1. Introduction

[1] The influence of climate on the relief of mountain ranges has long been supposed but has received little quantitative investigation. The simplified system of a longitudinal river profile in a bedrock channel coupled to a physically based model of orographic precipitation is used to explore how orographically influenced patterns of precipitation affect the relief of fluvially sculpted mountain ranges. It is shown that the impact on the orography depends on the nature of the precipitation regime. Significant changes in relief also occur for imposed temperature changes of realistic magnitude, although the ability of the relief to respond to any given change depends sensitively on its response time and the timescale of the forcing. INDEX TERMS: 1815 Hydrology: Erosion and sedimentation; 1824 Hydrology: Geomorphology (1625); 3354 Meteorology and Atmospheric Dynamics: Precipitation (1854); KEYWORDS: erosion, orography, precipitation, relief, climate, feedbacks


1. Introduction

[2] The structure and form of landscapes is ultimately controlled by the interplay between tectonics, erosion, and climate. Among geomorphologists, interest in the influence of climate variability has been largely overshadowed by interest in the competition or relative balance between erosion and uplift in mountain range evolution. While atmospheric scientists have long understood that the largest mountain ranges such as the Himalayas-Tibetan Plateau and the Rocky Mountains, cause hemispheric-scale changes in atmospheric circulation, and hence in the position of the jet stream and storm tracks [e.g., Charney and Eliassen, 1949; Hoskins and Karoly, 1981], they have he been recognized that climate, in turn, influences the form and evolution of large mountain ranges [e.g., Koons, 1989; Hoffman and Grotzinger, 1993; Willett et al., 1993; Brozovic et al., 1997; Willett, 1999; Whipple and Tucker, 1999; Montgomery et al., 2001].

[3] Much of the discussion of the relation of climate to landscape evolution has centered on the role of relief development. Ruddiman et al. [1989] noted that the long-term cooling of climate over the last five million years coincides with evidence for increased mountain uplift around the globe. They suggested that enhanced chemical weathering of exposed rock surfaces in uplifted mountain ranges plays an important role in reducing the long-term level of atmospheric CO₂ and hence that tectonic uplift may have led to the onset of a glacial climate. Molnar and England [1990] offered the opposing hypothesis and argued that evidence for such uplift primarily reflected greater rates of exhumation and isostatic rebound due to higher erosion rates accompanying climatic deterioration. They argued that evidence for increased late Cenozoic rock uplift simply recorded the isostatic uplift of mountain peaks in response to enhanced excavation of valleys. Gilchrest et al. [1994] and Montgomery [1994] showed that the erosional deepening of valleys, which create relief, could account for at most a roughly 25% increase in the height of mountain peaks. On the basis of a model of a steady state river profile, Whipple et al. [1999] argued that there is little potential for enhanced river incision to substantially increase relief in a landscape with landslide-dominated threshold hillslopes. Montgomery and Greenberg [2000] showed how in a real landscape excavation of valleys could substantially increase the height of local mountain peaks.

[4] Although these studies addressed the potential for interactions between climate and relief development, relatively little attention had been paid to the interaction between orographic precipitation and relief. In actively uplifting mountain ranges the relief, and ultimately the rate of landscape exhumation, are set by river and glacier incision into bedrock [Seidl and Dietrich, 1992; Burbank et al., 1996]. Given that a river’s (or a glacier’s) power to incise into rock is controlled by the conversion of precipitation into river discharge (or ice flux), and that a fundamental feature of mountain climates is strong orographically driven gradients in precipitation, there is a potential for an interaction between relief development and precipitation patterns. Tucker and Bras [2000] and Snyder [2001] studied the impact of storm frequency on river profile evolution, and a few landform evolution models have implemented simple orographic rules for precipitation [Beaumont et al., 1992; Masek et al., 1994], but the consequences of the implied feedback have not been explored in depth and the representation of climate tends to be crude.

[5] This paper explores how patterns of precipitation affect the relief along a bedrock river channel. Such rivers are important in setting the relief of many mountain ranges [e.g., Whipple et al., 1999], and hence the results have important implications for how local climate and climate
change interact with topography. The reduced system of the longitudinal profile of a bedrock river is the simplest realistic framework that allows for an examination of the behavior of the coupled precipitation-orography system. The obvious advantage of using the reduced system is that the interactions and dependencies are clear and understandable, but the results also point to how that which is left out of the model formulation might affect the conclusions. The most important simplifications made in this work are (1) fixed tectonic forcing in the form of an imposed uniform uplift; (2) an assumption that the erosional mechanisms are constant so the system adjusts solely by changing river profile gradients; and (3) a simplified representation of orographic precipitation is used that omits the role of storms.

The above simplifications notwithstanding, we demonstrate first that both the pattern and amount of precipitation distributed over a given river basin are important in setting the relief of the main river channel within that basin and that the relief is particularly sensitive to precipitation close to the channel head. Second, by introducing a parameterization of orographic precipitation that depends on the form of the river profile, we show that the feedback between topography and precipitation constitutes an important control on the relief of the profile, and by extension on the relief of mountain ranges. The results emphasize two important controls on the orographic precipitation distribution: slope and elevation. The dependency on slope gives rise to a negative feedback on profile relief, and the dependency on elevation gives rise to a positive feedback. Real precipitation distributions are a combination of these two and other factors, and so the sign of the overall feedback will depend on the geometry and orientation of the mountain range and the local climate regime, and may well change during evolution of the orogen. Last we note that climate change in the form of changing surface temperatures alters the moisture carrying capacity of the atmosphere, and so acts as an important control on precipitation rates. Our formulation of the orographic precipitation allows us to impose plausible temperature changes and to show how such changes can impact relief.

