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A one-dimensional model for the interaction between continental-scale ice sheets and atmospheric stationary waves

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Abstract The great continental ice sheets of the Pleistocene represented a significant topographic obstacle to the westerly winds in northern midlatitudes. This work explores how consequent changes in the atmospheric stationary wave pattern might have affected the shape and growth of the ice sheets themselves. A one dimensional (1-D) model is developed which permits an examination of the types and magnitudes of the feedbacks that might be expected. When plausible temperature perturbations are introduced at the ice-sheet margin which are proportional to the stationary wave amplitude, the equilibrium shape of the ice sheet is significantly altered, and depends on the sign of the perturbation. The proposed feedback also affects the response of the ice sheet to time-varying climate forcing. The results suggest that the evolution of a continental-scale ice sheet with a land-based margin may be significantly determined by the changes it induces in the atmospheric circulation.

1 Introduction

If the presence of a continental-scale ice sheet significantly alters the atmospheric circulation, then the possibility of a feedback mechanism exists: changes caused to the patterns of melting and accumulation will, over time, act to change the shape of the ice sheet. This will in turn lead to further changes in the circulation. Observations and theory show that when the midlatitude westerlies are forced to deviate around large-scale topography such as the Tibetan Plateau, then stationary waves, standing patterns in the winds and temperatures, are established within the atmosphere (e.g. Hoskins and Karoly 1981; Cook and Held 1988). Consequently, it is likely that such feedbacks existed during the ice ages. However, the complexities of both ice sheets and the atmosphere make it difficult to examine the interaction using an atmospheric general circulation model (GCM). Even the simplified three dimensional (3-D) model we have developed, which is presented in a companion paper (Roe and Lindzen 2000), is complex. We have therefore developed a simple 1-D (north–south) model which captures many important features of the interaction, and permits an understandable examination of the possibilities and their plausible magnitudes. It also provides a useful interpretive tool for the more complex model.

We use a perfectly plastic material approximation for the ice-sheet shape, and simple representations of accumulation and melting allow for an analytic expression for the equilibrium size of the ice sheet. The effects of the atmospheric stationary wave are represented in the 1-D model simply by allowing for a temperature perturbation over the ice sheet whose magnitude is proportional to the height of the ice sheet. The results demonstrate the potential for the stationary wave feedback to significantly affect the equilibrium size of the ice sheet. The change depends on whether the induced temperature perturbation acts to warm or cool the southern margin. In time dependent calculations, the model suggests that the inclusion of the stationary wave effect should destabilize the eastern slopes of an ice sheet, while stabilizing the western slopes.

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2 Ice sheet model

A relevant configuration for the northern continental ice sheets of the Pleistocene is shown in Fig. 1. The northern edge of the ice

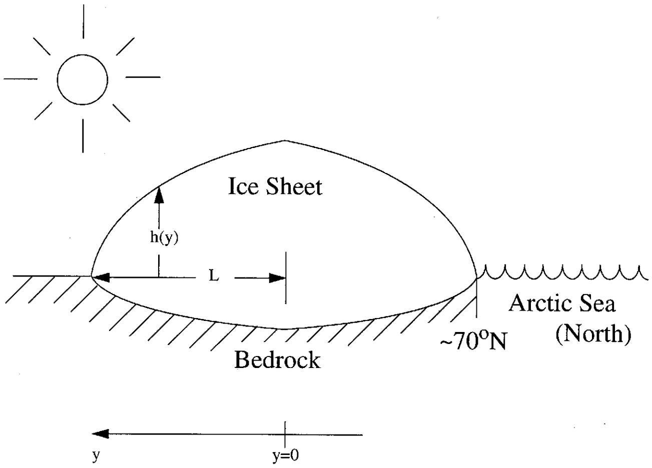

Fig. 1 Schematic of relevant ice sheet configuration for the Pleistocene ice sheets. Ice is grounded by the Arctic Sea at 70°N, and the southern margin is free to vary. The ice depresses the underlying bedrock. See text for explanation of symbols

sheet is grounded by the Arctic Sea. The southern margin is free to vary and is therefore located where the climate is warm enough for the summer melting to balance the southward spreading of ice as it flows under its own weight. The ice sheet is also sufficiently massive to depress the underlying bedrock.

