ is the assumption that near-failure conditions prevail, which gives rise to the significant rate increase.

[12] We illustrated the dependence of $\Re$ on the distribution of maturities, using a population of independent faults with maturities distributed as in our model of background and aftershock seismicity (see companion paper). We show later that a single fault may be described by an analogous model of a population of potential nucleation sites or patches, although with differing distributions of maturities. Figure 2 shows that the rate change calculated numerically for a positive stress step and a constant background rate as in $\Re_D$ (see also Figure 1 of the companion paper), and the maximum change decreases with progressively increasing fractions of the mature faults/nucleation sites removed from the population [also see *Parsons*, 2002, Figure 13a] The rightmost point in Figure 2b corresponds to the maximum rate change calculated for the full set of failure sources, and progressing to the left, points represent populations with increasing fractions of the most mature failure sources removed. We see that as we eliminate progressively more of the mature sources the size of the maximum transient rate change rapidly and significantly decreases. Our numerical simulations provide only a qualitative guide to the expected reduction in seismicity rate change that may result from a population lacking in mature faults or nucleation patches, because we model the failure process assuming it is quasi-static. However, while use of a fully dynamic, more computationally intensive, model would result in different recurrence times (independent of starting conditions), we have verified that the rate change will not be affected by this quasi-static assumption. Additional confirmation comes from the theoretical, fully dynamic modeling study of *Belardinelli et al.* [2003] that shows that close-to-failure conditions are reached after a fault matures for about 80–90% of its cycle time.

3.2. Seismicity Rate and Probabilities

[13] We now use equation (6) to obtain a general probability density function (PDF) that accounts for the effects of a stress perturbation. We begin by noting that in this context a PDF is a normalized recurrence rate,

$$f(T) = \frac{r(T)}{\mathrm{Num}} \tag{8}$$

Num is the total number of events, and Num = 1 for a single fault. Thus, from equation (6), the PDF following a stress perturbation may be written

$$\tilde{f}(T') = \frac{f(T)}{1 - \frac{dt_c}{dT}(T)} = \frac{f(T' + t_c)}{1 - \frac{dt_c}{dT}(T)} = f(T' + t_c)\Re(T') \tag{9}$$

This perturbed PDF accounts for both the permanent change in stress and the transient frictional response. We interpret this key result as follows. The unperturbed PDF represents a time-varying failure rate, or equivalently a distribution of potential failure times. A stress perturbation redistributes the failure times such that the perturbed PDF represents a new time-varying failure rate that is further modified by $\Re$. Notably $\Re$ depends only on the failure process and maturity of the fault at t_0. In other words, the perturbed rate or distribution is the product of two terms, one that depends on the original rate or PDF and the other only on the failure process. The perturbed PDF is derived by clock advancing the original PDF as in equation (4) but instead of t_{Coulomb} the more general t_c is used, which depends on t_0 (maturity) in a manner also dictated by the failure process. If the failure model of *Dieterich* [1992, 1994] is appropriate then equation (9) becomes

$$\tilde{f}(T') = f(T' + t_c)\Re_D(T' - t_0) \qquad T' > t_0 \tag{10}$$

We have used $T' - t_0$ as the independent variable in $\Re_D$ to highlight the fact that it depends only on the time since the perturbation and not the elapse time t_0 alone. As we will see, equations (9) and (10) provide continuous solutions for the numerical approach proposed by *Hardebeck* [2004].

3.3. The *Stein et al.* [1997] Probability Model

[14] *Stein et al.* [1997] present an approach for estimating the probability of an earthquake occurring on a single fault between times t_e and $t_e + \Delta t$ that accounts for both the permanent effect of a step (or static) stress increase, $\Delta\tau$, and transient frictional effects. In short, their approach assumes that locally, between the short time interval t_e to $t_e + \Delta t$, the failure process may be considered as Poissonian with failures occurring at a rate r_p, or with recurrence time $1/r_p$. If the stress is suddenly increased on the fault, this "local" equivalent Poissonian rate is increased by an amount that depends on t_0 (i.e., the fault's maturity) and