The discussion section at end of the paper outlines in more detail what the consequences of the simplifying assumptions are, which aspects of the results are likely to prove robust, and where future work would be useful in addressing some of the issues raised.

2. Model

The simplest form of the stream power erosion law models the erosion rate, \( E \), as a power law function of discharge, \( Q \) and slope, \( \frac{dz}{dx} \):

\[
E = KQ^m \left( \frac{dz}{dx} \right)^n,
\]

where \( z \) is height, and \( x \) is the along channel distance, with \( x = 0 \) defined as the drainage divide. \( K \) is the erosion coefficient and \( m \) and \( n \) are the governing exponents, whose ratio depends on the assumed underlying physics. If the erosion rate depends on the stream power then \( m/n = 1.0 \); if it depends on the basal shear stress or the unit stream power then \( m/n = 0.5 \) [Whipple and Tucker, 1999]. Alternative formulations of the erosion law represent transport-limited erosion, or posit a threshold discharge or basal shear stress below which no erosion occurs [e.g., Tucker and Bras, 2000; Tucker and Whipple, 2002; Snyder, 2001].

In steady state the erosion rate must everywhere balance the uplift rate, \( U \), which we will assume to be spatially uniform. Equating equation (1) to the uplift rate and rearranging yields

\[
\frac{dz}{dx} = -\left( \frac{U}{K} \right)^{\frac{1}{m}} Q^{-\frac{n}{m}}.
\]

From equation (2), it can be seen that all factors that affect the discharge are scaled by an exponent of \((m/n)\) in their influence on slopes (and therefore relief) and therefore do not depend on the separate values of \( m \) and \( n \). This stems from the mathematical separability of the \( dz/dx \) and \( Q \) in the erosion law, and would not necessarily be the case for a formulation of the erosion law with a critical stress [e.g., Snyder, 2001].

Equation (2) is frequently reformulated in terms of slope and upstream drainage area, \( A(x) \), by assuming a power law relation between discharge and area of the form

\[
Q(x) = k_c A^c,
\]

which, upon substitution into equation (2), gives

\[
\frac{dz}{dx} = -\left( \frac{U}{K} \right)^{\frac{1}{m}} A^{-n},
\]

where \( c \) is the profile concavity and equals \((m/n)\) for \( c = 1 \), and the constant \( k_c \) has been subsumed into \( K \). Equation (4) is cast in terms of readily measurable quantities and so is the form most often compared to observations. Under the assumptions of steady state and spatially uniform uplift and lithology, \(-\theta\) equals the gradient of the log-slope versus log-drainage area curve for the river profile. River profiles typically have concavities of between 0.3 and 0.8 (Table 1) [see also Tucker and Whipple, 2002].

However, specifying discharge by equation (3) precludes a priori the possibility of an interaction between orography and precipitation, and it is a strong functional constraint on the precipitation distribution. Moreover, \( c \) is frequently set equal to one [e.g., Pazzaglia et al., 1998], which implies uniform precipitation over the drainage area. In fact, neglecting evapotranspiration and ground water leakage, the (steady state) discharge at any given location is more properly represented as the integral of the precipitation rate over the drainage area upstream of that point:

\[
Q(x) = \int_0^x p(x') \frac{dA}{dx'} dx',
\]

where \( p \) is the precipitation rate and \( x' \) is a dummy variable of integration [Roe et al., 2002]. Note that equation (5) means the discharge-area relationship in equation (3) implies a singularity in the precipitation rate at the drainage divide for values of \( c < 1 \).

Roe et al. [2002] showed that incorporating a precipitation feedback into equation (1) via equation (5) necessarily means that, even in steady state, the profile concavity would not be equal to \((m/n)\) and that a significant
part of the variability between observed profiles may reflect the precipitation feedback.

Equations (1) and (5) encapsulate the feedback through which the form of the topography controls the spatial variation in precipitation and hence the river discharge. In turn, the discharge controls the development of the river profiles that drive topographic development (Figure 1).

### 3. Impact of a Specified Precipitation Distribution

Before introducing a quantitative feedback between the topography and the precipitation, it is useful to examine the impact on the river profile of a simple specified precipitation distribution. Accordingly, we take a precipitation field that is uniform except for a delta function anomaly located at some point, \( x_0 \), along the stream channel:

\[
p(x) = p_0 + p_1 \Delta \cdot b(x - x_0).
\]

Observations show that to a good approximation drainage areas across a wide range of scales are well represented by \( A(x) = k_x x^h \) with \( k_x = 1/3 \), and \( h = 2 \) [Montgomery and Dietrich, 1992]. Substituting this and equation (6) into equation (5) results in different expressions for \( Q(x) \) on either side of \( x_0 \), a point an infinitesimal distance upstream of \( x_0 \):

\[
Q(x) = \begin{cases} 
p_0 k_x x^h & 0 \leq x \leq x_0 \\
p_0 k_x x^h \left[ 1 + \frac{h p_1 \Delta}{p_0 x_0} \left( \frac{x}{x_0} \right)^h \right] & x_0 < x \end{cases}
\]

Upstream, the expression for discharge of the anomaly is the same as for uniform precipitation. Downstream, the relative impact on \( Q \) is greatest when \( x_0 \) is small, since the anomaly is a larger fraction of the total discharge close to the channel head, and Second, the impact on \( Q \) decreases with increasing \( x \).

