What do glaciers tell us about climate variability and climate change?

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Abstract

Glaciers respond to long-term climate changes and also to the year-to-year fluctuations inherent in a constant climate. Differentiating between these factors is critical for the correct interpretation of past glacier fluctuations, and for the correct attribution of current changes. Previous work has established that century-scale, kilometer-scale fluctuations can occur in a constant climate. This study asks two further questions of practical significance: how likely is an excursion of a given magnitude in a given amount of time, and how large a trend in length is statistically significant? A linear model permits analytical answers wherein the dependencies on glacier geometry and climate setting can be clearly understood. The expressions are validated with a dynamic glacier model. The likelihood of glacier excursions is well characterized by extreme-value statistics, though probabilities are acutely sensitive to some poorly-known glacier properties. Conventional statistical tests can be used for establishing the significance of an observed glacier trend. However it is important to determine the independent information in the observations, which can be effectively estimated from the glacier geometry. Finally, the retreat of glaciers around Mt. Baker in Washington State is consistent with, but not independent proof of, the regional climate warming that is established from the instrumental record.

1 Statistics and the interpretation of glacier variability

Climate is defined as the statistics of weather, averaged over some period of interest. The World Meteorological Organization takes 30 yr as the time interval over which those statistics should be determined, though other intervals are equally valid to choose, depending on the purpose. Obviously, the statistics of weather includes the average and the standard deviation, as well as higher-order statistical moments. By definition then, a constant climate means constant (or stationary) statistics. And therefore, variability, as manifest in the standard deviation, is inherent to a constant climate. What does this mean for how glaciers behave in such a climate? Of particular importance are the year-to-year stochastic fluctuations in accumulation and ablation. Glaciers are dynamical systems with a finite memory, and a fundamental property of such systems is that they will integrate such stochastic fluctuations to produce persistent fluctuations on longer time scales (e.g., Hasselmann, 1976; Roe, 2009).

Oerlemans (2000) and Reichert et al. (2002) model two well-studied glaciers in Scandinavia and the Alps, and conclude that Little Ice Age-scale fluctuations will occur every so often, even in a constant climate. Roe and O'Neal (2009) show that, for the setting of Mt. Baker in Washington State, glaciers will undergo kilometer-scale, century-scale fluctuations, again, even in a constant climate. Sorting out real climate change from the variability intrinsic to a constant climate is crucial to correctly interpreting the climatic cause of past glacier variations, and to the detection and attribution of modern climate change from the modern glacier record.

Roe (2009) and Burke and Roe (2010) give a spectral interpretation of this argument, which we review briefly here. The true, physical, measure of climatic persistence is whether climate variables are autocorrelated. In other words, does one year's climate bear any relationship to that of previous years? Consider a climate that has year-to-year variability (drawn randomly from the probability distribution of that climate), but no memory. The time series of such a climate is characterized by a ‘white noise’ power spectrum - that is to say, it has equal power at all frequencies. By construction then, a climate that has no persistence nonetheless has power at all frequencies. The reason is that the phase of individual frequencies in the spectrum is random. On average they destructively interfere, leaving no persistence in the time series constructed from that spectrum.

Now, a glacier can be thought of as acting as a low-pass spectral filter: the glacier’s response to this white-noise climate is characterized by a ‘red noise’ power spectrum - analogous to red light, higher frequencies are damped compared to lower frequencies. Because of this damping, different frequencies will no longer cancel out, and the time series of glacier length variations exhibits long-term fluctuations with a timescale related to the spectral filter of the glacier dynamics.

In the above example, the climate was chosen to be white noise (i.e., with no persistence). Weak interannual persistence in sea-surface temperatures does exist because the ocean mixed layer has some thermal intertia (e.g., Deser et al., 2003). Such weakly red persistence is captured in the 30-yr statistics, and so should properly be included in the definition of the climate statistics. The reponse time of the mixed layer is about one year or less, except near sites of deep ocean convection, of in the vicinity of the sea-ice margin (e.g., Stouffer et al., 2000). In other words, it is much less than typical glacier response times, and so the above argument is unaffected: the persistence of the glacier fluctuations is due to the memory intrinsic to the glacier and not any persistence intrinsic to the climate. Burke and Roe (2010) analyze the persistence of relevant climate fields and mass balance records for Europe and, after linearly detrending to account for anthropogenic trends, find no evidence in the instrumental or glacier mass-balance record for decadal-scale persistence.

Lastly, even where such persistence in climate does exist, it is typically only a small fraction of the overall variance. It is always possible to split the time series of climate forcing into the piece that is due to persistence (i.e., what can be related to previous years), and a piece that is a residual without persistence. Burke and Roe (2010) calculate the relative importance of these two pieces in driving the variance of glacier fluctuations in Europe, and conclude that the residual piece is of greatest importance.

The fundamental and important point is that glaciers can undergo large and persistent fluctuations in a constant climate that has little or no persistence. This fact is often overlooked in the climatic interpretation of past glacier fluctuations. A central goal of paleoglaciology (and paleoclimate in general) is to identify glacier fluctuations that are either unusually large or unusually persistent, and which are therefore 'interesting' to explain. They are interesting because we can then conclude some definitive change in the climate dynamics or climate forcing has occured, and try to identify the cause. The importance of framing the problem in this way is that the alternative is unsatisfying and violates basic tenets of statistical analysis. If climate is defined as the statistics of weather averaged over some period of choice, and if it is also established that a particular glacier fluctuation is quite likely to occur given those statistics, then it makes no sense to conclude that the glacier fluctuation reflects a climate change.

The interpretation of the climatic cause of glacier fluctuations can therefore be distilled into a classic statistical exercise of correctly identifying changes due to a signal (i.e., the glacier response to a climate change) versus changes due to noise (i.e., the glacier response to interannual variability). What factors control the relative magnitude of this signal and noise? Under the assumption that a glacier is a simple dynamical system relaxing back to equilibrium with a single dynamical response time (e.g., Jóhannesson et al., 1989a,b), some useful formulae can be derived (e.g., Roe and O'Neal, 2009; Huybers and Roe, 2009). This study extends these prior analyses to ask two more questions:

1. What factors govern the likelihood of seeing a glacier excursion of a given magnitude in a given interval of time?
2. How can the statistical significance of a trend in observed or reconstructed glacier length be evaluated?

In both cases, formulae can be derived from the linear equations, in which the dependencies on glacier geometry and climate setting can be clearly understood. The value of these formulae is that the parameters involved can be calculated from a glacier's geometry, and so they give guidance as to which glaciers are likely to be best for detecting past climate change. A second and key part of the present study is to establish whether the formulae successfully predict the behavior of a dynamic glacier model, which obeys a nonlinear rheology.

We find that maximum glacier excursions are governed by high frequency behavior of the glacier, and the linear formulae hold provided the short-term lag correlations are used to calculate the response time. The probabilities of a given excursion are very sensitive, however, to the magnitude of the natural variability. We also show that glacier trends can be evaluated using a standard Student's t-test, provided that the right degrees of freedom are used. These degrees of freedom can be accurately calculated from the linear model equations. Finally, we conclude that the current retreat of glaciers around Mt. Baker in Washington State is consistent with, but not by itself independent proof of, regional warming.

2 Linear and dynamic models

2.1 Linear model

Roe and O'Neal (2009) derived a simple linear model for describing variations in glacier length in response to variations in melt-season temperature and annual accumulation. It is similar in spirit to other earlier models (e.g., Jóhannesson, 1989a,b; Harrison et al., 2001), all of which boil down to essentially the same first-order ordinary differential equation:

$$\frac{dL'(t)}{dt} + \frac{L'(t)}{\tau} = \tilde{\alpha}T'(t) + \tilde{\beta}P'(t) \qquad (1)$$

Equation (1) represents a dynamical system in which glacier length, $L'(t)$, responds to fluctuations in annual accumulation, $P'(t)$, and melt-season temperature, $T'(t)$. Primes denote departures from the long-term climatological mean. τ is the e-folding timescale on which the glacier relaxes back to equilibrium, or equivalently it is the length of time over which the glaciers remembers its previous states. $\tilde{\alpha}$ and $\tilde{\beta}$ are coefficients relating climate forcing to the tendency on glacier length. Equation (1) is continuous in time. Its discrete form is

$$L'_{t+\Delta t} = \gamma L'_t + \alpha T'_t + \beta P'_t, \qquad (2)$$

where $\Delta t = 1$ yr, and $\gamma = 1 - \Delta t/\tau$, and is the lag-1 correlation coefficient. This form of the equation is convenient for deriving expressions for the statistics of glacier length fluctuations. As derived in Roe and O'Neal (2009), τ, α, and β are functions of the glacier geometry and some mass balance parameters:

$$\tau = \frac{wH}{\mu \Gamma \tan\phi A_{abl}},$$
$$\alpha = -\frac{\mu A_{T>0} \Delta t}{wH}, \tag{3}$$
$$\beta = \frac{A_{tot} \Delta t}{wH}.$$

A schematic illustration of the model is given in Figure 1. The geometric parameters for the glacier are: width, w; depth, H; total area, A_{tot}; ablation area, A_{abl}; melt area, $A_{T>0}$; and basal slope, $\tan\phi$. μ is the melt factor, relating melting rates to melt-season temperature, with units of [m yr^{-1} ^{o}C^{-1}]; Γ is the atmospheric lapse rate, 6.5 ^{o}C km^{-1}.

