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How sensitive is climate sensitivity?

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Estimates of climate sensitivity are typically characterized by highly asymmetric probability density functions (pdfs). The reasons are fundamental and well known, but the situation leaves open an uncomfortably large possibility that climate sensitivity might exceed 4.5°C. We explore what changes in the pdfs of the observations or feedbacks used to estimate climate sensitivity would be needed to remove the asymmetry, or to substantially reduce it, and demonstrate that such changes would be implausibly large. The nonlinearity of climate feedbacks is calculated from a range of studies and is shown also to have very little impact on the asymmetry. There is a strong expectation that the pdf of climate feedbacks should be approximately symmetric because of the intrinsic relationship between observed and model-derived estimates of climate sensitivity. There have to be strong linkages between uncertainties in the observed climate forcing and the climate's radiative response to that forcing (i.e. the feedbacks).

1. Introduction

Climate sensitivity ($\equiv T_{2\times}$), the long-term response of global-mean, annual-mean, near-surface air temperature to a doubling of carbon dioxide above preindustrial concentrations, is a conceptually convenient metric for comparing different methods of estimating climate change. However, both the observations from which $T_{2\times}$ is estimated and the climate simulations from which $T_{2\times}$ is derived are uncertain, so that we cannot establish a single value but only its probability density function (pdf), $h_{T_{2\times}}(T_{2\times})$. Both observations and simulations yield highly skewed pdfs, with finite probabilities of large sensitivities [e.g., *Knutti and Hegerl*, 2008].

Because the large asymmetry of $h_{T_{2\times}}$ has been questioned [e.g., *Hannart et al.*, 2009; *Ghil et al.*, 2010; *Solomon et al.*, 2010], it is appropriate to revisit the underlying assumptions on which its derivation rests. First, $h_{T_{2\times}}$ must be consistent with observations, so we analyze what modifications of those observations would lead to a significantly more symmetric pdf. Secondly, we examine the effect of relaxing assumptions underlying the simple model of *Roe and Baker* [2007, hereafter *RB07*], who derived an asymmetric $h_{T_{2\times}}$ from the pdf of the total feedback factor f.

2. Estimates of climate sensitivity from observations

A linearization of Earth's energy budget is $H = R - \lambda^{-1}T$, where H is ocean storage, R is radiative forcing, and $\lambda^{-1}T$ is the climate response in terms of the global-mean, annual-mean, near-surface air temperature change, T, and the climate sensitivity parameter, λ. Let $R_{2\times}$ be the forcing due to a doubling of CO_2 over pre-industrial values ($\simeq 3.7\text{Wm}^{-2}$). Computation of the distribution of $h_{T_{2\times}}$ can be made purely from observations of the modern state via the relationship:

$$T_{2\times} = \frac{T_{obs} R_{2\times}}{(R_{obs} - H_{obs})}, \tag{1}$$

since H is zero in equilibrium. Simplifying notation, let $F_{obs} = R_{obs} - H_{obs}$. Pdfs of these quantities are related by:

$$\begin{aligned} h_{T_{2\times}}(T_{2\times}) = \int_0^\infty h_{F_{obs}}(F_{obs}) \cdot h_{Tobs}\left(\frac{T_{2\times} F_{obs}}{R_{2\times}}\right) \\ \cdot \frac{F_{obs}}{R_{2\times}} \cdot dF_{obs}. \end{aligned} \tag{2}$$

where $h_{Fobs}(F_{obs})$ and $h_{Tobs}(T_{obs})$ are the pdfs of the observations. Both are found to be nearly normal distributions [e.g., Fig. 2.20, *Solomon et al.*, 2007], given by

$$\begin{aligned} h_{Tobs}(T_{obs}) = \frac{1}{\sigma_T \sqrt{2\pi}} Exp[-\frac{T_{obs} - \overline{T}_{obs})^2}{2\sigma_T^2}], \\ \equiv \phi(T_{obs}, \overline{T}_{obs}, \sigma_T) \end{aligned} \tag{3}$$

and $h_{F_{obs}}(F_{obs}) = \phi(F_{obs}, \overline{F}_{obs}, \sigma_F)$. Various estimates of F_{obs} and T_{obs} have been made. We use values from *Armour and Roe* [2011] (hereafter *AR11*) of $\overline{F}_{obs} \pm \sigma_F = 0.90 \pm 0.55 \mathrm{Wm}^{-2}$, and $\overline{T}_{obs} \pm \sigma_T = 0.76 \pm 0.11°\mathrm{C}$, which are the same as *Solomon at al.* [2007] but updated with new ocean storage observations [*Lyman et al.*, 2010; *Purkey and Johnson*, 2010, and see auxiliary materials). We assume independent errors.