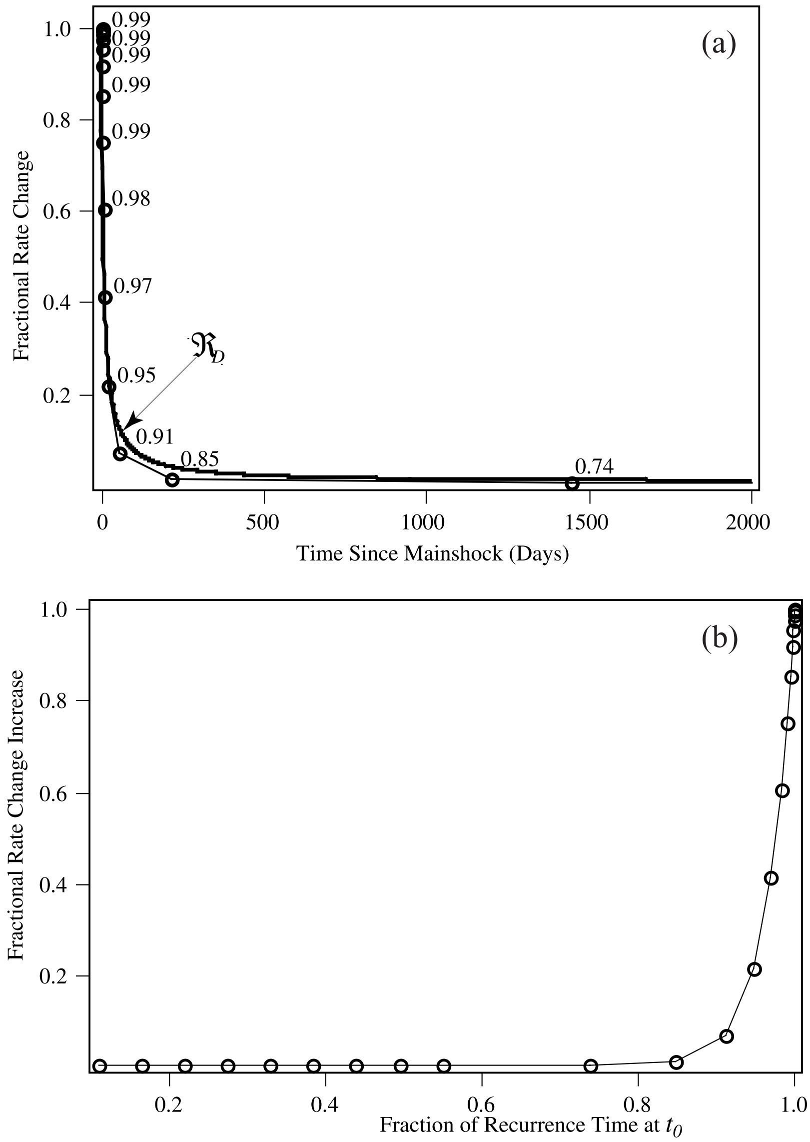

Figure 2. (a) Numerically calculated change in failure rate (circles connected by lines), normalized by the maximum change (y axis), due to a positive stress step affecting a population of independent faults that fail at a constant rate under tectonic loading alone. Also shown is the *Dieterich* [1994] rate change model, $\Re_D$ (darker curve). The distribution of initial and fault conditions was chosen to apply to background and aftershock seismicity. This is the same as Figure 1c in the companion paper [*Gomberg et al.*, 2005], which also contains the input parameters used. (b) Same fractional rate change (y axis) as in Figure 2a but plotted as a function of the maximum maturity found in the population at the time when the step occurs (x axis). These maturities are also noted as the fractions annotating the numerical results in Figure 2a. Both plots show that mature faults are required for a significant rate increase. For example, when the most mature fault in the population is 90% of the way to failure, the rate increase produced by the stress step is diminished to 10% of its maximum value. Although the distribution of initial conditions would differ for a population of patches, or equivalently for a distribution of potential recurrence times, on a single fault, the general conclusion is the same. That is, a significant rate increase requires that a perturbing stress occur when patches are near failure or close to the expected recurrence time. Note that because of the use of a quasi-static failure model this result is approximate, such that the significant rate increase probably requires even greater maturities than shown (see text).

size of the stress step. This local equivalent Poissonian rate will be greater for more mature faults (see Figure 2), and thus the probability of failure will be greater. Use of the equivalent Poissonian rate, r_p, which depends on t_0, is meant to account fully for the maturing nature of the failure process. The frictional response to a stress step further modifies this local rate, increasing it by $\Re_D$. Since failure is considered Poissonian, at least “locally”, the frictional response does not depend on the fault’s maturity.

[15] We summarize the analytic formulation of the *Stein et al.* [1997] recipe in order to tie it to our discussions in sections 3.1 and 3.2, particularly of the perturbed PDF $\tilde{f}(T') = f(T' + t_c)\Re_D(T' - t_0)$ (equation (10)). A probability model is used that considers earthquake occurrence as a locally nonstationary Poisson process with time-varying rate (i.e., seismicity rate) R. For such a model, the probability of an earthquake occurring between t_e and $t_e + \Delta t$ is

$$P(t_e \le T < t_e + \Delta t) = 1 - \exp\left[-\int_{t_e}^{t_e+\Delta t} R(T)dT\right] \tag{11}$$

P_c is obtained from equation (4) and equated to P, and hereafter $t_e = t_0$ for simplicity. The amplitude of the rate, $R(T)$, is assumed constant over the interval t_e to $t_e + \Delta t$ and is equated to the local Poissonian rate [*Toda et al.*, 1998]

$$r_p(t_0) = -\frac{1}{\Delta t}\ln[1 - P_c] \tag{12}$$

To represent the frictional response, the temporal behavior of $R(T)$ follows the *Dieterich* [1994] rate change model,

$$R(T) = r_p\, \Re_D(T - t_0) \tag{13}$$

When used in equation (11), this conveniently results in an analytic expression for the probability,

$$\begin{aligned} P(t_e < T < t_e + \Delta t) &= 1 - \exp\left[-r_p\int_{t_e}^{t_e+\Delta t} \Re(T - t_0)dT\right] \\ &= 1 - \exp[-N] \end{aligned} \tag{14a}$$

N has the form

$$N = r_p(t_0)\{\Delta t + F(\Delta t,\ t_e - t_0,\ \Delta\tau,\ \dot{\tau}, A,\ \sigma)\} \tag{14b}$$

In the language of *Stein et al.* [1997], $r_p(t_0)\Delta t$ describes the "permanent" or stationary probability and $r_p(t_0)F(T - t_0)$ the "transient" frictional response. We can relate this analytic expression directly to the conditional probability model by recasting r_p in terms of P_c (equation (12)), or

$$P(t_e < T < t_e + \Delta t) = 1 - [1 - P_c]^{[1+(F/\Delta t)]} \tag{15}$$

We see that as the frictional response (embodied in F) becomes negligible, the probability becomes identical to P_c (equation (4)). The approach of *Stein et al.* [1997] accounts for the permanent change in stress state by integrating over $f(T' + t_C)$ alone (equation (9) with $t_c = t_{\mathrm{Coulomb}}$) to estimate a conditional probability, P_c.