2.2 Dynamic model

A dynamic flowline model is also used in this study. We follow standard equations for the shallow ice approximation incorporating glacier sliding (e.g., Oerlemans, 2001):

$$\frac{dH(x)}{dt} + \frac{dF(x)}{dx} = \dot{b}(x),$$
$$F(x) = \rho^3 g^3 (f_d H^2 + f_s) H^3 \left(\frac{dz_s}{dx}\right)^3. \tag{4}$$

$H(x)$ is glacier thickness at position x, $F(x)$ is the vertically integrated flux of ice, and dz_s/dx is the surface slope. f_d and f_s are the coefficients governing deformation and sliding, respectively. Following Budd (1979) and Oerlemans (2001), we take $f_d = 1.9 \times 10^{-24}$ Pa^3 s^{-1} and $f_s = 5.7 \times 10^{-20}$ Pa^3 m^2 s^{-1}. $\dot{b}(x)$ is the local mass balance. For simplicity we assume a uniform accumulation pattern, and melt-season temperature is calculated as a function of x using the standard lapse rate. Equations 4 were solved using standard numerical techniques on a 50 m grid, though results with 20 m and 100 m grid spacing proved very similar.

2.3 Preliminary comparision of linear and dynamic models

In order to evaluate and compare the linear and dynamic models, we use the well-documented setting of Mt. Baker, a stratovolcano in Washington State flanked by five typical midlatitude glaciers. We use the same climate as Roe and O'Neal (2009), based on a combination of local station measurements, weather model output, and mass balance measurements. The annual mean accumulation is 5 m yr^{-1} with an interannual standard deviation of 1 m yr^{-1}. The interannual standard deviation in melt-season temperature is 0.8 °C.

Roe and O'Neal (2009) specified parameters and geometry representative of the Mt. Baker glaciers, and showed that the linear model was able to simulate historical glacier length variations fairly well. In the current study, rather than the extensive fiddling with the dynamic model that would be necessary to get this exact geometry and parameter set, we use a more efficient procedure for the comparison of the linear and dynamic models, which is our main purpose. We specify the accumulation and the basal slope, and adjust the mean melt-season temperature until the dynamic model is approximately right in terms of total area. We then diagnose from the model output the other geometric factors needed for the linear model (H, A_{abl}, $A_{T>0}$). This allows a more exacting comparison between the two models, which is the point here. Values for this geometry and standard parameters are given in Table 1. In particular the linear timescale calculated from the model geometry ($\equiv \tau_{lin}$) is about 7 yr.

In the remainder of this section we perform two preliminary comparisons of the two models. We first calculate the change in length due to step-function changes in mean climate forcing, for which the linear model has analytical solutions (Roe and O'Neal, 2009). The dynamic model must be integrated until the new equilibrium is reached. In general there is extremely good agreement between the dynamic and linear models, shown in Figure 2, with length changes differing by less than 5% for climate changes spanning ± 6 °C in melt-season temperature and ± 2 m yr^{-1} in accumulation. There is a suggestion in Figure 2 that the response to precipitation is slightly more linear than the response to temperature. On the whole though, for this range of climate forcing and for this glacier geometry and setting, Figure 2 strongly supports the validity of the assumptions made in deriving the linear model (Roe and ONeal, 2009).

The second comparison is of the models' response to a linear trend in climate forcing. We pick a warming trend comparable to that experienced in the Pacific Northwest during the 20$^{\text{th}}$ century (+0.1 °C decade^{-1}, e.g., Mote, 2003), and an increasing accumulation trend (0.1 m yr^{-1} decade^{-1}, though the significance of observed accumulation trends is unclear in this region, Mote, 2003). Analytical solutions are again available for the linear model, and do a good job of predicting both the rate and magnitude of the response of the dynamic model, as shown in Figure 3. For both trends, the dynamic model lags slightly the predicted linear response and, in the case of a temperature trend, the rate of retreat in the dynamic model appears slightly greater than predicted from the linear model, consistent with the results in Figure 2.

3 The response to climate variability

In what follows we characterize the nature of the glacier response to interannual climate variability in more detail. Equation (2) can be developed further to derive some useful properties of glacier variability, whose dependence on glacier geometry and climate parameters can be clearly understood. As noted in the introduction, a focus of this present study is to evaluate the degree to which these expressions also govern the behavior of the dynamic model.

Roe and O'Neal (2009) demonstrate that, after linearly detrending, the observed interannual variability in the Pacific Northwest in both the annual-mean accumulation and the melt-season temperature is consistent with random fluctuations that are Gaussian (i.e., normally-distributed), and white (i.e., uncorrelated in time). In other words, after accounting for the anthropogenic trends in climate, the remaining natural variability has no interannual persistence. The analysis in Roe and O'Neal is sufficient to prove that, even if some interannual persistence in climate does in fact exist for the region, it accounts for a statistically-insignificant fraction of the climate variability over the period of the instrumental record.

Two 10,000 year-long realizations of white noise were generated to simulate interannual variability in P' and T', characteristic of the Pacific Northwest (with standard deviations of 1 m yr^{-1} and 0.8 $^{\circ}$C, respectively). This is long enough to acquire good statistics on the glacier response. The two climate time series are assumed to be uncorrelated, also consistent with observations for the region (Roe and O'Neal, 2009). Both the dynamic and the linear models are then integrated forward in time, using the same realizations of this simulated climate variability. Figure 4 shows a 500 year segment of the climate and the glacier response. Both the linear and dynamic models undergo kilometer-scale, centennial-scale fluctuations in response to a climate that we reiterate has no persistence. The standard deviation of the linear model can be derived from model parameters (Roe and O'Neal, 2009), and is 360 m. The standard deviation of the dynamic model must be calculated from the numerical integration, and is 324 m, a difference of 10%, which is about what Roe and O'Neal (2009) found for a similar calculation. The smaller standard deviation of the dynamic model is evident in Figure 4, as is the fact the dynamic model is noticeably smoother. Finally, it is also clear that the response of the dynamic model lags behind the response of the linear model.

For comparison, Oerlemans (2001) estimates a 1σ of 660 m for typical glacier parameters, and Reichert et al (2002) model values of 550 m and 290 m for Nigardsbreen and Rhone glaciers, respectively. Of course the exact values do, and should, vary with geometry and setting.

A linear model like that of Equation (2) must have a normally-distributed response to normally-distributed forcing. For the dynamic model, one test of its linearity is to calculate the probability density function (PDF) from the histogram of its fluctuations. The PDFs for both models are shown in Figure 5. The smaller standard deviation of the dynamic model relative to the linear model is clear in its narrower clustering around zero. Visually, it appears there is a hint of skewness to central and negative values, though the skewness is in fact very slightly positive (0.06). The dynamic model PDF is not quite normally-distributed, however: a standard Kolomogorov-Smirnov test (e.g., Von Storch and Zwiers, 1999) rejects the normal distribution at greater than 95% confidence. The probable reason is that kurtosis of the dynamic model is 3.2, implying it is slightly more outlier-prone than a normal distribution, for which the kurtosis is 3.0.

Despite some small differences, the response of the linear and dynamic models to equilibrium climate changes, climate trends, and climate variability have differed by only a few percent. This generally solid agreement between the dynamic and linear models in these preliminary tests is a firm basis for proceeding to explore the response to climate variability, using the analytical power of the linear model to understand the reasons for the glacier behavior.

3.1 The autocorrelation and the spectral response of a glacier

The autocorrelation function and the power spectrum of a time series are powerful tools for revealing the time dependence of a dynamical system. There are of course closely related to each other since the periodogram spectral estimate is just the fourier transform of the autocovariance function (e.g., Von Storch and Zwiers, 1999). Both were calculated from the 10,000-yr integrations, a sample of which is shown in Figure 4.