A simple approach, which approximates an ice sheet's behavior, is to treat the ice sheet as a perfectly plastic material; no deformation occurs until the threshold (or yield) stress is reached, at which point there is instantaneous adjustment. In equilibrium therefore, all of the ice is at the yield stress. Paterson (1994) shows that the profile of such an ice sheet is described by the equation:

$$h = \lambda(L - |y|)^{1/2} \qquad (1)$$

where L is half the length of the ice sheet and y is the distance from the midpoint of the base of the ice sheet (see Fig. 1). $\lambda = 4\tau_0/3\rho g$, where ρ is the density of ice, and τ_0 is the yield stress. From observations of currently existing ice sheets τ_0 can take values between $0.5 - 1.0 \times 10^5$ Pa, giving λ between 2.65 and 3.74 $\mathrm{m}^{\frac{1}{2}}$ (Paterson 1994). Equation (1) also assumes the bedrock depression is governed by isostatic equilibrium with the overlying ice sheet, and that the density of ice is one third that of the bedrock.

Integrating Eq. (1) over y gives the volume of the southern half of the ice sheet, V, as

$$V = \lambda L^{3/2} \qquad (2)$$

Equation (1) describes the profile of ice sheets existing in today's climate reasonably well, and has been used to study ice-sheet growth and response to Milankovitch cycles (e.g., Weertman 1963, 1976).

2.1 Ablation rate

For the type of problem we are looking at, we do not need a very detailed representation of ablation (runoff of melt-water) at the ice sheet's surface. We take a simple temperature-based parametrization: *ablation* $= a + bT$, where T is the surface temperature, $a = 6\ \mathrm{myr}^{-1}$, and $b = 1.2\ \mathrm{myr}^{-1}\,{}^\circ\mathrm{C}^{-1}$. These values are consistent with more complex treatments (e.g., Pollard 1980; Calov and Hutter 1996). To produce an easily integrable expression over the year, we approximate the annual temperature cycle by a square wave.

$$\begin{aligned} T &= T_0 + 2 \cdot T_a/\pi \quad \text{for } 0 \le t < 0.5 \\ T &= T_0 - 2 \cdot T_a/\pi \quad \text{for } 0.5 \le t < 1 \ , \end{aligned} \qquad (3)$$

where t is the time in years. The factor of $2/\pi$ gives the same integral of temperature over the year as for a sinusoidal variation of amplitude T_a. Letting T_s be the annually averaged temperature at latitude y, the ablation model gives the following conditions:

- If $T_s + 2T_a/\pi < -a/b$ then no melting occurs.
- If $T_s + 2T_a/\pi > -a/b > T_s - 2T_a/\pi$ then melting occurs for half a year at a rate $a + b(T_s + 2T_a/\pi)$.
- If $T - 2T_a/\pi > -a/b$ then melting occurs for half a year at a rate $a + b(T_s + 2T_a/\pi)$ and half a year at a rate $a + b(T_s - 2T_a/\pi)$.

If $T_a = 15\,^\circ\mathrm{C}$ the annually averaged temperature at y would have to exceed $5.5\,^\circ\mathrm{C}$ for the third condition to apply. This does not happen in practice for reasonable values for the accumulation rate. Melting therefore occurs for half a year at most.

We now calculate the total annual ablation over an ice sheet whose profile is given by Eq. (1). Defining T_0 as the annually averaged temperature at the southern margin of the ice sheet, then:

$$T_s(y) = T_0 + \Gamma h(y) + \bar{T}_y \cdot (L - y) \qquad (4)$$

where Γ is the atmospheric lapse rate (which we take as $-6.5\,^\circ\mathrm{C\,km}^{-1}$), and $\bar{T}_y$ is the meridional temperature gradient (which we take as $-6.0\,^\circ\mathrm{C}/1000$ km). Ablation only takes place within a couple of hundred kilometers from an ice sheet margin. Within this distance the ice sheet rises to a height of around 1.5 km. Therefore in comparison to the vertical variation in temperature over the ablating zone, the meridional variation can be neglected.

By the stated condition for melting, and using Eqs. (4) and (1), the snow line (the level above which no melting takes place) will be at y_0, where y_0 satisfies:

$$a + b(T_0 + 2T_a\pi + \Gamma\lambda(L - y_0)^{1/2}) = 0 \ . \qquad (5)$$

So

$$L - y_0 = \left(\frac{a + 2bT_a/\pi + bT_0}{b\Gamma\lambda}\right)^2 \ . \qquad (6)$$

For the values already stated and taking $T_a = 15\,^\circ\mathrm{C}$, $T_0 = -5\,^\circ\mathrm{C}$, and $\lambda = 3.4\ \mathrm{m}^{1/2}$, the width of the ablating zone is 187 km.