To explore the effect of this precipitation pattern, fluvial erosion is assumed to take place in the domain defined by \( x_0 < x < L \). The channel length, \( L \), is taken to be 30 km, and the lower boundary condition is taken to be that \( z = 0 \) at \( x = L \). \( x_0 \) is the location of the channel head, and is the critical drainage length required for the formation of fluvial channels. It is set at 400 m. \( U \) and \( K \) are chosen such that for a uniform precipitation rate of 1 m yr\(^{-1} \), which is typical of midlatitudes, the relief of the fluvial portion of the channel (i.e., \( z(x) - z(L) \)) would be 3000 m. The river profile can be readily obtained by integrating equation (2) using the appropriate expression for \( Q \) in each part of the domain. For some combinations of \( h, m, \) and \( n \), an exact analytical solution can be found, but it is also straightforward to integrate numerically.

We take \( p_1 \Delta = 2000 \text{ m}^2 \text{ yr}^{-1} \), equivalent to an extra 1 m yr\(^{-1} \) over a 2 km segment of the channel. This is a moderate-amplitude anomaly when compared with observations of orographic precipitation, but nonetheless it has a significant effect. Figure 2 shows the change in relief for a variety of combinations of \( h, m, \) and \( n \). While the river profile adjusts smoothly to the precipitation anomaly over its entire length, the impact on the channel relief corresponds closely to the impact on the discharge: the greatest reduction in relief occurs when the anomaly is located near the channel head. For \( x_0 < 1 \text{ km} \), the reduction in relief is between 200 m and 1 km, depending on the values of the exponents. With the increase in discharge, a shallower slope can balance the same uplift rate in equation (2), and hence

\[
dz = \left( \frac{U}{K} \right) q \frac{m}{n} dz
\]

\[
p(x) = f \int z(x) dz
\]

**Figure 1.** Illustration of the feedback loop. Topography (i.e., the slopes and elevations) influences the precipitation which controls the river discharge, which in turn shapes the topography.

**Figure 2.** Fluvial relief versus location of imposed delta function in precipitation, for a variety of values of \( h, \) \( m, \) and \( k_x p_1 \Delta = 2000 \text{ m}^2 \text{ yr}^{-1} \). Channel length is 30 km. See text for details.
the overall relief is reduced. The effect of the anomaly on the relief does not depend strongly on the length of the channel chosen, or on the choice of $U$ and $K$. Larger values of $h$ cause a larger reduction in relief for small values of $x_0$, but the magnitude of the reduction falls off more quickly with increasing $x_0$.

[18] Taking log derivatives of equation (2) with fixed $K$ elucidates this behavior:

$$\frac{\Delta (dz/dx)}{dz/dx} = \left( \frac{m}{n} \right) \frac{\Delta Q}{Q} + \left( \frac{1}{n} \right) \frac{\Delta U}{U}. \tag{8}$$

Assuming $U$ to be constant, then fractional changes in discharge cause fractional changes in the slope, weighted by a factor of ($m/n$). Therefore the steeper the slopes (i.e., near the divide), the greater the impact of discharge changes will tend to be. Note that in the case where $U$ is not constant, then for fixed ($m/n$), the higher the value of $n$ the greater will be the influence of discharge changes relative to uplift changes.

[19] The particular importance of the precipitation near the channel head can be shown by specifying a second distribution. Precipitation is taken to be uniform and equal to 1 m yr $^{-1}$ over the whole domain, except for anomalies imposed in two segments between 0 and 5 km and between 10 and 15 km, and whose magnitude is either $-0.5$, 0, or $+0.5$ m yr $^{-1}$. Table 2 shows the results for the nine possible combinations, using the parameters $h = 2$, $k_d = 1/3$, and $m/n = 1/2$ (which are also used for all remaining results presented). Imposing the negative anomaly close to the divide reduces the discharge and causes an increase in the relief of 700 m. Similarly, the positive anomaly increases the discharge and lowers the relief by nearly 400 m. Imposing anomalies further downstream causes the same sign of change in the relief, but the magnitude of the change is very much reduced. While it is not surprising that it is the precipitation near the channel head that most affects the relief, it is important to emphasize that even modest variations in the upper reaches of the channel network can have significant impact on the relief.

### 4. Parameterization of Orographic Precipitation

[20] Precipitation is highly variable in mountainous regions [e.g., Smith, 1979; Houze, 1993; Barros and Lettenmaier, 1994]. For the question of river profile evolution, we have to consider what aspects of the precipitation distribution are likely to be robust on long ($>\sim 10^5$ years) timescales. This section begins by reviewing the mechanisms of orographic precipitation and then outlines a simplified parameterization that will be coupled to the river profile model.

[21] The most familiar instance of orographic precipitation is where a mountain range lies across the prevailing wind direction: forced ascent up the windward slopes cools the air column, leading to saturation and enhanced precipitation; conversely, descent over the leeward slopes warms the air and suppresses precipitation. This results in the well-known “rain shadow”, often associated with dramatic differences in vegetation and climate across a mountain range. A second mechanism occurs when low-level clouds sitting over small hills or within valleys act to locally enhance larger-scale rainfall, as the falling raindrops coalesce with the low-level cloud droplets (the so-called “seeder-feeder” mechanism) [Bergeron, 1960]. Third, in a conditionally unstable atmosphere, forced ascent or daytime heating may cause an air parcel to rise above its level of free convection [e.g., Smith, 1979]. This results in further unstable ascent, and precipitation. In addition to these mechanisms, orography may influence precipitation indirectly by affecting the airflow, for example by causing internal atmospheric gravity waves [Robichaud and Austin, 1988], or through blocking and variations in the atmospheric stability [Sinclair et al., 1997; Rotunno and Ferretti, 2001]. These airflow changes can influence the precipitation distribution. Last, although not directly related to orography, a complex array of cloud microphysical processes, strongly dependent on the ambient atmospheric conditions, determine the rate of formation and fall time of precipitation [e.g., Houze, 1993; Barros and Lettenmaier, 1994].