At low frequencies, with periods longer than a few decades, the spectra of the linear and dynamic models are identical. These timescales are much longer than the adjustment time of the glacier, and so both linear and dynamic models are in near-equilibrium with the forcing: dynamics don't matter and the glacier is simply acting as a reservoir of ice with nearly balanced input and output fluxes. The linear model physics captures exactly this. At higher frequencies the two spectra differ considerably. Consistent with the time series shown in Figure 4, high frequencies in the dynamic model are considerably damped compared to the linear model. In the linear model any mass imbalance is instantly converted into a tendency on the length (Equation (1)). However, in the dynamic model, and in a real glacier, there is some intertia to terminus movement: it takes time for mass to travel to the terminus, and the terminus slope has steepen to the point it drives a flux of ice forward.

The autocorrelation curve shows essentially the same information, but in a different light. For a linear model described by a single timescale the autocorrelation curve decays exponentially with a e-folding timescale of τ. Figure 6b shows that for lags longer than about 15 years ($\approx 2 \times \tau_{lin}$), the autocorrelations of the linear and dynamic models are identical, and closely approximate the exponential behavior. For lags shorter than 15 years, the dynamic model has much higher autocorrelations than the linear model. This reflects the more smoothly varying behavior of the dynamic model, evident in Figure 4. For this setting and geometry, $\sim$15 years is the true measure of the timescale that separates when dynamics does and does not matter. The power spectrum can be deceptive in this regard. The visual appearance from Figure 6a is that it is only at much longer periods that the behavior of the dynamic and linear models' converge. This appearance is because a factor of 2π enters when the exponential decay time is projected onto the sinusoidal components of the power spectrum (e.g., Roe, 2009).

There are various way of characterizing the glacier response time, and there has been substantial discussion in the literature (e.g., Nye, 1965; Jóhanneson et al., 1989a,b; Van de Wal and Oerlemans, 1995; Bahr et al., 1998; Jóhannesson, 1997; Raper et al., 2000; Oerlemans, 2001; Pelto and Hedlund, 2001; Harrison et al., 2001, 2003; Lesinger Vieli, and Gudmundsson, 2004; Schwitter and Raymond, 2003; Oerlemans, 2007; Raper and Braithwaite, 2009). Figure 6b shows that the autocorrelation function of the dynamic model cannot be represented by a single timescale. Using the ARFIT algorithm of Schneider and Neumeier (2001), we find that an 8th-order autoregressive process is needed to match it, suggesting that ice dynamics introduces a complicated structure of persistence to the glacier length record.

From Figure 6 it seems that best 'effective response time' depends on the timescale and question of interest. However several studies have assumed explicitly or implicitly a single response time in characterizing past, and predicting future glacier variations (e.g., Harper, 1992; Pelto and Hedlund, 2001; Oerlemans 2005, 2007). We find a single best-fit timescale, τ_{bf}, for the dynamic model by fitting an AR(1) process using the ARFIT algorithm. This is equivalent to doing a regression analysis of the dynamic model to Equation (2), using a least-squares minimization. The high autocorrelations at short lags are weighted heavily in the fitting, and the resulting timescale, $\tau_{bf} =$ 73 yr, is much longer than predicted from the linear model ($\tau_{lin} = 7$ yr). For comparison, Figure 4 shows the output from a linear model driven by the same climate forcing, but with $\tau_{bf} = 73$ yr. The longer timescale correctly captures much of the low frequency variability of the dynamic model, but cannot capture some of the decadal fluctuations.

4 The likelihood of a glacier excursion

One way of characterizing the expected natural variability of a glacier in a constant climate is to answer questions like: What is the expected return time, on average, of a particular glacier advance? How long, on average, does the glacier persist above or below its equilibrium length? How likely is an excursion of a given size in a given period of time?

For linear models of the form of Equation (2), answers can be derived using standard formulas for threshold crossings of stochastic processes, first laid out by Rice (1948). VanMarcke (1983) and Leadbetter et al. (1983) contain good summaries. In the Appendix it is shown that, as long as $\tau >> \Delta t$, the average interval between up-crossings of a particular threshold, L_0, is given by

$$R(L_0) = 2\pi\sqrt{\frac{\tau\Delta t}{2}} e^{\frac{1}{2}\left(\frac{L_0}{\sigma_L}\right)^2}. \tag{5}$$

$R(L_0)$ is also the average return time of a glacier advance of size L_0. σ_L is the standard deviation of natural fluctuations. Roe and O'Neal (2009) show that Equation (2) yields

$$\sigma_L = \sqrt{\frac{\tau}{2\Delta t}} \cdot \sqrt{\alpha^2\sigma_T^2 + \beta^2\sigma_P^2}, \tag{6}$$

provided that P' and T' are neither autocorrelated or correlated with each other, consistent with climate in the Pacific Northwest. A general expression for σ_L without these restrictions is also possible (Huybers and Roe, 2009).

From Equation (6), the exponent in Equation (5) contains τ, and therefore the return time of a given advance is a very sensitive function of the response time. A larger value of τ means that the glacier is slower to return to equilibrium, and has a weaker restoring tendency. All else being equal, it will tend to have larger excursions for a given return time, as shown in Figure 7. Secondly, the L_0^2 in the exponent in Equation (5) means the average return time lengthens extremely rapidly as the size of the advance increases. For $\tau = 12$ yr, for example, an advance of 1 km will happen on average every 250 yr, while for an advance of 1.5 km, the average return time balloons to 7500 yr (Figure 7).

4.1 Return time of zero-crossings

How often does a glacier return to its equilibrium length? For up-crossings across zero, $L_0 = 0$ in Equation (5), and the average return time is given by

$$R(0) = 2\pi\sqrt{\tau\delta t/2}. \tag{7}$$

For the linear model, with a $\tau = \tau_{lin} = 7$ yr, Equation (7) gives an average up-crossing interval of 12 yr, which is in excellent agreement with the linear model output in Figure 4 - it must be since it is an exact solution of the linear equations. Using the best-fit response time for the dynamic model, $\tau_{bf} = 73$ yr, Equation (7) gives a prediction for average up-crossing interval of 38 yr. The actual average interval from the dynamic model output is 41 yr, and thus compares well with the prediction.

It can be shown (e.g., VanMarcke, 1983) that the rate of zero-crossing depends only of the first and second statistical moments of the spectrum. Although it is hard to see from Figure 6a because of the log-log axes, the centroid of the spectrum and its other moments are dominated by the high frequency part of the spectrum, and so is consistent with using τ_{bf}.

4.2 Likelihood of maximum glacier excursions

Equation (5) presents the average return time of a particular advance or retreat. It is also possible to calculate the probability distribution of such return times. This is governed by the statistics of a Poisson distribution (e.g., Von Storch and Zwiers, 1999), wherein discrete stochastic events occur at a known rate, λ. A requirement of the process is that a time interval, $(t_f - t_i)$, can be identified in which the likelihood of one event occurring is proportional to $(t_f - t_i)$, and that the likelihood of two events occurring in that interval is negligible.

So assuming a Poisson process, the probability of observing zero advances (or retreats) of magnitude L_0 in an interval $(t_f - t_i)$ is given by

$$p(N(t_f - t_i) = 0) = \exp[-\lambda(L_0)(t_f - t_i)], \tag{8}$$

where $\lambda(L_0)$ is the reciprocal of the up-crossing interval, $R(L_0)$, in Equation (7). The probability of *at least* one occurrence of an L_0 advance (or retreat) is given by the complement of Equation (8):

$$p(N(t_f - t_i) \geq 1) = 1 - \exp[-(t_f - t_i)\lambda(L_0)] = 1 - \exp\left[-\frac{(t_f - t_i)}{2\pi} \cdot \left(\frac{2}{\tau \Delta t}\right)^{\frac{1}{2}} \cdot e^{-\frac{1}{2}\left(\frac{L_0}{\sigma_L}\right)^2}\right]. \tag{9}$$

Equation (9) reveals the dependencies clearly. The probability of seeing an advance or retreat is more sensitive to $(t_f - t_i)$ than to τ. The probability is also remarkably and acutely sensititve to the ratio of L_0 and σ_L: the exponent itself has an exponential dependence on the square of this ratio.

The advances or retreats considered so far have been relative to the equilibrium glacier position. In any given glacial valley, it is hard to know what the long-term average position of a glacier has been, especially in the face of a changing climate. A measure of more practical relevance is the total excursion of the glacier accounting for length changes of both signs (i.e., maximum advance minus maximum retreat). The probability of a total excursion of at least a size of ΔL occurring in a given period of time is given by the probability *density* of a maximum advance between L_1 and $L_1 + dL$, $f(L_1)$, multiplied by the probability the maximum retreat exceeding $L_2 \equiv L_1 - \Delta L$, integrated over all possible maximum advances (see Figure 8).