Letting ψ be the distance from the southern ice margin (i.e., $\psi = L - y$), and assuming that at all times and all locations on the ice sheet, $T_0 - 2T_a/\pi > -a/b$ (which may be checked) then melting takes place for half a year only, and the total annual ablation, A, over the ice sheet can be found:

$$A = \frac{1}{2} \cdot \int_0^{L-y_0} (a + b(T_0 + 2T_a/\pi) + b\Gamma\lambda\psi^{1/2})\mathrm{d}\psi \qquad (7)$$

which, after some manipulation, can be written as

$$A = \frac{2}{3}\frac{(a + 2bT_a/\pi + bT_0)^3}{(b\Gamma\lambda)^2} \qquad (8)$$

So the total annual ablation is independent of the size of the ice sheet (except to the extent that the size of the ice sheet determines the temperature of the southern margin). This is because the ablation rate is dependent on the surface temperature which only depends on the distance from the edge of the ice sheet and not on its size. A is a cubic function of the ice margin temperature because of two effects: the ablation rate at any given point depends linearly on temperature, and the area over which that ablation is occurring increases as the square of the margin temperature (Eq. 6). This strong functional dependence of A on T_0 suggests that the ice margin will be tied quite strongly to a particular isotherm: a change in accumulation rate in the interior of the ice sheet can be compensated for by only a small change in T_0, or equivalently, a small change in the southern margin of the ice sheet.

2.2 Equilibrium ice sheets

If T_n is the temperature at the edge of the polar sea, then T_0 can be expressed as:

$$T_0 = T_n - 2\bar{T}_y \cdot L \qquad (9)$$

Let $\bar{p}$ be the average accumulation rate in the interior. We might more properly take the accumulation rate at a specific latitude and height as proportional to the local saturation mixing ratio, which is an exponential function of surface temperature (e.g., Holton 1979). However, the results presented later show that the equilibrium halfwidth (i.e., L) is in fact rather insensitive to the mean interior accumulation rate. In addition, because of the assumption of perfect plasticity, the ice-sheet model adjusts instantly everywhere, so the details of the accumulation distribution do not affect the ice-sheet response in this model.

In equilibrium the southern margin is determined by where there is a balance between the accumulation rate over the southern half of the ice-sheet interior and the ablation rate at the southern margin. This gives a cubic equation for the equilibrium halfwidth of an ice sheet:

$$\bar{p}L = \frac{2}{3}\frac{(a + 2bT_a/\pi + b(T_n - 2\bar{T}_y L))^3}{(b\Gamma\lambda)^2} \qquad (10)$$

For $T_n = -15\,^\circ$C, $T_a = 15\,^\circ$C, $\lambda = 3.4$ m$^{1/2}$, and $\bar{p} = 0.3$ myr^{-1}, Eq. (10) gives an equilibrium halfwidth of $L = 707$ km, as well as two other physically meaningless solutions with negative Ls. However, the cubic form of the ablation function does allow more than one equilibrium solution for some sets of parameters. For example, for $\bar{p} = 1$ myr^{-1}, and $T_n = -10\,^\circ$C, then the possible equilibrium halfwidths are about 40 km and 530 km. The smaller ice cap is, however, unstable to small perturbations. It will either collapse to nothing or grow to the larger ice sheet, depending on the direction of the initial change. Other treatments of the mass balance have also yielded this small unstable ice cap solution (e.g., Weertman 1976).

3 The stationary wave feedback

Cook and Held (1988) modeled the LGM winter stationary wave pattern using a GCM and found that the temperature perturbations over the Laurentide ice sheet could be mostly accounted for by the linear atmospheric response to the topographic forcing of the Laurentide. Stationary waves are also caused by large-scale zonal asymmetries in thermal forcing, and because the patterns are global in scale, forcing in one location can affect the climate over the whole hemisphere. However, motivated by the results of Cook and Held (1988) we focus here on the local topographically-forced stationary wave response. Topography is also an important influence on the distribution of precipitation, and the consequences of this are discussed at the end of this section.

