ESS310 Mathematical Methods in Earth Sciences

Spring 2014: M 11.30-1.20, JHN117; TWThF 11.30-12.20; JHN026;

Instructor: Gerard Roe, JHN 361 gerard@ess.washington.edu, 206-697-3298

TA: Kat Huybers, ATH 219, khuybers@u.washington.edu, 206-616-5393
Office hours: Tuesday 1.30 to 3pm

Website:
- Will get sorted out, and we will let you know. It will also be linked from Roe's ESS website.
- As an experiment, videos of the lecturers will be available on the web site

(This is limited to the class list, let us know if you have trouble accessing it. It is also linked from Roe's homepage)

Credit hours.

Four one-hour lectures per week, one two-hour problem solving session.

Student evaluation.

Grades will be assigned based on performance, weighted in the following way: Attention/preparedness 10%; homeworks 40%; mid-term 25%; final 25%.

Note, for each part, the quality of the assignments will be a substantial part of the grade (~30%). Listen to the in-class rant.

Home works will be posted online by Wednesday, and will be due the following Wednesday. You are responsible for bringing a copy to the lab session on Monday.

Text.

The textbook for this class is J**. Stewart, Multivariate Calculus (7th ed)**, or **Stewart, Calculus (7th ed)**. The former book is a subset of the latter.

The text will be supplemented with other readings as appropriate; in particular, short sections from Snieder, R, Boas, Bostok et al. (see below)

Other useful textbooks
Crowell, simple physics, (free online)
Riley, Hobson, and Bence: Mathematical Methods for Physics and Engineering.
Boas, Mathematical methods in the physical sciences.

Bostok, Chandler and Rourke, Further Pure Mathematics (for matrics/vectors).
Snieder, R: A guided tour of mathematical methods for the physical sciences
Schey, H.M., Div Grad Curl and all that

Online math resources:
Good videos for brushing up on old topics: http://www.khanacademy.org/

Math Study Center: Go do your homework there – lots of resources
Clue: Tutoring in Math available: http://depts.washington.edu/aspuw/clue/home/

Syllabus.

Weeks 1 and 2: The language of math, and the power of the math you know.

Readings: These cover review topics. There are plenty of sources, and use the ones you like the best. Here are readings from **Stewart** that might help.

Functions and problem solving: Stewart: Chapter 1 (esp Sec 1.2, 1.3, and 1.4)
Derivatives: Stewart Chapter 2
Integrals: Steward Chapter 4, Chapter 7
Coordinates systems: Chapter 10
Vectors: Chapter 12

Also look at Kahn academy videos for anything you need to brush up on

These two sets of topics will be interwoven.

Mathematical solutions to physical problems:
I. Logic notation, mathematical symbols
II. Setting up physical equations,
III. Clarity of solutions,
IV. Dimensions/units,
V. Coordinate systems,

Earth Science applications of material from MATH124/125
I. Physically useful functions and applications
II. Motion; rates of change;
III. Frames of reference; exhumation rates;
IV. Areas and volumes of objects;
V. Scalar vs. vector objects; fluxes;
VI. Work, power, rotation, Lorentz force

Math tools (mainly review):
Derivatives; integrals (definite and indefinite); chain, product, and quotient rules; ODE's; vectors; dot and cross products.

- Week 3: Making approximations of Nature

Readings: Convergence of a series: Stewart 11.6, 11.7,
Power-law series and Taylor series: 11.8, 11.9, 11.10

Power series, MacClaurin series, Taylor series, Small number expansions, linearization, discrete derivatives, window onto numerical methods.

Applications:- Gradient forces (stress within the crust or glaciers); heat flux within the crust (1D); pressure gradients and currents; linearization of radiation & moisture; solution accuracy.