The probability density of a maximum advance between L_1 and $L_1 + dL$, $f(L_1)$ is:

$$f(L_1) = \frac{d}{dL_1}\left(N(t_f - t_i) = 0\right) = \frac{\lambda(L_1)\Delta L}{\sigma_L^2} e^{-(t_f - t_i)\lambda(L_1)}. \tag{10}$$

Hence the total probability of a maximum excursion exceeding ΔL in

$$p(L_{max} - L_{min} > \Delta L) = \int_0^\infty \frac{\lambda(L_1)\Delta L}{\sigma_L^2} e^{-(t_f - t_i)\lambda(L_1)} \left(1 - e^{-T\lambda(L_1 - \Delta L)}\right) dL_1. \tag{11}$$

Figure 9 shows the probability distribution of maximum excursions in any 1000 yr period calculated from the dynamic model output, and also calculated from Equation (11) using $\tau_{bf} = 73$ yr. There is good agreement, demonstrating that the model is closely behaving as a Poisson process with the best-fit timescale. In any 1000 yr period, it is very likely ($> 95\%$) to undergo an excursion of at least 1.4 km, driven just by interannual variability in a constant climate. On the other hand, it is very unlikely ($< 5\%$) to undergo an excursion exceeding 2.2 km. Also shown is the curve for $\tau = 7$ yr, which would predict longer excursions. This is because the greater power at higher frequencies and the shorter response time for the linear model (Figure 6a) makes it more likely to have a short, spikey fluctuation that takes the linear glacier across a given threshold. For the more smoothly varying dynamic model, these events are rarer.

Reichert et al. (2002) made some similar calculations for the Alps and for Scandinavia, but used a different definition of a glacier excursion and diagnosed λ from the output of a numerical model. These results reenforces their conclusions however, and also those of Roe and O'Neal (2009), for the Cascades: kilometer-scale, centennial-scale variations in glacier length will occur in a constant climate. An advantage of Equation (11) is that the dependency of the excursion probabilities on the underlying glacier properties can be clearly seen. Figure 7 makes clear that it is important to identify the correct timescale.

How robust are these excursion probabilities to different assumptions and model parameters? Figure 10a shows how the probabilities change for different time intervals. The change in the curves in going from a 500 yr to a 1000 yr interval is about the same as in going from a 2000 yr to 5000 yr interval. In other words for longer intervals, the probabilities of seeing large excursions begins to saturate. However the super-exponential dependency on σ_L in Equation (9) makes the excursion probabilities acutely sensitive to glacier properties. Figure 10b shows how the curves change for small changes in σ_L. It is clear that even $\pm 20\%$ variations have a very large impact. An error in estimating σ_L of even this small amount might well reverse the interpretation about whether an excursion could be caused by interannual climate variability or an actual climate change. It is doubtful that the σ_L of real glaciers can be known that accurately. It is probably appropriate therefore, to be cautious about studies that use such curves to conclude that modern retreats exceed natural variability (e.g., Reichert et al., 2002). It's certainly possible, but an exhaustive error analysis is needed to be confident.

5 Trend detection for glaciers

How big of a trend in glacier length is statististically significant? When does the trend exceed that expected from natural variability? We show here that two factors are of primary importance: 1) the magnitude of the trend relative to the the amplitude of the natural variability, and 2) the amount of independent information in the observations. This last factor depends on the degrees of freedom, which in turn depends on the length of observations and the glacier memory.

Let ρ be the correlation of the observations of glacier length and time at lag Δt, and let ν be the degrees of freedom in the dependent time series (i.e., glacier length in this case). A $\tilde{t}$ statistic can be calculated from the following combination of ρ and ν:

$$\tilde{t} = \frac{\rho\sqrt{\nu - 2}}{(1 - \rho^2)}. \tag{12}$$

The tilde is used to distinguish this variable from the one we've used for time. Basic text books on statistics (e.g., Zwiers and vonStorch, 1999) show that, in the absence of a real trend, the probability of finding a given value of $\tilde{t}$ will follow a Students $\tilde{t}$ distribution, and standard tables can be used to calculate how often $\tilde{t}$ would occur simply by chance. In general, the larger the absolute value of $\tilde{t}$ the greater the confidence that the observed trend is significant.

Equation (12) can also be written as:

$$\tilde{t} = \frac{b\sigma_t\sqrt{\nu - 2}}{\sigma_{res}}, \tag{13}$$

where b is the regression coefficient between time and glacier length, and σ_t and σ_{res} are, respectively, the standard deviations of time and of the residuals of glacier length after the time-correlated trend has been subtracted.

σ_t is the standard deviation of the independent variables (in this case time) over a given interval of time, $(t_f - t_i)$, and is given by

$$\sigma_t = \left\{ \frac{1}{(t_f - t_i)} \int_{-(t_f-t_i)/2}^{(t_f-t_i)/2} t^2 dt \right\}^{\frac{1}{2}} = \frac{(t_f - t_i)}{2\sqrt{3}}. \tag{14}$$

Therefore (15) becomes

$$\tilde{t} = \frac{\Delta L}{\sigma_L} \cdot \sqrt{\frac{\nu - 2}{12}}. \tag{15}$$

$\Delta L \equiv b(t_f - t_i)$, and is the change in glacier length that is attributable to the linear trend.

Equation (15) shows some basic and readily understood dependencies. The first factor on the right hand side of can be regarded as the signal-to-noise ratio: the greater the trend relative to the natural variability, the more significant the trend will be. Glaciers that exist in maritime climates are subject to a high degree of precipitation variability (e.g., Huybers and Roe, 2008), and have a muted sensitivity to temperature. As such, a warming trend in melt-season temperature may be obscured by the natural variability. A continental glacier with less precipitation variability and a higher sensitivity to temperature, may more directly reflect warming trends.

The second factor on the right-hand side of Equation (15) shows that the degrees of freedom (i.e., the number of independent pieces of information in the glacier record) is critical to assigning statistical confidence to an observed trend. If the glacier position were recorded annually, $\Delta t = 1$ yr, and there would be $N = (t_f - t_i)/\Delta t$ observations. However a glacier has a dynamical response time, and so it has memory of its previous positions. It is therefore autocorrelated and there are fewer than N degrees of freedom. Standard theory (e.g., VonStorch and Zwiers, 1999) gives the appropriate formula for the effective degrees of freedom as:

$$\nu = N\frac{1-\gamma}{1+\gamma}, \tag{16}$$

where, as noted above, γ is the autocorrelation coefficient at a lag time of Δt. Using the approximation that $\delta t << \tau$, then using (3) and taking only first order terms:

$$\nu \approx \frac{(t_f - t_i)}{\Delta t}\frac{\Delta t/2\tau}{(1-2\Delta t/\tau)} \approx \frac{t_f - f_i}{2\tau}. \tag{17}$$

So for a 100 yr glacier record and for a $\tau_{lin} = 7$ yr, there are about seven effective degrees of freedom.

An important assessment of whether the $\tilde{t}$ test can be used in practice is to establish whether random realizations of a dynamic glacier that is forced by a climate without a trend does, in fact, follow a Student's $\tilde{t}$ distribution. In other words, is glacier variability consistent with the assumptions of the $\tilde{t}$ test? To do this, the right number of degrees of freedom needs to be established.

Figure 11 shows the probability distribution of the $\tilde{t}$-statistic (Equation (12)), using 1000 randomly selected 100-yr intervals from the dynamic model output, and assuming that $\nu = 100$ yr$/(2\tau_{lin})$. Also shown are the theoretical Student's $\tilde{t}$ distributions calculating ν using both $\tau = \tau_{lin} = 7$ yr, and $\tau = \tau_{bf} = 73$ yr. It is clear that the output of the dynamic model is well characterized by a Student's $\tilde{t}$ distribution with ν calculated from τ_{lin} and not, as might be expected, from τ_{bf}. In other words, for a 100 yr period of length variations at Mt Baker, Figure 11 shows that are about 7 effective degrees of freedom, and the significance of a trend can be evaluated using a standard $\tilde{t}$-test.

The $\tilde{t}$ statistic was also calculated for 50, 200, and 1000 yr trends, and again using τ_{lin} for the degrees of freedom. For 50 yr trends (just 3.2 effective degrees of freedom), the agreement with the theoretical distribution was slightly worse than that shown in Figure 11, but at no point does the error exceed 5%. For 200 yr and 1000 yr trends, the $\tilde{t}$ statistic approximated the theoretical distribution even more closely than that in Figure 11. We also note that annual observations of glacier length are not required to estimate trends: provided observations are frequent enough to sample the effective degrees of freedom, there can still be a correct assessment of significance.