3.4. The *Hardebeck* [2004] Probability Model

[16] The *Hardebeck* [2004] approach relies only on a conditional probability model (equation (1)). In her approach the transient change in probability of failure of a single fault is associated with epistemic uncertainty, although she develops it by appealing to two analog physical models. In one, she considers a PDF, $f(t)$, as a histogram of the precisely known failure times of a suite or population of hypothetical faults, one of which actually represents the true fault although it is unknown which one. In the second, she considers a suite of possible, imprecisely known failure times of a single fault. Hardebeck's approach is most easily understood in terms of her first analog. To evaluate the effect of a perturbing stress on earthquake probabilities one simply has to know how the perturbation alters the failure time of each fault in the population, employing some failure model. A histogram of these perturbed times thus represents the perturbed PDF from which conditional probabilities can be calculated using equation (1).

[17] The key fact employed in Hardebeck's approach is that the failure probability does not change whether one considers an unperturbed or perturbed elapsed time interval. Hardebeck uses this to estimate the perturbed PDF numerically, considering a discrete population of faults ordered according to their failure or recurrence times. Thus the probability of failing between ith and jth faults, with $j > i$, is the same for both the unperturbed and perturbed loads, or

$$P\left(t_i < T < t_j\right) = P\left(t'_i < T < t'_j\right) \tag{16}$$

The perturbed PDF may be estimated by considering a sufficiently small interval such that the PDF's magnitude is approximately constant, or

$$\int_{T'_i}^{T'_j} \tilde{f}(T')dt' \approx \left(T'_j - T'_i\right)\tilde{f}(T') \tag{17}$$

Combining this with equation (16) and the known unperturbed PDF (e.g., the lognormal distribution of equation (2)), we see that the perturbed PDF may be approximated by a scaled version of the unperturbed probability, or

$$\tilde{f}(T') \approx \frac{P\left(t_j < t < t_i\right)}{\left(T'_j - T'_i\right)} \qquad T'_i < T' < T'_j \tag{18}$$

The scaling, i.e., the perturbed interval, may be calculated for some failure model that relates the perturbation to the change in failure time. *Hardebeck* [2004] then estimates the conditional probability from this numerically calculated PDF using the standard approach described by equation (1).

[18] We now show that *Hardebeck*'s [2004] approach is a numerical version of the general rate change and PDF developed in section 3.2 (i.e., equation (9)). If we assume sufficiently small time intervals, and recalling that $T' = T - t_c$, then equation (18) may be written

$$\tilde{f}(T') \approx \frac{f(T)\left(T_j - T_i\right)}{\left(T'_j - T'_i\right)} = f(T' + t_c)\frac{\left(T_j - T_i\right)}{\left(T'_j - T'_i\right)} \tag{19a}$$

Noting that the instantaneous rate is the inverse of the time between successive failures, we see that the ratio of unperturbed to perturbed intervals approximates the rate change. Thus

$$\tilde{f}(T') \approx f(T' + t_c)\Re(T) \tag{19b}$$

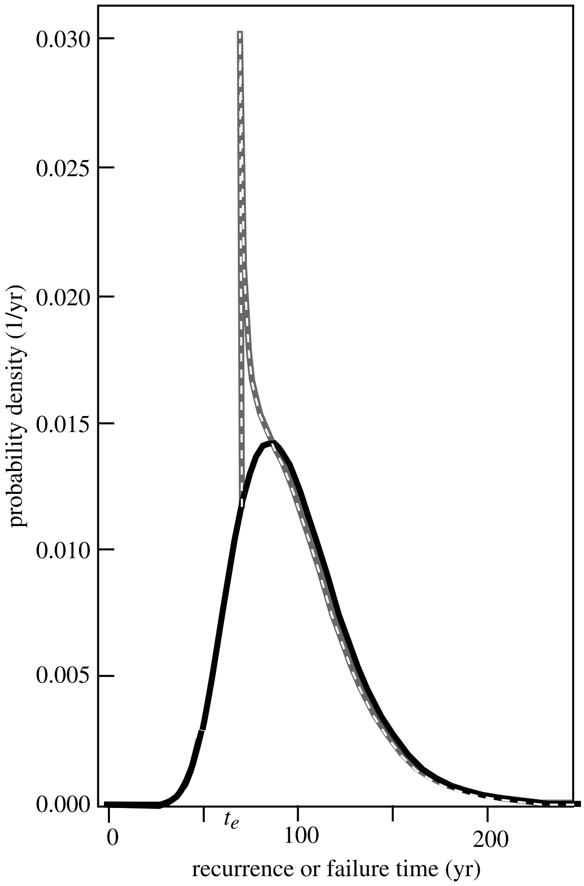

Figure 3. Comparison of the unperturbed PDF of Figure 1a (black) and perturbed PDFs for a positive stress step of $\Delta\tau = 0.5$ MPa imposed at $t_0 = 70$ years calculated using the numerical approach of *Hardebeck* [2004] (dashed) and the analytic solution described by equation (20) (gray). This example uses stressing rate $\dot{\tau} = 0.1$ MPa/yr, normal stress $\sigma_n = 100$ MPa, and $A = 0.005$, so that $t_a = a\sigma_n/\dot{\tau} = 5$ years.