The skewed nature of $h_{T_{2\times}}$ estimated from observations (Fig. 1) is an inevitable result of the fractional uncertainty in F_{obs} being much larger than the fractional uncertainty in T_{obs}. *Allen et al.*, [2006] present several other estimates for various time periods: in all cases, observations and reconstructions are more constrained for temperature than forcing.

2.1. Can observation-based $h_{T_{2\times}}$ be unskewed?

How different would the aforementioned assumptions have to be in order to significantly reduce the asymmetry of $h_{T_{2\times}}$? As a metric for the symmetry of the sensitivity pdfs, we define

$$S \equiv \frac{T_{95} - T_{50}}{T_{50} - T_{05}}, \tag{4}$$

where T_x is that value of T for which the cumulative probability of exceeding it, is given by

$$p_{cum}(T_x) \equiv \int_{Tx}^{\infty} h_{T_{2\times}}(T_{2\times}) dT_{2\times}. \tag{5}$$

S is the natural metric to pick, given the focus of many studies on the 90% confidence bounds of $T_{2\times}$. A symmetric distribution has $S = 1$, whereas for $h_{T_{2\times}}$ based on AR11, $S = 6.0$. We now focus on $h_{F_{obs}}$ because it matters much more than $h_{T_{obs}}$. Let $h_{F_{obs}}$ now be represented by the so-called 'skew normal' distribution:

$$\begin{aligned} h_F(F_{obs}) &= \phi(F_{obs}, \overline{F}_{obs}, \sigma_F) \\ &\times (1 + Erf[(\alpha_F(F_{obs} - \overline{F}_{obs})/(\sqrt{2}\sigma_F)] \\ &\equiv \Psi_{sn}(F_{obs}, \overline{F}_{obs}, \sigma_F, \alpha_F). \end{aligned} \tag{6}$$

For $\alpha_F = 0$ this is the normal distribution given by Eq. (3); for $\alpha_F \neq 0$ the skewness of $h_F(F_{obs})$ has the same sign as that of α_F.

The parameters necessary to achieve $S \approx 1$ are given in Table 1, and the corresponding pdfs are shown in Fig. 2a,c. It is obvious that to remove the skewness completely would require a drastically different $h_{F_{obs}}$. We can conclude that, without unfeasibly large reductions in forcing uncertainty, or compelling arguments why $h_{F_{obs}}$ has to be highly asymmetric, some skewness is inevitable in $h_{T_{2\times}}$. For the rest of the paper, we ask whether that skewness might perhaps be, if not completely removed (i.e., $S = 1$), then moderated substantially, and pick $S = 2$ as our measure. Table 1 shows this requires an approximate halving of σ_F, a large increase in $\overline{F}_{obs}$, or an $\alpha_F \simeq 2.0$. The accompanying distributions are shown in Fig. 2b,d. Table 1 gives guidance to the search for justification of lower S by means of new observations.

3. Estimates of climate sensitivity from models

Climate sensitivity may also be estimated by diagnosing feedbacks within climate models. Let f be the linear sum of individual climate feedbacks, $f \equiv \Sigma_i f_i$. There then is a one-to-one correspondence between values of this total feedback factor, f, and $T_{2\times}$ [e.g., *Roe*, 2009]. Thus the pdf of $T_{2\times}$ can be calculated from $h_f(f)$, the pdf of f. To derive estimates of $h_{T_{2\times}}$, RB07 further assumed: 1) $h_f(f)$ is Gaussian:

$$h_f(f) = \phi(f, \overline{f}, \sigma_f), \tag{7}$$

and 2) feedbacks are independent of temperature, which led to the relationship between sensitivity $T_{2\times}$ and f:

$$T_{2\times}(f) = \frac{T_0}{1-f} \tag{8}$$

where $\lambda_0 = 0.3, T_0 = \lambda_0 R_{2\times} \approx 1.2^{\circ}$C. Assumptions (7) and (8) yield an asymmetric $h_{T_{2\times}}$. For current best estimates $\sigma_f = 0.13, \overline{f} = 0.65$ the resulting pdf has $S = 4.0$.