The analysis highlights the importance of knowing the right effective degrees of freedom for evaluating glacier trends in practice. If degrees of freedom were calculated using $\tau_{bf} = 73$ yr, it would formally mean less than one degree of freedom in a 100 yr record, and Figure 11 shows that significant trends with high $\tilde{t}$-statistics would go unrecognized. The results also show that, for our chosen setting, the statistical significance of glacier trends can only be established on multi-decadal or longer timescales.

It is interesting that for evaluating trends τ_{lin} is the right timescale to use, whereas for the likelihood of large excursions, τ_{bf} works well. The reason seems to be that a trend, like an equilibrium step-change, is a low-frequency behavior of the glacier. It is thus well-described by the linear model, as seen in Section 2.3. In contrast, maximum excursions depend on relatively abrupt changes that cause a threshold to be crossed, and for that reason depend on the high-frequency behavior of the glacier, which is best characterized by τ_{bf}.

5.1 Is the observed trend significant?

Equation (15) provides a way of calculating how large a change in glacier length needs to be observed before the trend can be declared statistically significant:

$$\Delta L = \tilde{t}_{p=0.95,\nu} \cdot \sigma_L \sqrt{\frac{12}{\nu - 2}}. \tag{18}$$

Consider a 100 yr observing period (for which $\nu = 7.3$), a 95% significance level (for which $\tilde{t} = 1.88$), and let $\sigma_L = 324$ m, which was what we obtained from the dynamic model for typical Mt. Baker glaciers. From Equation (18), a change of 900 m would be necessary over that 100-yr period in order to declare a significant trend. The actual observed trend over the last eighty years is equivalent to 150 m per 100 yr (calculated from linearly detrending the compilation of results in O'Neal (2005) for Easton, Deming, Boulder, Rainbow, and Coleman glaciers). If a shorter period, the last 30 yr, is considered, the observed trend is larger (400 m per 30 yr), consistent with an anthropogenic climate signal emerging only since that time. However the degrees of freedoms in the observations are reduced to just 2.2, and so very much larger glacier changes of several kilometers would be required for statistical significance. Thus we conclude that the observed changes in Mt. Baker glaciers, by themselves, cannot be said to reflect a statistically significant trend.

It is important to be clear about the logic here. These results represent the difference between saying that the glaciers *by themselves* provide independent evidence of climate change, versus saying that they are merely *consistent* with the observed regional warming that is already established to be statistically significant from the instrumental record (e.g., Mote, 2003). There is obviously an important distinction between these two statements. The multi-year response time of glaciers means there is much less independent information in their history than in the instrumental record. Moreover these are maritime glaciers that experience large, and largely unrelated, interannual accumulation variability (e.g., Bitz and Battisti, 1999; Huybers and Roe, 2009), and so it should not be surprising that this variability obscures the effect of warming on the glaciers, and that the glacier record is therefore a less decisive demonstration of regional warming than is provided by thermometers.

5.2 How wrong could σ_L be?

The results above depend on estimating σ_L and ν from the dynamic model. In general, without long enough records of unforced natural variability, a model must be used. For example, general circulation models are used for estimating the natural variability of global mean climate. It is reasonable to ask whether a model adequately represents this natural variability, and in the case of global climate, this has been debated extensively.

Alternatively, one can turn the question around, and ask what would the value of σ_L have to be, in order for the observed trends, ΔL_{obs}, to be significant at the 95% level? That is, solve for

$$\sigma_L = \frac{\Delta L_{obs}}{\tilde{t}_{p=0.95,\nu}} \sqrt{\frac{\nu - 2}{12}}. \tag{19}$$

For the 100 yr and 30 yr trends given above, the answer is $\sigma_L = 52$ m and 20 m, respectively. It seems unlikely that natural variability is as low as this, or that the dynamic model is wrong by an order of magnitude. Larger values of σ_L could only come about if there were more degrees of freedom coming from a shorter effective response time. The agreement between the $\tilde{t}$-distributions in Figure 11 and the fact the response time would have to be less than 7 yr makes this, too, seem unlikely. The linear timescale is actually a lower bound on the timescale, since it assumes that glacier dynamics are instantaneous (all mass imbalances instantly transferred to the toe). It is hard to imagine, therefore, that there could be more degrees of freedom in the observations. Thus the conclusion – that the retreat of Mt. Baker glaciers is consistent with regional warming, but not independent proof of it – is very strong.

5.3 More than one glacier, more than one location

Most glaciers are reported to be retreating around the world. Does this constitute independent evidence of climate change? The $\tilde{t}$-test is a simple and powerful statistical measure that works well, even when the underlying process departs significantly from the test assumptions (e.g., Boneau, 1960). Rather than detailed modeling of individual glaciers, one could use global data sets of observed glacier length variations (e.g., from the World Glacier Monitoring Service, Haeberli, 1998), and use Equation (19) to solve for the combination of σ_L and ν required for the observed trend to be significant at the 95% level. Those values could be compared to existing estimates of a glacier's response time (e.g., Oerlemans, 2005), and historical or reconstructed estimates of its natural variabilty. This might be path to more rigorous estimates of statistical significance than obtained by varying model parameters (e.g., Oerlemans, 2005). It would also identify which glaciers are more decisive indicators of climate change than others. The fact that many glaciers within a single region are observed to be retreating or advancing does not necessarily provide much independent information, since they are experiencing essentially the same climate. The independence of individual glaciers can be estimated from the spatial coherence of patterns of natural climate variability (e.g., Bretherton et al., 1999) of the fields that are most relevant for glaciers (e.g., Huybers and Roe, 2009; Burke and Roe, 2010).

6 Summary and Discussion

Stochastic fluctuations are inherent to a constant climate. Distinguishing between climate records that just reflect these fluctuations and those that reflect a true climate change is a central challenge in climate science. For the case of global mean temperature, for example, there is widespread agreement that the instrumental record shows a significant warming trend that exceeds the natural variability of the last 30 yr. An identical issue arises in interpreting the climatic cause of past glacier fluctuations, which are almost always attributed to climate change. This study expands on earlier work and confirms that interannual variability alone can cause century-scale, kilometer-scale fluctuations in glacier length.

We presented results for the geometry and setting of the glaciers on Mt. Baker in the Pacific Northwest. These are relatively small and steep, and so to evaluate the impact of glacier geometry we also repeated the analyses for a glacier with double the total area and half the basal slope, and also for a glacier with quadruple the total area and one quarter the basal slope (and double the width). For these cases, we obtained comparable agreement between the linear and dynamic models to that presented here.

We focussed on glacier length because that is typically the clearest signature of past fluctuations. All of the metrics and formulae discussed in this study could instead be applied to glacier volume. The linear model under-predicts the response of glacier volume to climate variability because it does not account for thickness changes. For the spectrum of ice volume fluctuations (i.e., the equivalent of Figure 6a), the linear and dynamic models shows very good agreement at high frequencies, but the linear model under-predicts at low frequencies (by ∼30% for the parameters presented here). At high frequencies mass balance fluctuations are simply added to the existing volume, and the dynamics has no time to respond. At low frequencies the linear model does not allow for the thickness changes that amplify the volume fluctuations in the dynamic model. The autocorrelation timescale for volume fluctuations in the dynamics model is much shorter than that for length fluctuations (15 yr vs. 73 yr), and so is closer to the predictions of the linear model. Understanding volume fluctuations might find relevance in settings where glacier history is recorded in sediments of proglacial lakes or in trimlines on valley sidewalls, or when the impact on sea-level is of interest (e.g., Raper and Braithwaite, 2009).

The important principle in this study is that stochastic interannual climate variability can cause large and persistent glacier fluctuations that should not be misinterpreted as being driven by a climate change. This principle is fundamental and does not depend in any way upon the details of the models used. These models are sufficient to gauge the magnitude of the effects, and sensitivity to different conditions has been reported herein. Glaciers are, of course, complicated beasts, and no model can capture all their facets. We note that our linear model does not incorporate the mass-balance (e.g., Harrison et al., 2001) or thickness feedbacks (e.g., Bahr et al., 1997) that are sometimes included in linear glacier models, though such a modification is easily possible. Other approaches to glacier response time have used volume-area scaling ratios that represent some nonlinearities and asymmetries (e.g., Bahr et al., 1998; Raper and Braithwaite, 2009). For our geometry and setting, and also for the sensitivity experiments, we did not find it necessary to introduce these additional factors in order to successfully emulate a dynamic glacier model for our purposes. Further work to establish when such factors or others, such as more complicated geometric setting, cause a breakdown of the relationships derived here, would be useful. In general, such complications will always be hard to model, and it may be best to identify settings where those complications are minimized.