which is identical to equation (9). This derivation provides another interpretation of the perturbed PDF; it is a distorted version of the unperturbed PDF and a change of the variable being integrated over, from the unperturbed to perturbed failure (or recurrence) time. The change of variable is accomplished by scaling the integrand by $\Re(T)$.

[19] The example provided by *Hardebeck* [2004] employs the same time to failure frictional model of *Dieterich* [1994] and the lognormal unperturbed PDF (equation (2)). For this case the perturbed PDF may be written analytically as

$$f(T') = f(T' + t_c)\Re_D(T' - t_0)$$
$$f(T') = \frac{\Re_D(T' - t_0)}{\sigma(T' + t_c)\sqrt{2\pi}} \exp\left(-[\ln(T' + t_c) - \mu]^2/2\sigma^2\right) \qquad T' \geq t_o \tag{20}$$

Finally, we verify these results by comparing PDFs calculated analytically following equation (20) and using Hardebeck's numerical scheme (Figure 3). The two appear essentially identical.

[20] As in the *Stein et al.* [1997] formulation, the notable aspect of the *Hardebeck* [2004] method is that all of the dependence of probability on elapse time (maturity) derives from the original PDF rather than from the friction model. The initial PDF, $f(t)$ in equation (20), is time shifted by t_c for $T > t_0$, which is specified by the particular failure relation assumed, and depends on t_0 for a rate-state frictional model. The rate change, in equation (20), $\Re_D$, does not depend on the elapsed time, t_0, but only the time since the stress change, $T' - t_0$.

4. Conceptual Models of Seismicity and Recurrent Large Earthquakes

[21] We now present several conceptual models to provide more insight into what the above probability models may imply physically. The first conceptual model illustrates a population of individual faults in which failure rate corresponds to successive failures of different faults, as might describe background and aftershock seismicity (discussed in detail by *Gomberg et al.* [2005]). However, our real interest is in understanding how seismicity rate change applies to the probability of failure of a single fault that fails repeatedly, rather than a population of faults in which each successive failure occurs on a different fault. The models we present are meant to show the connection between these applications, and to capture the key elements of the specific models of *Dieterich* [1994], *Stein et al.* [1997], and *Hardebeck* [2004]. Thus we chose model assumptions to best accomplish this, regardless of our opinions about their reasonableness.

[22] As noted in sections 2 and 3, the estimation of a failure probability requires specification of some probability distribution (density function or PDF), in which the distribution describes some population of potential recurrence times. The PDF may reflect our potentially imperfect knowledge of the last earthquake occurred on a fault (epistemic uncertainty), the true natural variability in failure times, or some combination of both. If a PDF is associated entirely with epistemic uncertainty then we need only consider the response to a stress change of a single fault (represented by a single set of properties) with some range of possible maturities (discussed in section 4.2.1).

[23] Alternatively, we consider two end-member single-fault models in which recurrence times vary due to heterogeneity in fault properties and rupture processes, and suggest how they might be distinguished observationally. In these a fault surface is composed of a population of "nucleation patches", which are analogs to individual faults in the aftershock application. Since any patch represents a potential nucleation site for rupture of the entire fault, the properties of the patch population, such as their failure rate and response to a stress change, determine how often and regularly the entire fault is likely to fail. In other words, they determine the PDF describing the variability in recurrence. Such a model also might include small faults or fault patches in the immediate vicinity of the fault that ruptures as major event, as long as they are sufficiently close that their failure can initiate rupture of the major fault. We cannot specify a precise distance over which this may happen however, because we still do not understand all the possible stress transfer mechanisms. We conclude this section with a

discussion of what all these mean for conditional probability estimates.

4.1. Aftershock Seismicity

[24] *Dieterich*'s [1994] rate change model describes the change in failure times of a distribution of nucleation sites affected by a stress perturbation. When applied to aftershocks the distribution naturally may be considered in discretized form as a population of faults affected by a stress step. Figure 4a illustrates our conceptual model of this population of faults. As the properties and behavior of this model are explained in detail in the companion paper, we