The skewed nature of $h_{T_{2\times}}$ is an inevitable result of the asymmetric amplification by the feedback response on the high side of the mode of $h(f)$, given our basic assumptions. This amplification serves to underscore the magnitude of the challenge of refining model-based estimates of the high side of $h_{T_{2\times}}$. It requires a high degree of confidence in the shape of the high side of $h(f)$ and, moreover, how that shape changes with mean climate state.

In previous work [*RB07* and *Roe and Baker*, 2011, hereafter *RB11*], we have shown that a model based on Eqs. (7) and (8) is supported by its ability to reproduce the multi-thousand member ensemble results of *climateprediction.net* results; by observational studies that find an approximately Gaussian distribution to the total feedback factor [*e.g., Allen et al., 2006*]; and by the fact that for a system of many feedbacks, the Central Limit Theorem would suggest that the distribution of $h_f(f)$ would converge on a Gaussian.

Despite these successes of the model, assumptions (7) and (8) have been questioned. *Hannart et al.*, [2009, hereafter *HDN09*] take issue with the RB07 result that it is hard to reduce the likelihood that $T_{2\times}$ is higher than the IPCC 'likely range' (i.e., $> 4.5^o$C) by reducing uncertainty in climate parameters, or equivalently in observations [*Allen et al.*, 2006]. They point out that Eq. (7) allows the possibility that $f \geq 1$, which they feel is an indictment of the model. However, in our view, if some combinations of model parameters that cannot be ruled out *a priori* do in fact lead to a total feedback factor that exceeds one, this should not be trivially or immediately dismissed since it may point to some real or possibly artificial compensation between model feedbacks [e.g., *Huybers*, 2009]. Eq. (8) has also been questioned by *HDN09*, by *Zaliapin and Ghil* [2010], and others. It is therefore appropriate to examine the effect of relaxing assumptions (7) and (8) on the symmetry parameter S.

3.1. Can model-based $h_{T_{2\times}}$ be unskewed?

We consider the following set of analyses, taken one at a time:

- Vary $\overline{f}, \sigma_f$, keep relationships (7), (8). We extend the arguments of *RB07* here.
- Let the pdf of feedbacks be asymmetric: $h_f(f) = \Psi_{sn}(f, \overline{f}, \sigma_f, \alpha_f)$: in order to decrease the asymmetry in $h_{T_{2\times}}$, α_f must be negative.

- Let the feedbacks be nonlinear: $f(T) = f_0 - 2a\lambda_0 T$, where f_0 is independent of temperature, and the constant a must be positive to reduce the asymmetry of $h_{T_{2\times}}$.

Table 2 shows that it is virtually impossible to achieve $S \to 1$ by any realistic single parameter change in the RB07 model: either the width of $h_f(f)$ must be extremely narrow, or the feedback distribution must be very asymmetric. The lowest value of S achievable for nonnegative $\overline{f}$ is 1.2. Table 2 also shows single parameter variations in the model that result in $S \approx 2.0$. The corresponding h_fs and $h_{T_{2\times}}$ are shown in Fig. 3, as well as the *RB07* model for comparison.

3.1.1. Nonlinear feedbacks

Allowing for nonlinearities (see RB11, and auxiliary materials), Eq. (8) is replaced by

$$T_{2\times} = \frac{-(1-f_0) + \surd((1-f_0)^2 + 4a\lambda_0^2 R_{2\times})}{2a\lambda_0}. \qquad (9)$$

The auxiliary materials derive the value of a from a large number of published studies. We find $a \leq 0.06$, from which $S \geq 2.8$. To achieve $S \approx 1$ requires a to be 20 times greater (Table 2). Fig. 3b shows the $h_{T_{2\times}}$ implied by Eq. (9) after adjusting f_0 so all curves pass through $f = 0.65, T_{2\times} = 3.5$°C, the best linear estimate for today's climate (see auxiliary materials). For $a = 0.11$, $S = 2$ and the high sensitivity tail ($T_{2\times} \gtrsim 8$°C) is cut off, while at lower values of a, $h_{T_{2\times}}$ is virtually identical to the linear model.