[22] Despite the myriad complexities of orographic precipitation, relatively simple parameterizations have some success in reproducing observed distributions of precipitation both for individual storms [e.g., Colton, 1976; Alpert, 1986; Sinclair, 1994], and in the time average [e.g., Alpert, 1986; Barros and Lettenmaier, 1992; Roe, 2002]. This lends confidence that at least in certain regions such parameterizations either capture directly the relevant physics, or reasonably represent the time-mean effects of the various precipitation-producing mechanisms. The parameterization used here is the same as Roe et al. [2002]. It is similar to that used in previous work [e.g., Sanberg and Oerlemans, 1983; Alpert, 1986; Sinclair, 1994], and emphasizes the moisture content of the air and the prevailing wind direction.

[23] To a good approximation the vertically averaged atmospheric moisture content is proportional to the saturation vapor pressure at the surface, $e_{sat}(T_s)$, which is given by the Clausius-Clapeyron equation, an approximation to which is $e_{sat}(T_s) = e_0 \exp(a (T_s - T_s))$, where $e_0$, $a$, and $b$ are constants [e.g., Emanuel, 1994]. $T_s$ is the surface temperature, given by $T_s(x) = T_s(L) - \Gamma z(x)$, with $\Gamma$ the atmospheric lapse rate, assumed herein to be $-6.5 \, \text{°C km}^{-1}$. (25)

[24] Letting $\nabla \cdot F$ represent the convergence of vertically averaged moisture flux, we take

$$-\nabla \cdot F = \left( c_0 + c_1 \frac{dz}{dx} \right) e_{sat}(T_s). \tag{9}$$
The parameter $\alpha_0$ represents a background convergence that occurs in the absence of orography. The term containing $\alpha_1$ is the convergence associated with upslope or downslope flow; the greater the prevailing wind, $v$, or the steeper the slopes, $dz/dx$, the greater is the moisture convergence. We pick values for $\alpha_0$ and $T_c(L)$ such that $-\nabla \cdot \mathbf{F} (x = L) = 1 \text{ m yr}^{-1}$ for $dz/dx = 0$. Equation (9) may be compared to a steady state water vapor budget [Roe, 1999], from which we take $\alpha_1 = 110 \text{ m yr}^{-1}/\text{m s}^{-1}$.

Spatial smoothing is used to account for the finite formation time of raindrops, their advection by the prevailing wind, and their descent to the ground. A Gaussian-shaped upwind weighting function is applied, so that the precipitation is given by

$$p(x) = \frac{2}{\Delta x \sqrt{\pi}} \int_{-\infty}^{\infty} -\nabla \cdot \mathbf{F}(x') \exp \left[ -\left( \frac{x - x'}{\Delta x} \right)^2 \right] dx', \quad (10)$$

$\Delta x$ is a smoothing scale which can be regarded as also incorporating some variation of the wind speed over time. We treat the wind speed and smoothing scale here independently whereas for any given storm they are related. However, in reality, the climatological precipitation pattern is the cumulative result of many separate storms and is thus not directly a function of the average winds and temperatures per se. In effect, $\Delta x$ and $T$ are used as tunable parameters to obtain precipitation rates consistent with observations. In the context of mountain range evolution, the timescales and other uncertainties involved mean that any precipitation parameterization is best regarded as encapsulating the qualitative physics of orographic precipitation and its functional dependence on climate and surface parameters.

In the quasi one-dimensional geometry of longitudinal river profile analysis, specifying the precipitation rate by equations (9) and (10) implies certain assumptions. First, we have assumed that longitudinal variations in precipitation dominate, that the contribution of discharge from tributaries can be ‘smeared out’ to contribute smoothly to the main river discharge, and that the tributaries have not carried precipitation from areas remote from the main channel. Second, since precipitation has been parameterized as a function of the slope along the river profile only, the influence of ridge profiles has been neglected. These are likely to be a serious flaws for complicated basin geometries where there is no correspondence between the ridge and river profiles. However, we will explore the consequences of a feedback of either sign, and so the results presented can be thought of as encompassing the range of behaviors of the coupled system.

5. Impact of a Precipitation Feedback on Relief

We seek to generate a coupled model which contains a plausible response of precipitation to orography, rather than a model able to capture the full complexities of orographic precipitation. In the results presented below we draw out the role of each factor by considering two precipitation regimes, illustrated schematically in Figure 3. We focus on the windward side of the range. The first regime can be considered most typical of smaller, narrower ranges [e.g., Smith, 1979] where precipitation is dominated by the prevailing upslope winds and precipitation maximizes at or near the drainage divide. The second regime is more characteristic of broader, taller ranges (or a plateau), where precipitation at high elevation is determined more by the moisture content of the air and decreases with elevation near the divide [Smith, 1979; Alpert, 1986]. For example, the Sierra Nevada [Colton, 1976], the Alps [Frei and Schar, 1998], and the Southern Alps of New Zealand [Watt et al., 1996] all show climatological precipitation maxima which are displaced significantly windward of the divide.

Figure 3. Schematic illustration of two different precipitation regimes. Smaller, narrower mountain ranges tend to experience precipitation which maximizes at or near the divide. Larger, broader ranges have precipitation which tends to be displaced significantly from the divide. The gray lines indicate the profiles of major river channels. See text for more details.