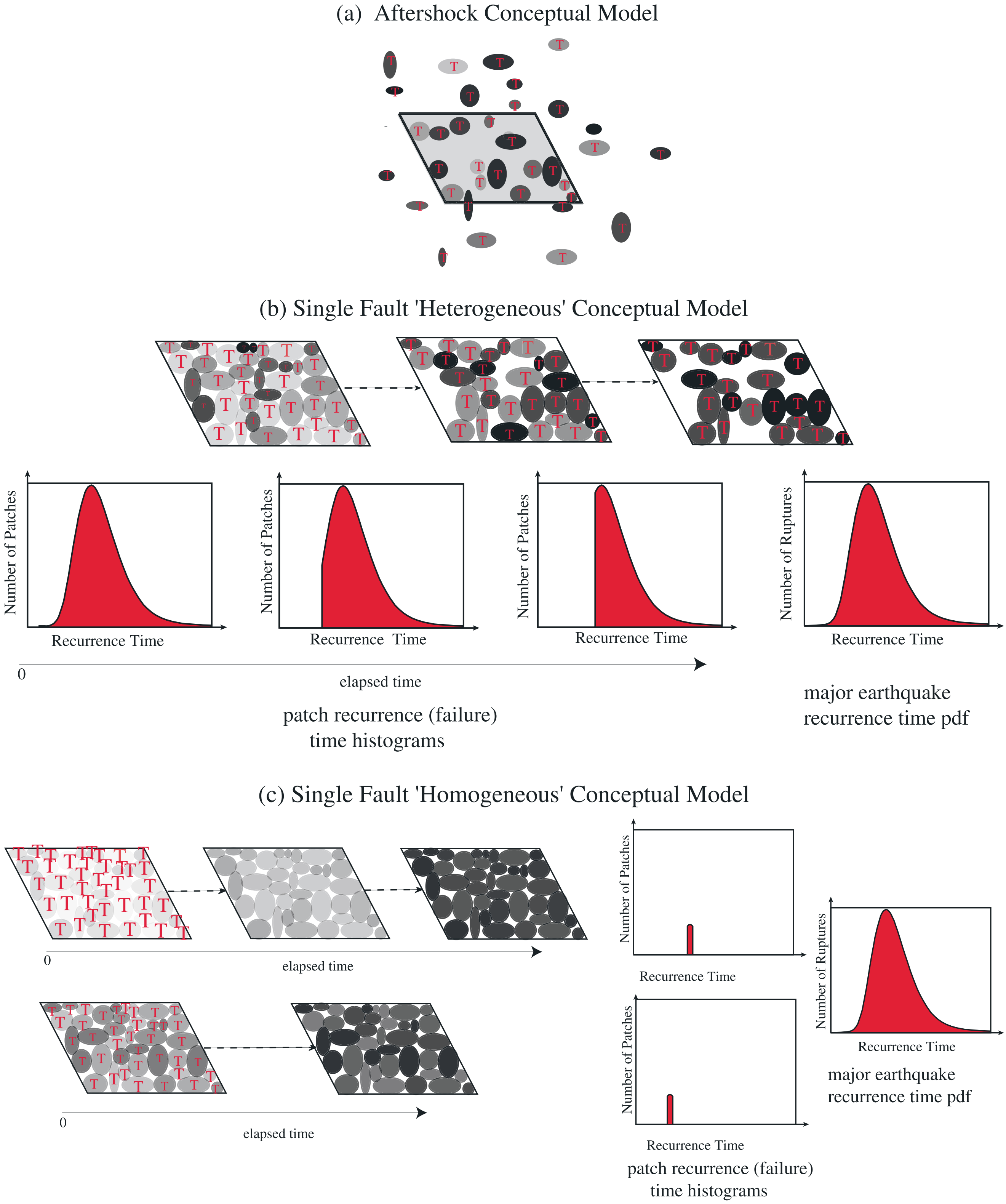

Figure 4

simply summarize them here. Table 1 in the companion paper also provides a summary of the meaning of various time parameters in the context of this model and the single fault model. Each fault fails just once so that the rate is determined by the time between failures on different faults. In this model the recurrence times of all faults have similar lengths but start (and thus failure) times that are offset to produce an approximately constant background rate. This implies that at any given time (e.g., the time of a perturbing stress change at t_0) there will be an approximately uniform distribution of maturities. Because clock advance depends on maturity, perturbed failures no longer occur at a constant rate and for a static stress step perturbation the change in failure rate follows the Omori law. The most mature faults give rise to the largest rate increase (see Figure 1 in the companion paper). Note that unlike the single fault models discussed in section 4.2, here recurrence time (durations) and failure times (absolute times) differ. We discuss in the companion paper how different faults with different frictional properties still yield a predictable rate change model, which might be used to compute probability changes due to stress interactions.

4.2. Recurrent Failure of a Single Fault

4.2.1. Epistemic Uncertainty

[25] This is perhaps the simplest single fault model, in which a fault surface may be described by a single set of properties and conditions, but for which its maturity (or equivalently, the failure times of previous earthquakes) is imprecisely known. Thus the population required to invoke rate change models corresponds to potential recurrence times, with a PDF describing the likelihood that any given one is correct. *Hardebeck* [2004] also proposes this model, noting that a PDF may represent a suite of possible, imprecisely known failure times of a single fault. She further notes that, even though we are concerned with a single fault, an easier way to understand the effects of a stress change on a recurrence PDF is to consider the equivalent view of a population of hypothetical faults with precisely known failure times (equivalently, maturities at the time of a stress perturbation). As for the aftershock model, this range of maturities leads to a rate change in response to a stress perturbation. While a population of physical entities (faults with a range of maturities) has been invoked, this is for conceptual ease only and the transient change in rate and probability really arises purely from considering a range of potential failure times (i.e., the uncertainty).