4. Why are observation-based and model-based estimates of $h_{2\times}$ so similar?

A striking feature of Fig. 1 is that observation-based and model-based estimates of climate sensitivity are very similar. If they differed wildly, it might perhaps imply that there was important unused information, or that there were troubling biases among different methods. Another reason for their similarity is also worth emphasizing. From Eq. (1) and the fact $\lambda = \lambda_0/(1 - \Sigma_i f_i T)$, we can write

$$\underbrace{\lambda_0 R}_{\text{(i)}} - \lambda_0 H = \frac{\lambda_0 T}{\lambda} = T - \underbrace{\Sigma_i f_i T}_{\text{(ii)}}. \tag{10}$$

λ_0 is known, H and T are well constrained in the current climate, and estimating λ is the goal. Term (i) on left-hand side of Eq. (10) reflects the principal source of uncertainty in observation-based estimates (the radiative forcing of aerosols), and term (ii) on the right-hand side reflects the principal source of uncertainty in model-based estimates, namely feedbacks. Eq. (10) therefore shows that these two approaches are equivalent to each other: uncertainty in the modern radiative forcing necessarily implies uncertainty in a climate model's radiative response. That is, a range of feedbacks are consistent with observations, and we lack the information in the global-scale energetics to constrain them better. Because $h_{R_{obs}}$ is nearly Gaussian [e.g., *Solomon et al.*, 2007], Eq. (10) is another reason to expect that h_f should be too. Moreover it is critical for future climate projections to appreciate that uncertainties in forcing are not independent of uncertainties in λ, though this is sometimes overlooked [e.g., Ramanathan and Feng, 2008; Hare and Meinhausen, 2006]. In fact, estimates of $h_{T_{2\times}}$ (or equivalently, h_λ) based on models are already somewhat narrower that than permitted by modern observations (Fig. 1), and would be narrower still if correlations among feedbacks were accounted for [*Huybers*, 2009]. If model-based estimates of $T_{2\times}$ are to improve to the point that they are significantly narrower than observation-based estimates, it requires a great deal of confidence that models represent the relationship between other aspects of the climate system and the global-scale energetics with sufficient skill [e.g., *Knutti et al.*, 2010]. A measure of whether such confidence exists is whether model-based estimates of climate sensitivity become formally used as a constraint to narrow uncertainties in climate forcing [*AR11*].

5. Discussion

We have developed a framework for examining how asymmetry in $h_{T_{2\times}}$ might be reduced. While we have only varied the parameters one at a time, we've shown that the asymmetry cannot be eliminated by any realistic change to the parameters of either the observed uncertainty distribution or the *RB07* model (see auxiliary materials for multiple parameter changes). We have also shown that, via global energetics, modeled and observed uncertainties in $T_{2\times}$ are intrinsically linked. Therefore *HDN09*, for example, overreach in asserting that the analysis of *RB07* is "a mathematical artifact with no connection whatsoever to climate".

We have not considered Bayesian approaches that try to combine multiple estimates of $h_{T_{2\times}}$. While in principle such techniques might lead to narrower and less skewed distributions, and while efforts still continue [*Annan and Hargreaves*, 2006, 2009], there are formidable challenges to objectively establishing: 1) the independence of different observations; and 2) how structural uncertainties within and among ever-more complex models affect the answer [e.g., *Lemoine*, 2010, *Henriksson et al.*, 2010; *Knutti et al.*, 2010].

Ominous consequences have been thought to follow from the skewness of $h_{T_{2\times}}$ [e.g., *Weitzman*, 2009]. The argument has been made that we should focus our efforts on decreasing the probabilities of high $T_{2\times}$ by making more accurate observations. Our results provide clear targets in terms of improved observations or more certainty among models.

However, this focus is to some extent misplaced. Firstly, because, as shown by RB07 and the present analysis, it would take large decreases in observed or modeled uncertainties to have much of an impact. Also, a reduction of uncertainty in F_{obs} or f moves the mode of $h_{2\times}$ to higher values. So, as noted in *RB07*, while the probabilities become more focussed, in other words the range – however measured – gets less, the cumulative likelihood beyond 4.5 °C remains stubbornly persistent. Secondly, and more fundamentally— $T_{2\times}$ is only a metric of a hypothetical global mean temperature rise that might occur thousands of years into the future. Very high temperature responses, if they develop, are associated with the very longest time scales [e.g., *Baker and Roe*, 2009]. On the other hand, in this century we face the very real threat of climate changes that will have very damaging impacts on life and society. While understanding the basic relationship between radiative forcing, climate feedbacks and climate sensitivity is important, arguments about the details of the pdf shape are not.