[26] In the quasi one-dimensional geometry of longitudinal river profile analysis, specifying the precipitation rate by equations (9) and (10) implies certain assumptions. First, we have assumed that longitudinal variations in precipitation dominate, that the contribution of discharge from tributaries can be ‘smeared out’ to contribute smoothly to the main river discharge, and that the tributaries have not carried precipitation from areas remote from the main channel. Second, since precipitation has been parameterized as a function of the slope along the river profile only, the influence of ridge profiles has been neglected. These are likely to be serious flaws for complicated basin geometries where there is no correspondence between the ridge and river profiles. However, we will explore the consequences of a feedback of either sign, and so the results presented can be thought of as encompassing the range of behaviors of the coupled system.

[27] We seek to generate a coupled model which contains a plausible response of precipitation to orography, rather than a model able to capture the full complexities of orographic precipitation. In the results presented below we draw out the role of each factor by considering two precipitation regimes, illustrated schematically in Figure 3. We focus on the windward side of the range. The first regime can be considered most typical of smaller, narrower ranges [e.g., Smith, 1979] where precipitation is dominated by the prevailing upslope winds and precipitation maximizes at or near the drainage divide. The second regime is more characteristic of broader, taller ranges (or a plateau), where precipitation at high elevation is determined more by the moisture content of the air and decreases with elevation near the divide [Smith, 1979; Alpert, 1986]. For example, the Sierra Nevada [Colton, 1976], the Alps [Frei and Schar, 1998], and the Southern Alps of New Zealand [Watt et al., 1996] all show climatological precipitation maxima which are displaced significantly windward of the divide.

[28] For the results presented below, the domain size is kept the same as in section 3. The lower boundary condition is $z = 0$ at $x = L$, and over the range $0 < x < x_c$, the profile slope is kept fixed at its value at $x_c$. Equations (2) and (10) are readily solved numerically using a straightforward downslope differencing scheme. Unless otherwise noted the parameters used in the erosion model are $m = 1/3$, $n = 2/3$, $U = 2 \text{ mm yr}^{-1}$, $K = 4 \times 10^{-2} \text{ s}^{-2/3}$, and $L = 30 \text{ km}$. For a uniform precipitation rate of $1 \text{ m yr}^{-1}$ this gives a relief along the channel (i.e., the fluvial relief) of $2650 \text{ m}$ between $x = x_c$ and $x = L$.

[29] The first regime, referred to as the full feedback case, is represented in the model by taking $\beta = 0.5 \text{ m s}^{-1}$ and $\Delta x = 30 \text{ km}$. The prevailing upslope winds produce a precipitation distribution that reflects the shape of the river profile, increasing toward the divide and reaching a maximum rate of $3 \text{ m yr}^{-1}$ (Figure 4a). Such a pattern and range of annual-mean rates are seen on west coast ranges in midlatitudes, and the maxima is considerably less than rates that can occur along the southern flank of the Himalayas, for example.

[30] Figure 4b shows the impact of this precipitation distribution on the river profile. Compared to uniform precipitation of $1 \text{ m yr}^{-1}$, the increase in discharge near the divide causes the river profile to flatten out somewhat, and the total relief is reduced by $950 \text{ m}$ to $1700 \text{ m}$. It is both the amount and the pattern of precipitation that act to control the relief: the average precipitation rate over the channel domain for the full feedback is $1.65 \text{ m yr}^{-1}$, and applying this uniformly over the channel produces a relief of $2050 \text{ m}$ (not shown).
As was shown by Roe et al. [2002], the distribution of precipitation affects the river profile concavity ($q$ in equation (4)), which can be seen on a log slope-log area graph (Figure 4c). In the case of uniform precipitation (of any magnitude) the profile concavity equals $m/n$, the ratio of the exponents in the governing erosion law (i.e., equation (1)). If the precipitation rate increases toward the channel head however, the profile concavity is reduced (to a mean value of about 0.43 in this case). Roe et al. [2002] point out therefore that since the magnitude of the orographic precipitation differs between locations, the presence of a feedback complicates the interpretation of river profiles.

The second regime considered is the case of precipitation controlled by the atmospheric moisture content via the Clausius-Clapeyron relation (hereinafter referred to as the C-C feedback). This is achieved in the model by setting $v = 0$ and $\Delta x = 10$ km. The effect of this is that precipitation decreases with elevation, with a minimum of $0.52$ m yr$^{-1}$ at the divide (Figure 4b). The resulting reduction in discharge necessitates steeper slopes to balance the same uplift rate, and consequently this causes the fluvial relief to increase to 3200 m (Figure 4b). This precipitation pattern also produces an increase in the curvature of the river profile, and in this model integration $\theta$ has an average value of about 0.54 (Figure 4c).

### 5.1. Varying the Feedback Strength

The strength of the feedback in this model is determined completely by the prevailing wind strength and the assumed smoothing scale. Figure 5 shows the precipitation...
rate at the divide and the corresponding relief for a range of values of $v$ and $C_1 x$. For $v = 0$, precipitation decreases with height, and the feedback is determined by $C_1 x$. For a smoothing scale greater than around 15 km, the reduction in the precipitation at the divide is moderate (less than 40 cm yr$^{-1}$), and the increase in relief is limited to a few hundred meters or less. As the smoothing scale becomes shorter, the increase in relief becomes progressively larger. If no smoothing is used there is no steady state solution: the exponential decrease in moisture availability more than offsets the effect of the steepened slopes, and therefore the uplift rate cannot be balanced by fluvial erosion. However, as the relief increases, slopes will either eventually steepen sufficiently for landsliding, or rise above the permanent snow line, in which case glacial erosion will act to limit relief [Schmidt and Montgomery, 1996; Hallet et al., 1996; Burbank et al., 1996; Brozovic et al., 1997; Montgomery et al., 2001; Burbank, 2002].