4.2.2. Natural Variability

[26] Figures 4b and 4c illustrate the two conceptual models of a single fault that fails repeatedly with a varying recurrence time. In one, variability in recurrence time among major earthquake cycles arises entirely from initially heterogeneous patch conditions that statistically share the same characteristics. In the other, patch conditions are relatively homogenous but their mean characteristics may differ for each major earthquake cycle. Conditions on the fault are initialized each time the entire fault ruptures as a major earthquake, although not necessarily uniformly. The fault also may have spatially variable frictional properties. Conceptually the fault surface is composed of patches with different frictional properties and initial conditions, which determines the recurrence time of each patch. Precedent exists for such a patch model, in publications describing both observational and theoretical studies [e.g., *Boatwright and Cocco*, 1996; *Bouchon*, 1997]. In this application the recurrence times of individual patches serve as proxies for their conditions and properties. Because the recurrence times of individual patches vary, their abundances and thus the probability of one nucleating rupture of the entire fault evolves as the population (fault surface) is loaded tectonically and by any stress changes. These populations of patches may be thought of as analogs to population of

Figure 4. All models illustrated share some common formatting. Ovals represent individual faults in Figure 4a, or nucleation sites or patches of a larger single fault (Figures 4b and 4c), which may differ physically from one another. Oval sizes differ only to illustrate the irregularity of real world faults. The relative recurrence times are indicated by the sizes of the T in each fault or patch. Patch or fault maturity (proximity to failure) is shown by the shading, with black indicating very close to failure conditions and lighter shading farther from failure. (a) Aftershock model in which an earthquake occurs on a large fault (rectangle), increasing the static stress on a population of small faults nearby. The faults have similar frictional properties, as indicated by their similar recurrence times. See text. (b) Evolution of a population of nucleation patches with initially variable or "heterogeneous" properties, or recurrence times, on a single large fault (rectangles) under tectonic loading alone for the first conceptual patch model (see text). From left to right the faults show the distribution of patch properties at increasing elapse times; just after a major earthquake, sometime during the interseismic period, and just prior to failure of the entire fault as a major earthquake. In this model, the distribution and evolution of patch properties is the same statistically for all major earthquake recurrences (cycles) and for any perturbation as long as $t_0 > t_e$ there are always mature patches. Below each fault, the corresponding histograms (i.e., distributions) of patch failure times at each elapse time are shown. The PDF for major earthquake recurrences (right PDF) is just a scaled version of the distribution of patch population failure time histograms for $t_e = 0$. (c) Single-fault homogeneous conceptual model. In the second or homogeneous model, for each major earthquake recurrence, there is not a large spread of patch properties (patch recurrence times). As in Figure 4b, each individual major earthquake recurrence may be viewed as a deterministic case with true recurrence time T (e.g., each row) corresponding to the mean patch failure time. Unlike the heterogeneous model, the histograms of patch failure times do not vary with elapse time except when $t_e \sim T$ (none are ready to fail otherwise). The mean patch failure times for individual major earthquake recurrences vary so that the PDF of all potential major earthquake recurrence times is identical to that of the heterogeneous model (i.e., the PDF is the sum of the histograms shown and those for all other possible major earthquake recurrences). When considering an individual major earthquake recurrence (the deterministic view) in this model for mature patches to exist at the time of a perturbation, in addition to the requirement that $t_0 > t_e$, the perturbation must also occur near the true recurrence time or $t_0 \sim T$.

individual faults as in aftershock seismicity, so that under certain circumstance both can be described by the same seismicity rate change model. Individual patches, or groups of patches, may fail during the interseismic period producing background seismicity. We distinguish these from failure of the entire fault by referring to the latter as a major earthquake. Both models described here lead to the same unperturbed recurrence PDF (e.g., $f(T)$ as described by equation (2)) but differ in the heterogeneity of fault properties and initial conditions that give rise to a distribution of recurrence times and their background seismicity rates.

[27] In the first end-member conceptual model (Figure 4b) the PDF of major earthquake recurrence times arises entirely from the heterogeneity of patches within a single major earthquake cycle. Patches are distributed and evolve similarly for each repeat of the major earthquake cycle, and have recurrence times that vary from zero to beyond the mean recurrence time of major earthquakes. The greatest numbers of patches must have recurrence times close to that of the mean recurrence time of major earthquakes. Figure 4b shows the patch distribution after various elapse times, t_e, with corresponding histograms (PDFs) of unperturbed patch recurrence times. For each major earthquake cycle the histogram of patch failure times may be described by $f(T)$, having the same characteristics as the PDF assumed for the conditional probability calculation that describes the distribution of all potential cycles (e.g., $f(T)$ as in equation (2)). Of note in this model is that at any t_e there will always be mature (ready to fail) patches; e.g., at short t_e patches with short recurrence times (slightly greater than t_e) will be mature and at longer t_e patches with longer recurrence times will be mature, etc.

[28] At any time, patches will reach failure and produce background seismicity. However, fewer mature patches exist at shorter t_e producing a lower background seismicity rate. This evolving rate may be quantified using the histogram (PDF) such that the number of patch failures in some interval Δt equals the area beneath the histogram from t_e to $t_e + \Delta t$, which clearly increases as the mean recurrence time is approached (Figure 4b). This rate may be thought of as corresponding to the equivalent Poisson rate employed in the *Stein et al.* [1997] probability approach. As patches also represent potential nucleation sites for a major earthquake, this changing rate also implies an increasing probability of occurrence of a major earthquake as the elapse time approaches the mean recurrence time. A nonzero background seismicity rate grows with elapse time as patches fail but do not cascade into a major earthquake.