The success of the linear model at emulating the dynamic model at low frequencies means that, when climate change happen on time scales longer than a couple of glacier response times (meaning longer than about 15 years for Mt. Baker), the glacier's response to climate variability (such as the excursion probabilities) can be combined linearly with, and superposed directly on, the glacier's response to the climate change.

Extreme events and zero-crossings depend on short, rapid advances and retreats, and so are governed by the high frequency characteristics of the glacier. The dynamic model is highly autocorrelated on short timescales (Figure 6b), and hence the longer decorrelation timescale must be used in the formulae. We also note the extreme sensitivity of threshold crossing statistics to σ_L. It may be very hard to determine the value of σ_L for a real glacier to within the accuracy needed to formally establish whether a given glacier advance exceeds that expected from natural variability (c.f., Oerlemans, 2000; Reichert et al., 2002).

On the other hand, trends, $\tilde{t}$-tests, and equilibrium changes depend on the low frequency characteristics of the glacier, for which it is acting as an essentially passive reservoir of ice. Therefore the shorter timescale of the linear model provides excellent agreement, and moreover it can be efficiently estimated from the glacier geometry. Furthermore, the analyses are far less sensitive to parameter uncertainties. Lastly, although not formally as rigorous, by solving for the σ_L required for the observed trend to have a nominal confidence level of 95%, the expression for a $\tilde{t}$-statistic can be used to roughly gauge the significance of a trend, and so circumvent the need for a comprehensive simulation of the natural variability. Non-parameteric tests, that do not rely on the glacier adhering to a particular theoretical pdf might be applied for trend detection (Morell and Fried, 2008; Cotter, 2009). Such tests are more flexible, though typically less powerful, than parametric tests, and care is needed in accounting for serial correlations.

Various methods have been used to estimate the glacier response time from observations (e.g., Harper et al., 1992; Oerlemans, 2001; Pelto and Hedlund, 2001; Harrison et al., 2003; Klok and Oerlemans, 2004; Oerlemans, 2007). A useful exercise would be to repeat those various methods on the output from the dynamic model, and see which best captures the correct effective degrees of freedom. A concern is that the results here suggest that short-term lag correlations, which are the most easily estimated from observations, may underestimate the actual degrees of freedom.

Finally, this study evaluated the observed retreat of glaciers around Mt. Baker. We conclude there are about seven effective degrees of freedom in a 100 yr long record, and that the retreat would have to much larger than is observed to be considered *independent* evidence of regional warming. It can certainly be said that the retreat is *consistent* with the observed warming that is already established to be significant from the instrumental record. It should be made clear that the detection of a trend in glacier length is different exercise from the detection of a trend in glacier mass balance, which is in many ways more closely related to the instrumental record of climate. Where available, local instrumental and local mass-balance records have more statistical power to resolve climate change than glacier-length records. In the case of mass balance records, changes in glacier area must be factored in (e.g., Oerlemans, 2001).

Glaciers are consequential and captivating elements of the earth system. Correctly understanding their dynamics and interpreting their history and is a worthwhile challenge. Provided that care is taken is to identify the correct timescale, the linear model and the formulae derived from its equations do an excellent job of characterizing some important properties of a glacier's behavior. Such formulae can be used to give guidance as to which glaciers and settings are most sensitive indicators of warming trends or precipitation trends, and which paleo-reconstructions are likely to be most indicative of past climate changes. Identifying such conditions is an important prerequisite for realizing the fullest potential of glacier records.

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Appendix A: Threshold crossing rates

Let $\dot{L}$ refer to the rate of change of the glacier. Rice (1948) (also Vanmarcke, 1983) showed that, for a general random process the expected rate, $\langle\lambda(L_0)\rangle$, at which it crosses up over a given threshold, L_0, is given by

$$\langle\lambda(L_0)\rangle = \frac{1}{2}\int_{-\infty}^{\infty} sgn(\dot{L})p(\dot{L}, L_0)d\dot{L}. \tag{A-1}$$

The term inside the integrand is the joint probability density of the glacier having a length between L_0 and $L_0 + dL$ and, simultaneously, a rate of change which would cause it to cross L_0. The total probability is the integral over all possible rates of change. The primes have been dropped from the Ls for the sake of convenience. If $\dot{L}$ and L can be considered independent of each other, Rice further showed that the expected rate of up-crossings past L_0 is

$$\langle \lambda(L_0) \rangle = \frac{1}{2\pi} \frac{\sigma_{\dot{L}}}{\sigma_L} e^{-\frac{1}{2}\left(\frac{L_0}{\sigma_L}\right)^2}. \tag{A-2}$$

where σ_L and $\sigma_{\dot{L}}$ are the standard deviations of the glacier length and its rate-of-change, respectively. We next derive an expression for $\sigma_{\dot{L}}$, and show that the correlation between $\dot{L}$ and L can indeed be considered small.

From (1)

$$\langle \dot{L}^2 \rangle = \langle L^2 \rangle + \frac{\alpha^2 \sigma_T^2}{\Delta t^2} + \frac{\beta^2 \sigma_P^2}{\Delta t^2}, \tag{A-3}$$

which, using (6), becomes

$$\sigma_{\dot{L}}^2 = \sigma_L^2 \left(\frac{1}{\tau^2} + \frac{1-\gamma^2}{\Delta t^2} \right). \tag{A-4}$$

Since we are dealing with typical conditions where $\tau >> \delta t$ this simplifies to

$$\sigma_{\dot{L}} = \sigma_L \left(\frac{2}{\tau \Delta t} \right)^{\frac{1}{2}}. \tag{A-5}$$

Next, we determine correlation coefficient between $\dot{L}$ and L, which is given by

$$r_{\dot{L},L} = \frac{\langle \dot{L} \cdot L \rangle}{\sigma_{\dot{L}} \sigma_L}. \tag{A-6}$$

From (1) it follows directly that $\langle \dot{L} \cdot L \rangle = \langle L^2 \rangle / \tau$. Therefore, using (A-5) the correlation coefficient becomes

$$r_{\dot{L},L} = \left(\frac{\Delta t}{2\tau} \right)^{\frac{1}{2}}. \tag{A-7}$$

For the typical Mt. Baker parameters, $\tau \approx 12$ yr giving $r_{\dot{L},L} \approx 0.2$. Using a Monte Carlo test (Figure 7), we show that this correlation is indeed small enough to be neglected, and that therefore (A-2) provides an accurate description of threshold crossings.

Substituting (A-5) into (A-2) gives

$$\langle \lambda(L_0) \rangle = \frac{1}{2\pi} \left(\frac{2}{\tau \delta t} \right)^{\frac{1}{2}} e^{-\frac{1}{2}\left(\frac{L_0}{\sigma_L}\right)^2}. \tag{A-8}$$

$R(L_0)$, the average interval between up-crossings across L_0 is the reciprocal of the rate, $\lambda(L_0)$.

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Figure 1: Idealized geometry of the linear glacier model, based on Johanneson et. al. (1989). Precipitation falls over the entire surface of the glacier, A_{tot}, while melt occurs only on the melt-zone area, $A_{T>0}$. The ablation zone, A_{abl}, is the region below the ELA. Melt is linearly proportional to the temperature, which, in turn, decreases linearly as the tongue of the glacier recedes up the linear slope, tanϕ, and increases as the glacier advances down slope. The height H of the glacier, and the width of the ablation area, w, remain constant. From Roe and O'Neal (2009).

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Figure 10: Probability of maximum excursions for different assumptions. a) probability of exceeding a given excursion, for different periods of time. Note the uneven time increments; b) probability of exceeding a given excursion, for different values of σ_L in Equation (11). All curves use the standard parameters for the dynamic model (except where σ_L is varied).

Figure 11: The $\tilde{t}$ statistic calculated from 1000 randomly selected 100 yr-long from the dynamic model output. Also shown are the theoretical t-distributions for degrees of freedom calculated from τ_{lin} and τ_{bf}. It is clear that τ_{lin} best characterizes the degrees of freedom.