For positive values of $v$, the precipitation increases toward the drainage divide. The shorter the smoothing scale, the more the heavy precipitation is focused near the channel head, and the greater is the reduction in fluvial relief. Annual-mean precipitation rates in excess of 5 m yr$^{-1}$ are not often observed though and so this probably represents a rough upper limit on the strength of the feedback. Even a relatively weak feedback ($v = 0.1$ m s$^{-1}$) reduces the relief by hundreds of meters in this example. [35] The curves in Figure 5 were chosen to span the plausible range of parameter space for the feedback, and show quite how strongly mountain relief is controlled by its interaction with the precipitation distribution.

5.2. Varying the Channel Length

Figure 6 shows the effect of varying the channel length, $L$. In these calculations $K$ was chosen so as to keep the no-feedback fluvial relief fixed at 2650 m as $L$ varied. For the full feedback and the standard parameters, the fluvial relief varies between 1550 m for $L = 3$ km to 1820 m for $L = 60$ km. As the channel length increases the profile slopes decrease and the maximum precipitation is reduced (Figure 6a). In the case of the C-C feedback, a larger domain size reduces the influence of the background precipitation rate near the divide (which is felt via the applied smoothing). Therefore, as $L$ increases from small values, near-divide precipitation is reduced and the relief increases. For channel lengths exceeding two smoothing scales the influence of the background precipitation is minimal, and the effect of the feedback asymptotes to a fixed value. [37] The fluvial relief resulting from uniform precipitation equal to the average of that produced by the feedbacks is also shown (dashed lines, Figure 6). For small domain sizes most of the relief change results from changes to the
average precipitation. For larger domain sizes, the pattern of precipitation becomes increasingly important. This simply reflects that as $L$ increases the average precipitation tends to background rate. This is partly a consequence of the simplified model geometry we have assumed.

5.3. Effect of Variations in Uplift Rate

Uplift rates may vary over time in response to tectonic changes or there may be local spatial gradients due to deformation within a mountain range. For spatially uniform precipitation the channel relief varies as $\sim U^{1/n}$ (i.e., integrating equation (2)). However, for the general case where the precipitation distribution (and therefore the erosion rate) is a function of the shape of the mountain (or river profile) the relief will show a different dependence on uplift rate.

The C-C feedback (i.e., only taking the column moisture into account) is positive. Accelerating the uplift elevates the profile, which in turn reduces the precipitation and consequently the discharge. Again, steeper slopes are required to balance the uplift in steady state, and the relief is therefore greater than it would have been without the C-C effect on the precipitation. For the case of $n = 2/3$, a doubling of the uplift rate from 1.5 to 3 mm yr$^{-1}$ increases the relief by a factor of 2.8 (i.e., $2^{2/3}$) for the no feedback case. When the C-C feedback is included, then for the same increase in uplift, relief roughly quadruples (Figure 7). Because the dependence of the precipitation on temperature (and therefore elevation) is exponential, the effect of the feedback increases with uplift rate. However, as noted in the previous section, this feedback will ultimately be limited by when nonfluvial processes come to dominate the erosion rate. When the prevailing winds are included, the full feedback is negative, and for a given change in uplift rate the change in relief is reduced to about half that of the no feedback case.

Our example shows an effect of the orographic precipitation feedback is that the dependence of steady state relief on uplift may be from as little as one half up to as much as twice that for uniform precipitation. It is important to note that for a given mountain range the sign of the feedback can depend on the specifics of the precipitation regime. For example, the general situation can exist where the precipitation near the divide occurs mainly due to advection of rain formed over slopes at lower elevations. Thus while the observed precipitation might decrease with elevation, an
increase in uplift rate could lead to steepening of the low elevation slopes and increase the advected precipitation (and thus the erosion) near the divide. So while it is clear that the precipitation feedback can have significant impact on relief, it is not easy to answer how it might have influenced relief over the uplift history of any given range.

6. Response of Relief to an Imposed Temperature Change

[41] The previous sections have shown that precipitation and its interaction with orography act as an important controls on relief. It is natural therefore to ask how relief may have responded to large-scale climate changes, such as the quasiperiodic ice ages of the Pleistocene. The model presented can begin to address these questions. Because precipitation has been parameterized in terms of prevailing winds and temperatures, we can readily explore how the river profile relief can respond to plausible imposed changes in those fields. Temperature, acting via the Clausius-Clapeyron relation affects how much water the atmosphere can hold, and is a fundamental control on how much precipitation falls. For example, it largely accounts for the latitudinal distribution of precipitation on the Earth [e.g., Peixoto and Oort, 1992]. Glacial to interglacial temperature changes are thought to be around 10°C in mid to high latitudes which corresponds to changing the saturation vapor pressure (and hence the atmospheric moisture content) by roughly a factor of two. A similar temperature change can be associated with the long term climate cooling over the last fifty million years which lead up to the onset of the ice ages [e.g., Zachos et al., 2001].

[42] To be sure, other factors also have an important role in precipitation rates, especially regionally. The locations of storm tracks (i.e., the regions of maximum storminess) affect both the frequency and intensity of precipitation events on interannual timescales. Results from global climate models suggest that the storm track locations vary in different climate regimes [e.g., Hall et al., 1996; Kageyama et al., 1999]. On longer timescales, the presence and evolution of the large mountain ranges themselves (e.g., the Himalaya-Tibetan Plateau, or the Rockies) affect the hemispheric-scale atmospheric circulation, the position of the storm tracks, and in turn the precipitation.