It needs to be emphasized that the Cook and Held (1988) simulations were for a wintertime climate, whereas summer is the important season for melting on ice sheets. GCM simulations of the LGM show similar circulation patterns over the ice sheets to Cook and Held (1988), although with weaker amplitude (e.g., Hall et al. 1996). It is thus not certain that the linear topographic response will dominate. However any process by which the ice sheet influences the atmospheric circulation will create analogous feedbacks to the one suggested here, although the amplitude and spatial distribution over the ice sheet would likely differ.

The classic solution for a planetary scale, topographically-forced stationary wave in midlatitudes is a high-pressure ridge over the western slopes of the forcing mountain and a low-pressure trough over the eastern slopes (e.g., Hoskins and Karoly 1981). The amplitude and phase of the wave depend on the details of the zonally averaged climate, as well as the size and location of the mountain. Generally though, the western slopes experience warmer temperatures than the zonally averaged basic state, and the eastern slopes colder temperatures. In the linear regime the magnitude of these temperature perturbations is proportional to the height of the mountain. We consider the 1-D model to be a north–south slice through the ice sheet at a particular longitude. The effect of the topographically forced stationary wave at that longitude can then be included in the model equation. Due to the uncertainty about any east–west spreading of the ice sheet, the results are best thought of as the upper limit on the latitudinal response of the ice sheet to the stationary wave feedbacks. The extent to which the conclusions drawn here can be carried over to the three-dimensional case depends on the phase and magnitude of the stationary wave generated by the ice sheet.

We model the effect of the stationary wave on the ice sheet by assuming that it creates a temperature perturbation over the ablating zones, T'_{sw}, which is linearly proportional to the maximum height of the ice sheet, h_{max}:

$$T'_{sw} = \gamma h_{max} = \gamma\lambda L^{1/2} \qquad (11)$$

where γ is the constant of proportionality. The actual temperature perturbation induced in response to the ice sheet will also depend on its east–west extent, which we do not account for here. However, all we are trying to do is qualitatively include the effect that the topographic influence of the ice sheet might make the surrounding climate warmer or colder. A linear dependence is the easiest way to introduce the effect.

For our purposes, the magnitude of γ need only be approximate. For a 2 km mountain a reasonable amplitude for a stationary wave would be a change in the 1000 mb–500 mb thickness of 100 gpm (geopotential meters), corresponding to a change in the mean temperature in the lower troposphere of about 5 °C. Therefore we consider two values for γ of $\pm 5\,^\circ$C/2 km, representing the effects of both a warm temperature ridge on the western slopes of the ice sheet, and a cold temperature trough on the eastern slopes. Obviously though, the stationary wave temperature field varies smoothly over the real ice sheet. For ease of notation we call the cold temperature trough the CTT feedback, and the warm temperature ridge the WTR feedback.

T'_{sw} can be incorporated directly into the ablation rate parametrization in Eq. (8), and it is straightforward to solve for the equilibrium halfwidths. Figure 2 shows the results for the different stationary wave feedbacks as a function of mean interior accumulation.

The CTT feedback more than doubles the halfwidth for all accumulation rates considered. An ice sheet of halfwidth 1000 km has a height of 3.4 km, and the CTT