Table 1: Parameters and geometry of standard case glacier. The first set of parameters are imposed, the second set are calculated from the dynamic model and used for the linear model formulae. Also included is the linear model timescale. The simplified, pseudo one-dimensional geometry means that not every aspect of the typical Mt. Baker glacier can be matched at the same time. In particular, the standard glacier has a nominal length of 8 km, and the accumulation area ratio is one half, rather than two thirds. See text for more details, and compare with values given in Roe and O'Neal (2009) for Mt. Baker glaciers.

parameter	value
μ	0.65 m yr^{-1} ^{o}C^{-1}
Γ	6.5 ^{o}C km^{-1}
tanϕ	0.4
w	500 m
A_{tot}	4.0 km^2
A_{abl}	2.0 km^2
$A_{T>0}$	3.4 km^2
H	44 m
τ_{lin}	7 yr

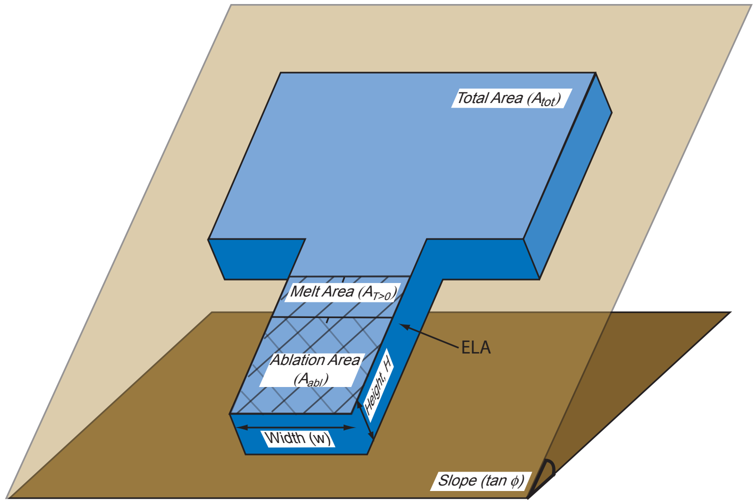


Figure 1: Idealized geometry of the linear glacier model, based on Johanneson et. al. (1989). Precipitation falls over the entire surface of the glacier, A_{tot}, while melt occurs only on the melt-zone area, $A_{T>0}$. The ablation zone, A_{abl}, is the region below the ELA. Melt is linearly proportional to the temperature, which, in turn, decreases linearly as the tongue of the glacier recedes up the linear slope, tanϕ, and increases as the glacier advances down slope. The height H of the glacier, and the width of the ablation area, w, remain constant. From Roe and O'Neal (2009).

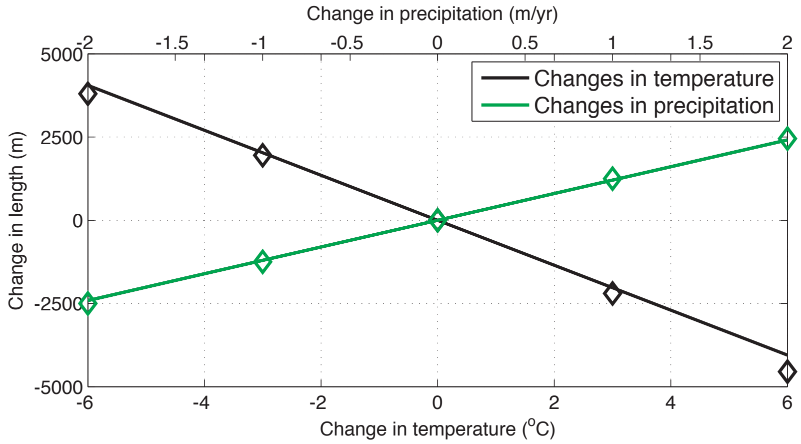


Figure 2: Response of glacier length to step function changes in accumulation and melt-season temperature. Solid lines show analytic solutions from the linear model, and the symbols show results from the dynamic model.

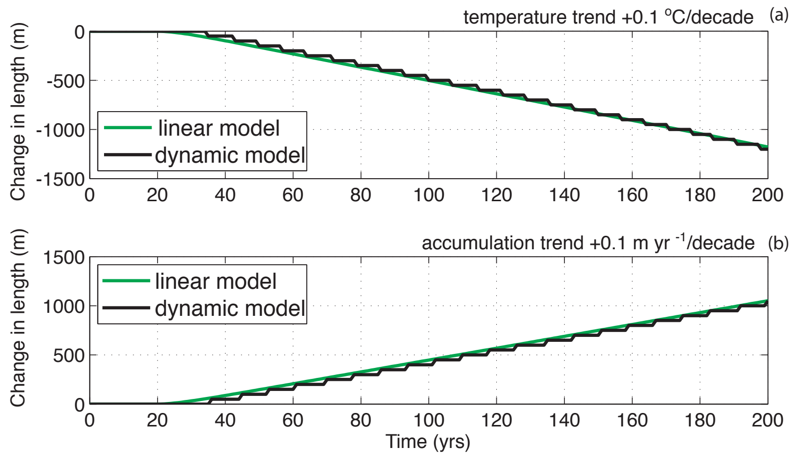


Figure 3: Response of glacier length to increasing trends in (a) accumulation (+0.1 °C decade^{-1}), and (b) melt-season temperature (+0.1 m yr^{-1} decade^{-1}), imposed beginning in model year 20. There is good agreement between the linear and dynamic models.

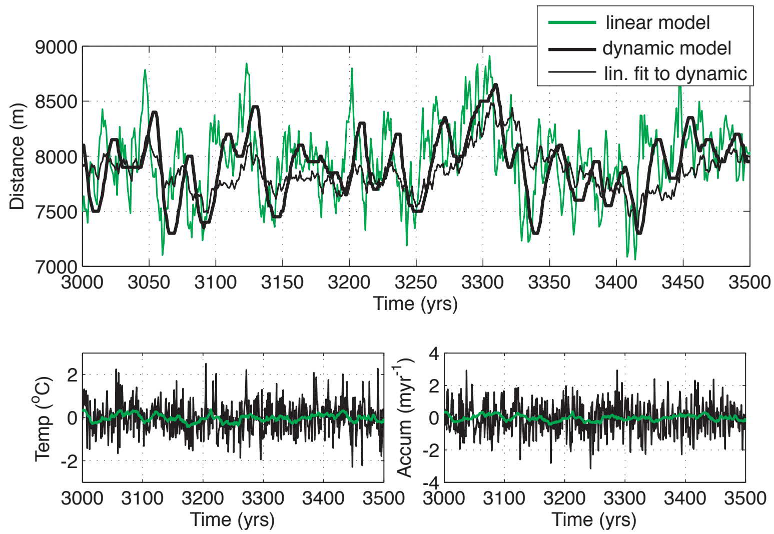


Figure 4: A 500 year segment of a 10,000 yr simulation of the glacier response to interannual climate variability. The lower panels are white-noise realizations of interannual fluctuations in accumulation and melt-season temperature, and for which a 30-yr running mean is also shown. The upper panel shows the response of the two glacier models. Kilometer-scale, century-scale glacier fluctuations occur in this simulated climate that by construction has no persistence. Also shown in the thin black line is a linear fit to the dynamic model, using the best-fit τ_{bf} of 73 yr.

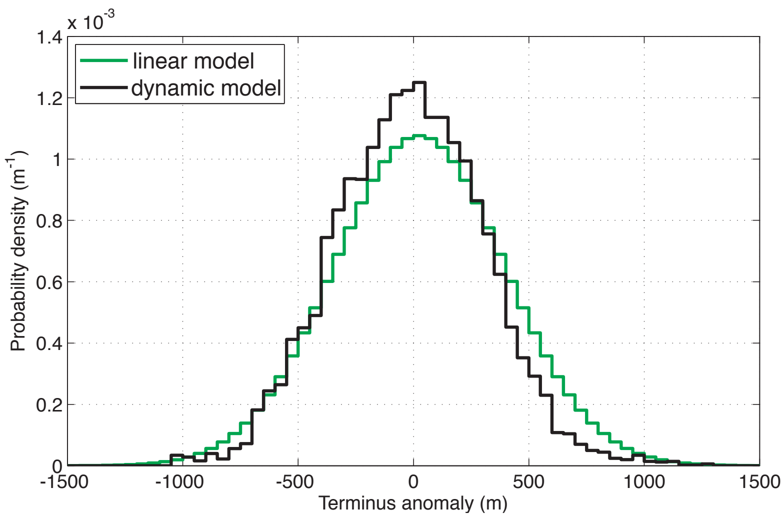


Figure 5: The probability density functions (PDFs) of the linear and dynamic models. The linear model follows a normal distribution, the dynamic model has a slightly non-normal distribution.