[43] At a given time, the local rate of change in elevation is just the imbalance between uplift and erosion, so the evolution of the profile can be determined by integrating

\[
\frac{dz}{dt} = U - KQ^n \left( \frac{dz}{dx} \right)^{n-1} \frac{dz}{dx},
\]

This is a nonlinear kinematic wave equation. With no feedback \( Q \) is a function of position only, and changes to the erosivity or uplift are manifest as an upstream-migrating wave of erosion [e.g., Luke, 1972; Whipple and Tucker, 1999], moving with speed \( c(x) \). However, the presence of the feedback allows information to propagate downstream as well. A change in the slope or elevation near the channel head is felt throughout the profile via its impact on precipitation and hence on the discharge.

[44] We present results below of how relief changes in response to imposed temperature changes. For integrations including the feedback, the precipitation is affected via equations (9) and (10). In the integrations without a feedback the precipitation is kept uniform, but multiplied by the ratio of the new to the old saturation vapor pressures. The magnitude of the feedback depends on the initial relief and so \( K \) was adjusted in the different integrations such that the initial steady state relief was in each case about 3000 m. All other parameters were kept at the values used in section 5, and results are presented as a percentage change from the initial relief.

[45] A 10°C increase approximately doubles the available moisture. At \( t = 0 \), before the profile has a chance to respond, the discharge is doubled and the instantaneous erosion rate increases by a factor of \( 2^m \), or 1.25 for \( m = 1/3 \). Eventually the erosion rate must return to its previous value since in steady state it must still balance the same uplift rate. Figure 8 shows that the warming-induced increase in precipitation lowers the equilibrium relief 25% to 35%, depending on the feedback. Similarly, the cooling leads to an increase in relief of between 40% and 65%. The effects of the feedbacks are consistent with their signs as argued for in section 5.2. The C-C feedback (positive) amplifies the response to the imposed climate change, and the full feedback (negative) reduces the response.

[46] Changes in wind strength have a similar effect to the results shown here. An increase in prevailing wind increases the orographic precipitation and reduces the relief consistent with the results of the sensitivity tests shown in Figure 5.

6.1. Response Time

[47] For both feedbacks the time rate of change is initially faster than for no feedback case. However, for these parameters the difference is slight and the time taken to reach the new steady state is determined more by what the new relief is, consistent with the results of Whipple and Tucker [1999].

[48] Whipple and Tucker [1999] show further that for uniform precipitation the response time, \( \tau \), for a sudden drop in base level is determined by how long it takes for a
knickpoint to propagate from the base to the channel head. This scales as

\[ \tau \propto U^{(\frac{1}{1+C})} L^{(\frac{2}{C})} \left( 1 - \frac{x}{C_1^{t/2}} \right) \]  

(12)

with a slightly different expression for the special case of \( \delta m/n = 1 \). So \( \tau \) depends on the basin size, the uplift rate and erosivity, as well as the exponents in the erosion law. The latter, for example, are poorly constrained by observations and subject to large uncertainties [e.g., Stock and Montgomery, 1999; Whipple and Tucker, 1999]. With \( n = 2/3 \) the timescale is on the order 10^5 years (Figure 8). However, for \( n = 2 \) (which lies within the uncertainty) and keeping \( m/n = 0.5 \), the timescale is more nearly 10^5 years.

[49] This sensitivity of the response time to the governing parameters complicates understanding the role of climate feedbacks in the evolution of mountain ranges. For example, the presence C-C feedback lowers discharge along the channel, and so for a given area and slope the local erosion rate is reduced. But the feedback also means that in steady state a given relief can be produced by a lower uplift rate. And hence by the scaling of equation (12), the response time for a river profile in that range will be reduced for \( n < 1 \), and increased for \( n > 1 \).

[50] Nonetheless, our analysis does show that substantial changes in relief (of around 50%) could be forced by climate changes of realistic amplitude. However, the characteristic timescale of the system is important in determining the timescale of climate forcing that the relief will respond to. For a system with a response time less than \( \sim 10^5 \) years a large change in relief is possible in response to the quasiperiodic 10^5 year ice ages of the late Pleistocene, as suggested by Koons [1989]. The relief of a system with a longer timescale will reflect less the cyclic ice age forcing and more the longer-term climate changes. Note though, that even if the profile shape and relief respond slowly, the discharge and erosion rates respond immediately to precipitation changes and, hence, will still reflect the timescale of the climate forcing [see also Whipple, 2001].

7. Summary and Discussions

[51] The amount and pattern of precipitation both play a major role in setting the relief of mountain ranges. By examining the reduced system of a longitudinal profile of a bedrock river channel governed by the stream power erosion law, we have demonstrated that the precipitation near the divide is of particular importance in controlling steady state relief. In order to maintain a balance with the uplift, a change in the river discharge forces a change in the slopes of the opposite sign. Since the profile slopes are steepest near the channel head, a given fractional change in the slopes there results in the larger change in relief than for slopes lower down the profile.

[52] To some extent, any mountain range also shapes the local climate in which it exists, leading to a pronounced feedback between orographic precipitation and the relief. Introducing a plausible coupling between the river profile evolution and the pattern of precipitation demonstrated the significance of the feedback. We focused on two important factors influencing the precipitation pattern: prevailing upslope winds, giving rise to a negative feedback on relief, and elevation effects which give rise to a positive feedback.

[53] The use of a simplified model framework needs to be emphasized. One can easily imagine a more complex set of interactions in reality. For example, as a mountain range grows, it is reasonable to suppose that in some circumstances an increasing number of storms are blocked on the flanks of the range, rather than traversing it [e.g., Smith, 1979]. This would reduce rainfall in the interior, but quite possibly by a different amount than that predicted by the simple elevation effect shown in section 5. Our key point, therefore, is not the specifics of our examples, but that they demonstrate that as long as there is some robust relationship between topographic form and precipitation, then a feedback will exist comparable to that demonstrated here.