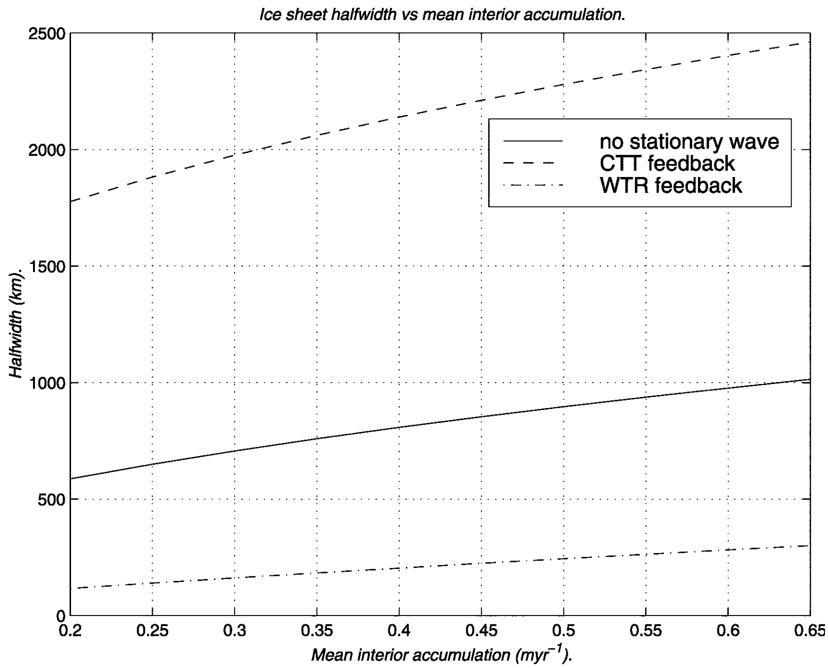

Fig. 2 Equilibrium halfwidth as a function of mean interior accumulation for runs including stationary wave feedback described in text. *CTT* is the cold temperature trough feedback, *WTR* is the warm temperature ridge feedback

feedback gives a cooling of 8.5 °C. Using $\bar{T}_y$, this means the isotherms are located 1400 km further south than for the no feedback case, and this cooling allows the ice to exist at lower latitudes. Additional accumulation over the extended ice sheet pushes the margin even further south. The WTR feedback is equally striking; the size of the ice sheet is strongly limited to only a few hundred kilometers by the warm temperatures that the stationary wave generates. An ice sheet of halfwidth 200 km has a height of 1.5 km, so the warming effect of the stationary wave (3.75 °C) is already quite pronounced at these scales. The 1-D results for such small ice sheets have to be treated skeptically: for ice sheets which are of limited north–south extent, the magnitude of the induced stationary wave will depend mostly on the ice sheet's east–west extent (the larger it is, the larger the stationary wave). The profiles of the ice sheets for $\bar{p} = 0.3$ myr^{-1} are shown in Fig. 3, showing the dramatic differences in the ice-sheet size and volume, depending on which of the feedbacks is included.

It is possible that one or other of the feedbacks could dominate over the full three-dimensional ice sheet, depending on the phase of the stationary wave. If, for example, the cold temperatures extended over most of the ice sheet then only a small region will feel the WTR feedback. In this case we then might expect that the CTT feedback would determine the overall behavior of the whole ice sheet.

The results only take into account the effect of the stationary wave on temperatures. There will also be some change in the moisture supply to the interior of the ice sheet. We expect a high-pressure ridge to bring warmer, moist air up onto the western sector of the ice sheet, and the upslope flow would create enhanced precipitation on the western slopes. The consequent increased accumulation rate in the western interior of the ice sheet would lead to more ice spreading southward and would tend therefore to partially off-set the higher ablation rate at the southern margin due to the the warm temperatures there. Conversely on the eastern sector of the ice sheet where dry northerly air and downslope flow predominate, we would expect to find decreased precipitation. This would again tend to counteract the effect of the stationary wave temperatures. Roe and Lindzen (2000) presents results from a 3-D model, which includes a treatment of topographic precipitation.

4 Time dependent behavior

Weertman (1963) used the perfectly plastic model to determine a time scale for the growth of an ice sheet. We can examine the effect of including the stationary wave feedback by first expressing the model in time dependent form:

$$\frac{dV}{dt} = \frac{3}{2}\lambda L^{1/2}\frac{dL}{dt} = \text{accumulation rate} - \text{ablation rate} \tag{12}$$

where accumulation and ablation refer to the integrated totals over the ice sheet. Hence for an active ice sheet with both accumulating and ablating regions,

$$\frac{dL}{dt} = \frac{2}{3\lambda}L^{-1/2}\left[\bar{p}L - \frac{1}{6}\frac{(a + 2bT_a/\pi - 2b\bar{T}_y L + bT_n - b\gamma\lambda L^{1/2})^3}{(b\Gamma\lambda)^2}\right] \tag{13}$$

Note that the feedback term, γ, enters in the right hand side of Eq. (13). We have still assumed that at all times the ice sheet is described by its equilibrium profile; that is, the process of adjustment to changes in mass balance