[29] In our second patch model the patch properties and initial conditions are relatively homogenous. The PDF of major earthquake recurrence times arises entirely from differences in the mean characteristics of the patch populations for each major earthquake cycle. In any single major earthquake cycle all patches have similar, but not necessarily identical, frictional properties and initial conditions (i.e., patch recurrence times) and the mean patch recurrence time determines the major earthquake recurrence time. These vary in accord with the PDF of major earthquake recurrence times. This variability from major earthquake to major earthquake may come from changing stress drops, evolving fault properties, strain partitioning, etc. Figure 4c shows two major earthquake failure cycles, and how the patches may be initialized and evolve in each. Unlike the first model, histograms of the patch recurrence times differ for each major earthquake cycle and span a small range, and for most of the major earthquake's cycle there are no mature patches. In other words, for each major earthquake cycle the histogram of patch failure times spans a much smaller range of failure times, although the aggregate of these for all potential cycles, the PDF assumed for the conditional probability calculation, has the same characteristics as $f(T)$. During the interseismic period this model predicts no background seismicity (i.e., it is locked) except until just prior to a major earthquake.

[30] Observationally these two models may be distinguishable. Faults represented by the heterogeneous patch model that generate seismicity during the interseismic period also should do so at an increasing rate as a major earthquake is approached, and should experience a rate increase whenever affected by a positive stress step. A stress step acting on a locked fault, represented by the homogeneous patch model, should cause an increase its background seismicity rate only if it occurs when the fault is near failure. Observations from the faults that broke in the Hector Mine earthquake may be consistent with the model of a heterogeneous patch distribution. These faults appear to generate background seismicity. *Parsons* [2002] found that the seismicity rates within 1 km of them increased, and then decayed according to an Omori law, starting at the time the Landers earthquake generated a positive shear stress step on them. The Hector Mine earthquake occurred seven years later suggesting that the faults were near failure at the time of the Landers event. Independent evidence is consistent with the inference of mature faults, although with large uncertainties. *Rymer et al.* [2002] note that the cycle times of the Hector Mine faults range between 5000 and 15,000 and find no evidence of faulting prior to 1999 in three trenches cut through ~7000 year old sediments across the faults. *Parsons et al.* [1999] made similar observations of positive stress steps coinciding with increased seismicity rates for volumes within 1 km of the San Gregorio and Hayward faults, following the 1989 $M7.1$ Loma Prieta earthquake. These observations and the fact that these faults also appear to generate background seismicity are consistent with the heterogeneous patch distribution model regardless of the maturity of the faults. (Estimates of the mean recurrence times and definition of the extent of past ruptures suggest that both these faults are probably not early in their cycle [*Working Group on California Earthquake Probabilities*, 2003].)

[31] Although we have not done a comprehensive search, we cite several possible examples of the homogenous or locked fault model. *Bouchon* [1997] inferred low stress levels on the fault that ruptured as the 1979 Imperial Valley, California, earthquake in areas that experienced large slip during the 1949 earthquake, consistent with a maturing fault only 30 years along in its cycle. He noted that in the 3.5 months prior to the 1979 earthquake the only (well-located) seismicity along the 35 km Imperial Valley fault that eventually ruptured occurred on 6 km segment where stresses were inferred to be at near critical levels. *Bouchon* [1997] also studied the stresses associated with the 1989

Loma Prieta, California earthquake and found that the one region of the fault that did not appear to be nearly critically stressed at the time of the earthquake showed a very low rate of background seismicity. *Boatwright and Cocco* [1996] studied more complex patch models that also predict various levels of background and precursory seismicity. They suggest that sections of the Calaveras fault in California may be locked, although they consider the possibility that it creeps aseismically.

4.2.3. Probabilities

[32] These single-fault patch models differ significantly in their physical attributes and in their responses to stress perturbations at time t_0 when viewed deterministically. However, these differences are generally insignificant for the conditional probabilistic estimates discussed herein. When viewed deterministically (i.e., considering a single failure cycle of a fault with a known unperturbed recurrence time), in the heterogeneous patch model there will always be mature patches on the fault, or a finite likelihood of nucleation of a large earthquake, regardless of when t_0 occurs relative to the true, unperturbed recurrence time, T (e.g., for any frame or elapse time in Figure 4b the fault always has ready-to-fail patches). Similarly, for any t_0 there is a significant rate and failure probability change. In the second model mature patches exist only when t_0 is close to the expected recurrence time of the major earthquake, and only then is there a nonzero unperturbed rate, or likelihood of failure (e.g., ready-to-fail patches exist only in the last frames of the cycles shown in Figure 4c). Additionally the change in rate and probability is only significant when the perturbation occurs near the true unperturbed recurrence time, or when $t_0 \sim T$.