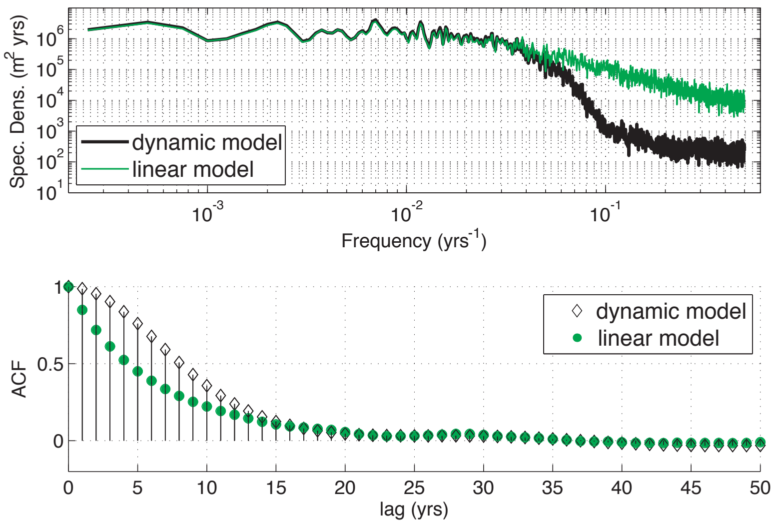


Figure 6: a) Power spectral estimate for linear and dynamic models, calculated using a windowed periodogram (a 20-kyr Hanning window). b) Autocorrelation function for linear and dynamic models. Both panels show that the dynamic model is damped at high frequencies compared to the linear model.

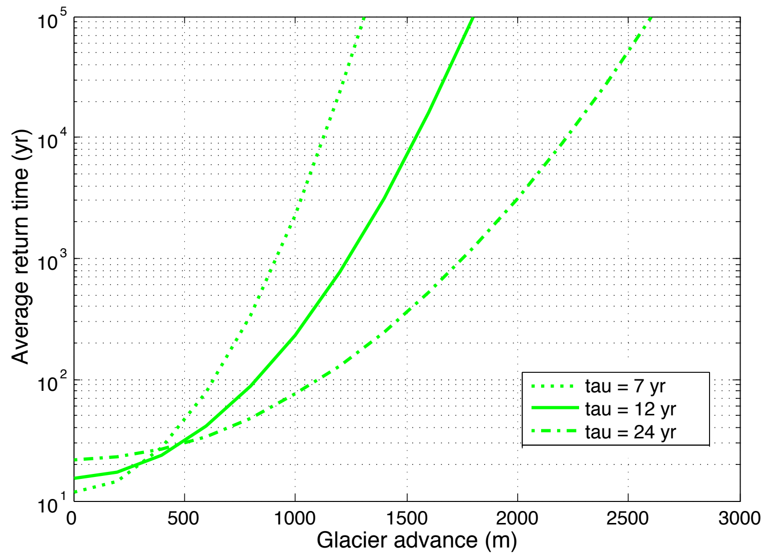


Figure 7: The average return time of a glacier advance (i.e., the interval between up-crossings of glacier length beyond a given threshold), calculated from (5). The three curves are for the range of parameters appropriate for a typical glacier on Mt. Baker, Cascades, WA. Note the logarithmic scale on the y-axis, and the acute sensitivity of the average return time to changes in glacier properties.

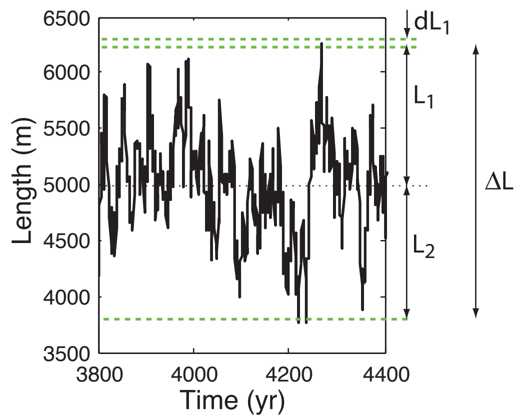


Figure 8: Schematic illustration for the calculation of the likelihood of exceeding a given total excursion.

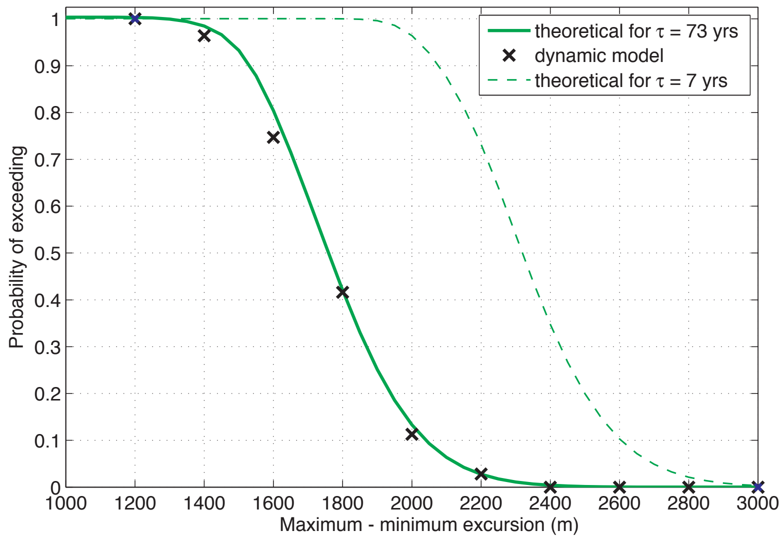


Figure 9: The probability of exceeding a given maximum total excursion (i.e., maximum advance minus maximum retreat), in any 1000 yr period. Crosses shows calculations from the dynamic model output. The curves are calculated from Equation (11) for two different response times.

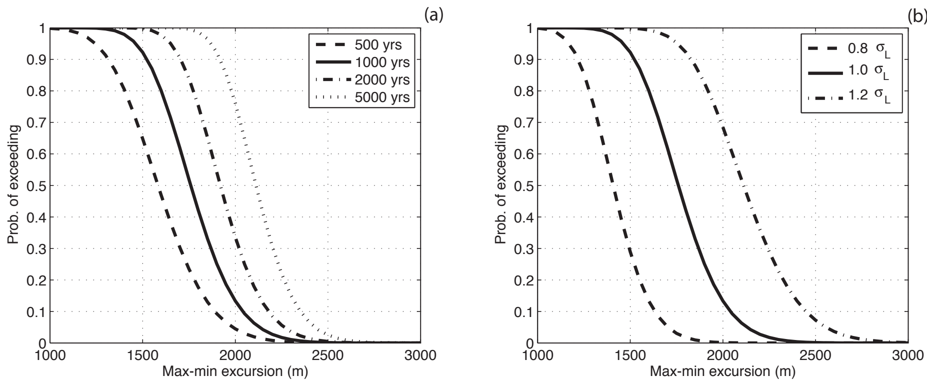
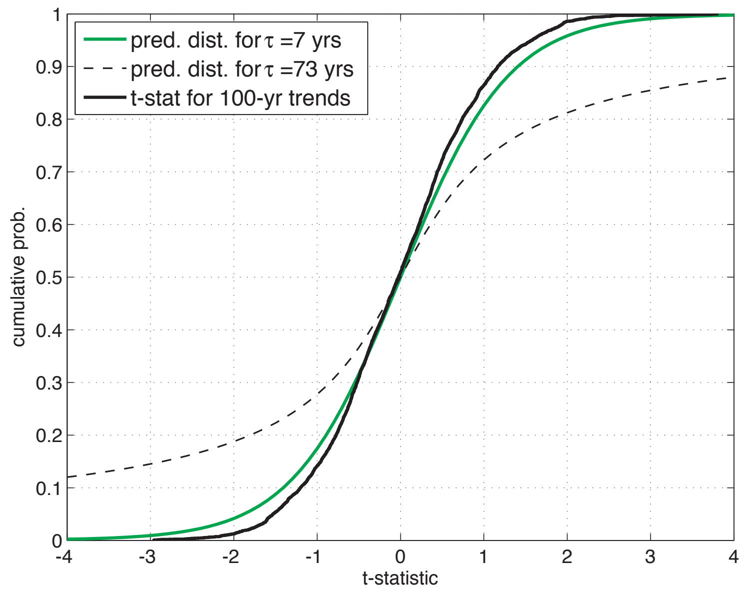


Figure 10: Probability of maximum excursions for different assumptions. a) probability of exceeding a given excursion, for different periods of time. Note the uneven time increments; b) probability of exceeding a given excursion, for different values of σ_L in Equation (11). All curves use the standard parameters for the dynamic model (except where σ_L is varied).



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