- Week 4: Slopes, gradients, surfaces and volumes

Readings

Stewart:
14.1 Functions of more than one variable
14.2 Partial derivatives
14.4 Tangents and linear approximations
14.5 Chain rule
14.6, 14.7 Gradient operator maxima and minima
15.1 to 15.9 Integration in more than one variable

Math tools: functions of more than one variable; partial and total derivatives; gradient operator acting on a scalar field; simple integrals of multivariate functions; conceptual extension to three dimensions.

Topics
1. Cartesian and vector equations for line/plane in 3-D (part review)
2. Topography
    Slopes in 2D
3. Partial derivatives
4. Total slope in an arbitrary direction
5. Chain/product rules for functions of more than one variable
6. Gradient operator

7. Taylor series in 2-D (maxima, minima, saddle points)
8. Integration to give total altitude change, to give cross-sectional area under a curve, worked done along a pathway.
9. Gradient fields.

- Week 5: Fluxes, convergence, and rotation

Readings: Stewart.
15.1, 15.2, 15.3, 15.4, 15.5, 15.6, 15.7 double/triple integrals
16.8 Stokes theorem
16.9 divergence theorem
16.5 curl and divergence

Readings (photocopy, scan, fair-use posting - Nick we need to do this)
Boas 6.10 Definition of divergence

Topics: Gradient transports of heat, momentum, dirt, and water. Fourier's law, Darcy's law, Fick's law, Ohm's law, Newtonian fluids, soil creep. Convergence and divergence of a flux; circulation and vorticity; nondivergent and irrotational flow; streamlines.

- Week 6: Conduction and diffusion

Readings: No readings cover this in Stewart!!!
Boas 13.1 to 13.5; selected readings from Crowell, and fair-use readings from Riley et al. posted on class web site

Topics: conservation laws for mass, energy, and momentum; diffusion of heat, mass, and momentum.

Math tools: differential forms of conservation laws; diffusion equation; concept and simple solution methods for a PDE; separation of variables; initial conditions and boundary conditions.

- Week 7: Oscillations and waves

Readings:
Fair use readings of Boas to be posted on web site

Topics:
Recap of sine waves
Oscillations, (on a spring, buoyancy oscillations)
Wave equation (derivation, physical interpretation)

d'Alembert solution
Separation of variables solution
phase speed group speed
Standing waves, fourier series

- Weeks 8 and 9: Operating on the real world - matrices

Readings: Fair use readings from Bostok and Chandler
Topics:
Matrices as 2 coupled equations, as a transformation of space
Matrix addition/multiplication
Determinant, Special matrices, Orthogonal matrix
3-D Matrices, 3-D determinant
Symmetric/skew-symmetric/diagonal
Eigenvectors,values
Diagonalization
Tensors
2D example of a material under strain
tensors can be diagonalized (principle stress directions, principle coordinate system)
Stress tensor has invariants
Linear systems of equations (maybe)
Numerical methods (maybe)

- Week 10: Solving the problem without solving the problem.

Readings: Fair use readings from Snieder

Estimation methods,
scale analysis;
nondimensional analysis;
Buckingham pie theorem; Reynolds/Prandtl numbers.

Does the mantle convect? Is flow turbulent?

ESS310: Guidance for doing homework problems, and laying out solutions:
When we talk to employers (companies, governments, universities, military, etc.) we hear over and over again that an ability to communicate clearly is the most critical factor in decision over hiring and promoting. It is absolutely essential to develop these skills. Tens of thousand of dollars of your future salary (cumulatively into the hundreds of thousands) will be contingent on whether you have these skills. In science, or in fact any quantitative endeavor, it is vital to be able to lay out a clear, concise, cogent, and correct analysis. This is one of your rapidly diminishing set of opportunities to develop these skills. Do please make the effort to do it right – your future depends on it.
Basically good scientific answers boil down to some very simple rules: a) be clear; b) include units; c) don't write crazy numbers; and d) be only as precise as is needed.
In more detail, here are a few tips:-

1. *Hand in neat work. If you have to rewrite a messy solution take the two minutes needed to do that.*

2. *Always include units! Do the units make sense? If your answer is supposed to be a length, time shouldn't be in there.*

   Doing this is called *dimensional analysis* and is a powerful tool. Sometimes, even without knowing the theory, you can guess at what the answer must be because there is only one way of combining the variables in a way that makes the dimensions correct for the answer you want.