[33] Keeping in mind that these are highly idealized models that rely on some significant assumptions (chosen to match those in a specific rate change model and applications of it), differences usually should have insignificant impact on estimates of the conditional probability of failure that attempt to account for a transient frictional response, as in the studies by *Stein et al.* [1997] and *Hardebeck* [2004] or more generally using the perturbed PDF of equation (19) or (20). This is because estimates of conditional failure probabilities integrate over potential recurrence times between $T = t_e$ and $T = t_e + \Delta t$, which can be thought of as considering a range of deterministic cases of major earthquake recurrences each with true recurrence times each equal to T. For example, if t_e to $t_e + \Delta t$, was small relative to the expected (mean, median, etc.) recurrence time of the PDF, this would be like considering deterministic models with short values of T, like those on the bottom row of Figure 4c. As noted above, in either model when $T \sim t_0$ there are mature patches, the two models respond similarly to a perturbing stress step, and the estimated probabilities also should be similar. Since typically one is interested in estimating the probability immediately after a perturbing earthquake (stress step), or between $T = t_0$ and $T = t_0 + \Delta t$, then typically the conditional failure probability estimates should not differ for these end-member models.

5. Discussion and Conclusions

[34] Here we note two additional significant assumptions made in these models, which we have not discussed. The first is that members of the population of nucleation sites do not interact. The potential importance of such interaction is evident in the literature on epidemic models of aftershocks, in which Omori's law (and the temporal behavior of foreshocks) are explained as a consequence of one aftershock triggering subsequent aftershocks, usually without consideration of frictional processes. In some cases, these models have been used to generate short-term probabilistic forecasts of seismicity (see summary comments of *Helmstetter and Sornette* [2002]). Are such stress transfers important when considering nucleation patch models and failure of a single fault? *Ziv and Rubin* [2003] have studied the effect of static stress transfer on the *Dieterich* [1994] seismicity rate change model, $\Re_D$, but did not consider how it might relate to the probability of failure of a single fault. The second assumption is that the distribution of nucleation patch sizes, or its possible evolution, does not affect the probability of failure of a fault. Observational and theoretical studies show that the distribution of aftershock and background earthquake sizes may change in response to (and perhaps in preparation for) a major earthquake [e.g., *Wiemer and Katsumata*, 1999; *King and Bowman*, 2003; *Ziv and Rubin*, 2003]. Evolution of the distribution of earthquake sizes is not considered in the derivation of $\Re_D$. We believe it may have important implications about how large earthquakes nucleate and should be considered in future studies.

[35] The potential for earthquake recurrence probabilities to affect public policy requires that the methods used to estimate them be fully understood. We have attempted to provide a careful examination of a class of strategies to estimate the probability of recurrence of a single large earthquake. These methods are based on the relationship between a probability (described by a probability density function, PDF) and a time-varying failure rate (i.e., a distribution of potential failure times). They also attempt to quantify how changes in failure rate, caused by some change in loading stress perhaps, affect the probability of failure. Two examples of such strategies are described by *Stein et al.* [1997] and *Hardebeck* [2004].

[36] In this paper we present a more general strategy based on a simple, generalized rate change formulation, and suggest that this generalization provides insight into how a load perturbation, or stress change, affects probability estimates. We also show how this general formulation relates to the approaches described by *Stein et al.* [1997] and *Hardebeck* [2004] and how they relate to one another. In short, the perturbed probability may be described as a product of two terms. The first depends on the original rate or PDF, such that a stress perturbation redistributes the failure times and the perturbed PDF represents a new time-varying failure rate. The perturbed PDF is further modified by the second term, which represents a rate change that depends on the physics of the failure process and maturity of the fault.

[37] We also have attempted to show heuristically what strategies employing probabilistic models, and rates and rate changes, might imply physically. Such strategies implicitly assume the existence of some distribution or population. To some degree, a PDF reflects epistemic uncertainty, but we also consider what might give rise physically to natural variability in recurrence times. We suggest that this may result from a population of initial conditions that vary from

earthquake to earthquake, and/or of fault properties that vary spatially over the fault surface. We represent these using two end-member models in which a fault surface is composed of populations of nucleation sites or patches. Such patch populations serve as analogs to populations of independent faults employed in models of main shock/aftershock seismicity rate changes [*Dieterich*, 1994]. These rate change models have been employed in the probability strategies of *Stein et al.* [1997] and *Hardebeck* [2004]). Thus we attempt to show the connection between models of seismicity rate changes for populations of independent faults as in main shock/aftershock seismicity and how rate changes apply to changes in probability density functions describing the likelihood of failure of a single fault. We conclude that while the response of a fault to a stress perturbation may depend significantly on its maturity (proximity to failure) and physical state, prediction of such a response requires knowledge of these characteristics that generally does not exist (i.e., requires a deterministic model!). However, our qualitative assessment indicates that the probabilistic models we have examined, which effectively integrate over a range of deterministic cases, appear in most applications to be robust.

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M. E. Belardinelli, Settore di Geofisica, Dipartimento di Fisica, Universita' di Bologna, Viale Berti-Pichat 8, I-40127 Bologna, Italy. (elina@ibogfs.df.unibo.it)

M. Cocco, Istituto Nazionale di Geofisica e Vulcanologia, Via di Vigna Murata 605, I-00143 Rome, Italy. (cocco@ingv.it)

J. Gomberg, U.S. Geological Survey, 3876 Central Ave., Suite 2, Memphis, TN 38152, USA. (gomberg@usgs.gov)

P. Reasenberg, U.S. Geological Survey, MS 977, 345 Middlefield Road, Menlo Park, CA 92045, USA. (reasen@usgs.gov)