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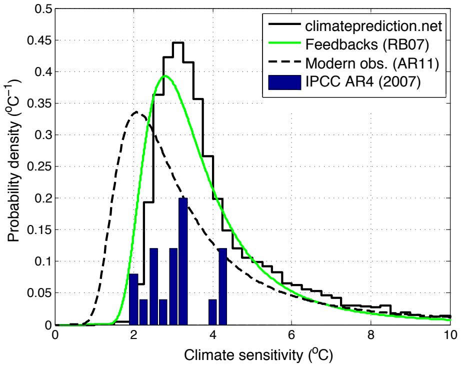

Figure 1. Pdfs of $T_{2\times}$ computed from perturbed physics ensembles [*Sanderson et al., 2008*], model-estimated climate feedbacks [*RB07*], modern instrumental observations [*AR11*]. A histogram of $T_{2\times}$ from IPCC AR4 [*Solomon at al., 2007*] models is also shown. The pdfs are normalized between 0 and ∞.

$\overline{F}$	σ_F	α_F	S
0.9	.55	0.	6.0
0.9	$\underline{3.6e\text{-}3}$	0.	1.0
$\underline{7.2}$	.55	0.	1.0
0.9	.55	$\underline{12}$.	1.3
0.9	$\underline{.21}$	0.	2.0
$\underline{4.9}$	.55	0.	2.0
0.9	.55	$\underline{2.1}$	2.0

Table 1. Variations in pdf of forcing, $h_{F_{obs}}$, and the impact on the asymmetry, S, of $h_{T_{2\times}}$. The first line are the standard combination of parameters for $h_{F_{obs}}$ in Eq. (6), and subsequent lines show the changes in parameters necessary to obtain the given value of the asymmetry parameter, S. In each case only a single parameter has been altered (shown underlined).

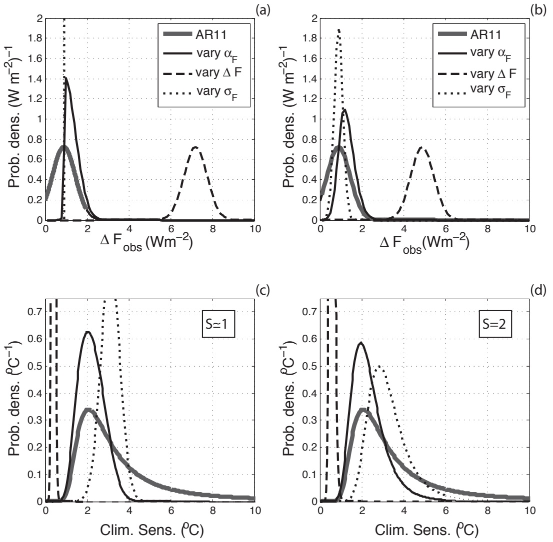

Figure 2. The effect of altered pdfs of radiative forcing observations (top panels) on the asymmetry of $h_{T_{2\times}}$ (bottom panels). The thick grey curve shows current uncertainties (AR11, $\alpha_F = 0, S = 6.0$) for comparison. a) and c) correspond to $S \simeq 1$. b) and d) correspond to $S = 2$. The pdfs are normalized between 0 and ∞.

Table 2. Variation of feedback model parameters and the impact on S.

σ_f	$\overline{f}$	α_f	a	S
0.13	0.65	0.	0.	4.0
1.1e-5	0.65	0.	0.	1.0
0.13	0.65	-5.1	0.	1.0
0.13	0.	0.	0.	1.2
0.07	0.65	0.	0.	2.0
0.13	0.65	-1.3	0.	2.0
0.13	0.31	0.	0.	2.0
0.13	0.65	0.	0.06	2.8
0.13	0.65	0.	0.11	2.0
0.13	0.65	0.	1.2	1.0

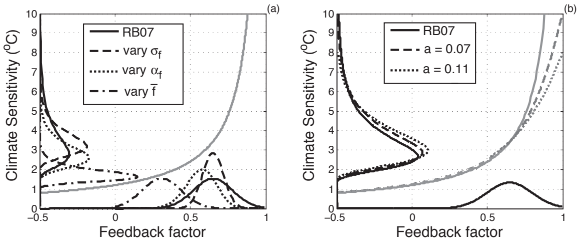

Figure 3. a) The effect on $h_{T_{2\times}}$ (y-axis) of varying the parameters controlling the shape of h_f (x-axis). Parameters correspond to those given in Table 2 for $S = 2$. Solid line shows the *RB07* model. (b) The effect of feedback nonlinearity parameter, a, on $h_{T_{2\times}}$. The grey lines show the $f - T_{2\times}$ relationships. See auxiliary materials for calculations of a from previous model studies. The pdfs of $h_{T_{2\times}}$ are normalized between 0 and ∞.