   A silly example: suppose you are told that heat of Q = 1000 J of energy is applied to a metal bar of mass of m = 10 kg, that a specific heat capacity of c = 1000 J kg^{-1} $^{o}C^{-1}$. What is the change in temperature? Even if you forget the formula you can deduce that it is given by DT = Q/mc: that is the only way that Q,c, and m can be combined to give an answer that has ^{o}C as its units.

3. *Check the magnitude of your answer. Does it make sense? Are your speeds, distances, whatever, at least <u>plausible</u>, given the problem and the physical situation?*

   If the answer does not make sense and you can't figure out why not, *make a note to that effect on your work*. It is much better to comment that the answer seems wrong, than to have a crazy number down there. If the method is correct, and you only made a typo on the calculator, you'll still get full credit, *unless* you could have reasonably seen that the number was crazy off.

   Putting down a crazy answer with no comments implies you are not thinking about what you are doing.

4. *Never, ever give an answer with more significant figures than is in the information in the question.*

   Round to the same order as the *least* number of significant figures in the problem. For example, if gravity is given as 9.80665 m/s^2, and the question states that ball is dropped from 12 meters, the answer for the time it takes to fall should only have 2 significant figures in it.

   Logically, there is no extra information in the higher significant figures. Also keep in mind how accurate an answer makes sense. Is it a high precision kind of answer, or is something good to 10% good enough to make the point. A physical sense for what matters is important. Normally, no more than 2 or 3 significant figures (i.e., tenths of a percent) are worth calculating in a class like this.

5. *Lay out your answers/arguments clearly in a logical/systematic order. Show all but the very simplest of steps.*

   There are many good reasons to do this. It is much easier to think about and find errors if all the steps are clearly laid out. Secondly it is much easier for someone else to follow your thinking.

6. *Use symbols and variables for as long as possible when working through math questions.*

   It is much, much easier to work through a problem using symbols (e.g., h for height, A for area, etc.) rather than numbers. Tracking what may be long numbers through several lines of working is very likely to cause errors. Much better is to create your own symbols if need be (and clearly define them). Only put numbers in at the very end to get the quantitative answer.

7. *Explain in words what your steps are.*

   Unless it is obvious, indicate how you get to the next step. If you use new information, say so. For example ”conservation of mass means that…”, or “from eqn 1 above”, or “neglect the 2^{nd} term”. This is all about sympathizing with your audience. Think about whether someone can easily follow your lines of working. Again it helps you to find errors also.

8. *Sketch out the situation in the problem. Label the various aspects*.

   It can really help clear thinking by making a little sketch of what’s going on, so you can see the relationship between the different parts of the problem.

9. *Begin before the night before! Give yourself plenty of time, ahead of time.*

Kinda obvious, but if you wait until there is not chance to think/learn, its very hard to do problem sets under pressure.

10. *Carry around only as many significant figures as you need for the answer.*

    Don't barf out all 12 digits from your calculator every time you use it, if you are only going to cite 2 sig. fig. in your answer. It may not seem like it, but taking only 2 sig. fig. through out the working of the problem, if you round correctly, will get you the right answer in the end. Also, always convert into scientific notation if you have, say, more than 4 decimal places. i.e. always right 4.2×10^{-6} instead of 0.0000042, which can be prone to producing error.

11. *Give yourself lots of space when you do math*.

    Crunched up working makes it hard to correct things and to see where errors might lie.

12. *Where possible, give your answer in units that make physical sense.*

    Thus if it takes 19×10^7 s to cool down, make it into years (1 year = pi x 10^7 s (accurate to about 3 sig. fig., and just a coincidence). In other words it is much more intuitive to know that it takes about 6 years to cool, instead of 19×10^7 s.