[54] The degree to which the different feedbacks operate in different climate regimes needs to be studied further. In particular, the role of climate feedbacks in the evolution of mountain ranges is important in determining the timescale of climate forcing that the relief will respond to. For a system with a response time less than \( \sim 10^5 \) years a large change in relief is possible in response to the quasiperiodic 10^5 year ice ages of the late Pleistocene, as suggested by Koons [1989]. The relief of a system with a longer timescale will reflect less the cyclic ice age forcing and more the longer-term climate changes. Note though, that even if the profile shape and relief respond slowly, the discharge and erosion rates respond immediately to precipitation changes and, hence, will still reflect the timescale of the climate forcing [see also Whipple, 2001].

Figure 8. Time response of fluvial relief to an instantaneous imposed temperature change of \( \pm 10^\circ C \) at \( t = 0 \), for the different feedbacks considered. Relief increases correspond to temperature decreases and vice versa. The profiles were all in steady state before \( t = 0 \), and all had an initial relief of 3000 m.
of the relief to channel length (Figure 6) suggests the orographic precipitation would still be an important feedback on relief. We also note that incorporating an isostatic response in the form of a point-by-point rebound to the erosion rates would not change any of the steady state results presented here: it would simply change the effective value of K uniformly across the channel, and here K is an imposed parameter.

While we have focused on the spatial pattern of precipitation it has been suggested that most erosion takes place during large storms, and that there is a threshold basal stress at the river bed that is required for incision to take place [e.g., Costa and O’Connor; 1995; Tucker and Bras, 2000; Snyder, 2001]. In this case it will be the precipitation distribution during those large storms together with their intensity and frequency that determines the strength of the feedback. It is quite possible that because of the nature of rare events, patterns of precipitation in the large, erosion-causing storms are quite variable. This would blur the effective (i.e., cumulative) precipitation pattern and deemphasize its importance relative to the total amount of precipitation. This seems worthy of further exploration.

Last, an important simplification is that the erosional mechanism and channel geometry has also been assumed constant: in these calculations all adjustment to the climate feedback takes place via changes to the channel slope. It is possible and likely that when the system has more degrees of freedom to respond, the adjustment via relief changes alone will be reduced, and instead may in part occur via changes to drainage basin size, channel width-discharge relationships, or the relative importance of different erosional mechanisms within the channel.

The results in section 6 showed that imposing a decrease in temperature led to an increase in profile relief, because precipitation was reduced. This runs counter to the often stated idea that erosion should increase in colder climates [e.g., Molnar and England, 1990]. This claim is made in part because there is evidence that an increase in sedimentation rates occurred in the Pleistocene [Zhang et al., 2001], and in part because glaciers, where present, appear to be relatively effective erosive agents [e.g., Hallet et al., 1996]. It is also assumed that a glacial climate, with a larger pole-to-equator temperature difference, results in a stronger jet stream and a more unstable and stormier atmosphere. In fact, the relationship between wind strength, storminess, and precipitation is not at all straightforward [e.g., Kageyama et al., 1999]. For example, a robust characteristic feature of a colder atmosphere is reduced moisture availability. So more frequent or larger storms do not necessarily carry with them greater precipitation. Atmospheric general circulation models have been used to simulate glacial climates [e.g., Gates, 1976; Manabe and Broccoli, 1985; Hall et al., 1996; Kageyama et al., 1999], and while there are substantial differences between models, most do not show very significant increases in storm track intensity (and many show a decrease). Shifts in the location of the storm tracks are more important for the large-scale precipitation patterns than are changes in intensity. Moreover, the models generally show lower precipitation rates globally and even within storm tracks, as a direct consequence of the colder temperatures.

For fluvial erosion therefore, decreased moisture availability associated with a long-term climate cooling would be a way to reconcile theories invoking a climate-induced global increase in relief [i.e., Molnar and England, 1990] with the arguments of Whipple et al. [1999] requiring decreased erosivity to accomplish this. Counter to this, if climate cooling produces glacial erosion at higher elevations that is sufficiently widespread it might offset the climate signal of lower precipitation rates. However, the point highlights the need to distinguish carefully between different and competing erosion mechanisms in understanding relief changes.

While we have restricted our considerations herein to fluvial incision, we note that similar interconnections between topography and precipitation extend to regions subjected to glacial erosion. The pattern of precipitation in the form of snow dictates the spatial variations in ice flux, which directly impacts of rates of glacial erosion. In existing models of glacial erosion at the landscape scale [e.g., MacGregor et al., 2000; Tomkin and Braun, 2002], the rate of erosion scales with the rate of glacier sliding and the area over which sliding is rapid, both of which tend to increase with ice flux, all other parameters remaining fixed. Other important factors, primarily the temperature and hydrologic regime at the glacier bed, affect glacial erosion [Henson, 1991; Hallet, 1996; Alley et al., 1999] as well as the sliding rate, and make a simple glacial erosion law elusive. Both of these parameters, however, are also dependent on climatic variables and, hence, are likely to further enrich interactions between topography and climate, making them attractive targets for future research.

The role of orographic precipitation feedbacks on mountain range relief has been postulated in earlier work, but it is frequently neglected in geomorphological studies. The results presented here demonstrate that in nonglaciated landscapes the interaction between orography and precipitation is a primary control on the relief of mountain ranges and must have been so during their evolution over Earth’s history.

Acknowledgments. The authors would like to thank two anonymous reviewers and Kelin Whipple for constructive suggestions which much improved the manuscript.

References