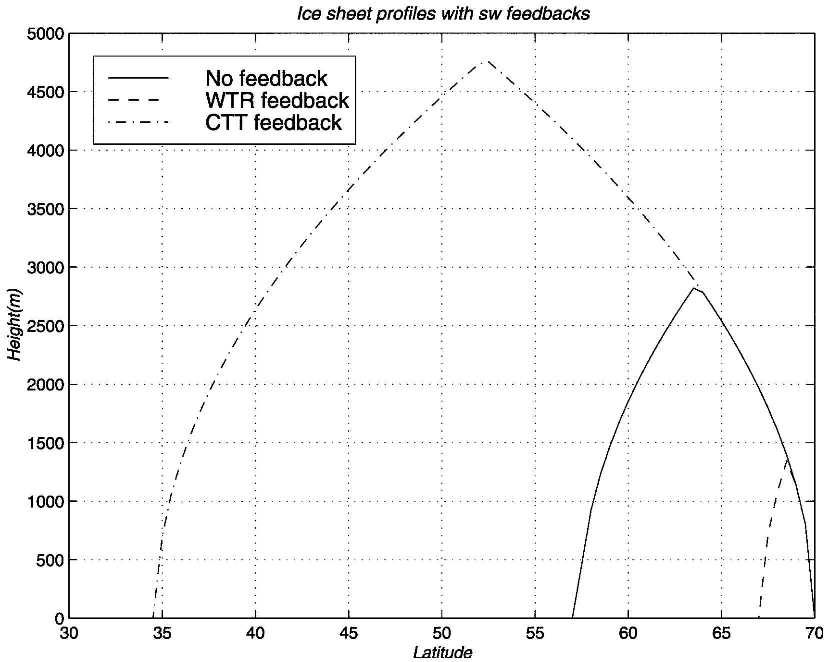

Fig. 3 North-south profiles for ice sheets with the different feedbacks. The mean accumulation rate is set at 0.3 myr^{-1}. Feedbacks labeled as for Fig. 2

is instantaneous. In talking about deviation from equilibrium we mean that the ice sheet is not in equilibrium with the surrounding forcing climate, or in other words, the growth rate, dL/dt, is non-zero.

Equation (13) was integrated forward in time from an initial ice sheet halfwidth of 200 km using the same climate parameters as the last section and with $\bar{p} = 0.3$ myr^{-1}. Three cases were considered: no stationary wave feedback ($\gamma = 0$ in Eq. 13), and the CTR and WTR feedbacks ($\gamma = \pm 5\,^{\circ}C/2$ km). The resulting growth curves are shown in Fig. 4. In the case of no stationary wave feedback, the ice sheet reaches 95% of its equilibrium size within 25 kyr (thousands of years). The WTR feedback causes the ice sheet to actually shrink a little but the initial size is close to its equilibrium value, which it attains quickly. The CTT feedback leads to a much larger ice sheet and it takes about 45 kyr before the ice sheet gets to 95% of its equilibrium size. In contrast to the no stationary wave case, this is enough time to see that the growth rate increases slightly with time, as given by Eq. (13). However the inclusion of the stationary wave feedback changes the time taken to attain equilibrium mainly by changing what that equilibrium size is. The growth is determined by the length of time it takes for sufficient accumulation to produce a given ice sheet. This can be seen from the growth rates over the first few thousand years for the no stationary wave and CTT feedback cases. Since their sizes differ so little at that stage, the integrated accumulation rates are very similar, and the growth rates are also approximately equal.

These results apply only to ice sheets growing in a prescribed and fixed climate. If however the growth is caused by some long-term climate cooling, then the tendencies on the growth rates of the different feedbacks will still be the same as explained, but the time to reach equilibrium, and what that equilibrium is, may be constrained by the climate cooling occurring.

4.1 Stability of the ice sheet

The most recent ice ages of the Pleistocene have had a longer duration (~100 kyr) than the external forcing driving them (the ~20 kyr and ~40 kyr Milankovitch cycles in insolation). It is interesting to ask whether the ice sheet – stationary wave feedback might affect the persistence of ice sheets. On the other hand, the climate record also shows Heinrich events, in which large deglaciations have occurred in relatively short periods of time, and that the terminations at the end of most ice ages take place in only a few thousand years (e.g., Crowley and North 1991). This suggests that significant collapses of the ice sheets are possible on relatively short time scales. Such events are generally attributed to changes in the bottom boundary conditions of the ice sheet (MacAyeal 1993) or to positive feedbacks on the ablation process such as melting into pro-glacial lakes (Pollard 1983), or snow ageing (Gallee et al. 1992). A possible alternative explanation is that atmosphere-ice sheet feedbacks might contribute to rapid retreat.

In general, we expect that if the evolving ice sheet modifies the surrounding climate, then the growth or decay of that ice-sheet will also be affected. We can use the 1-D model to investigate how the inclusion of a stationary wave feedback might influence the ice-sheet stability. If the growth rate is increased by the inclusion of a feedback we say it has been destabilized;

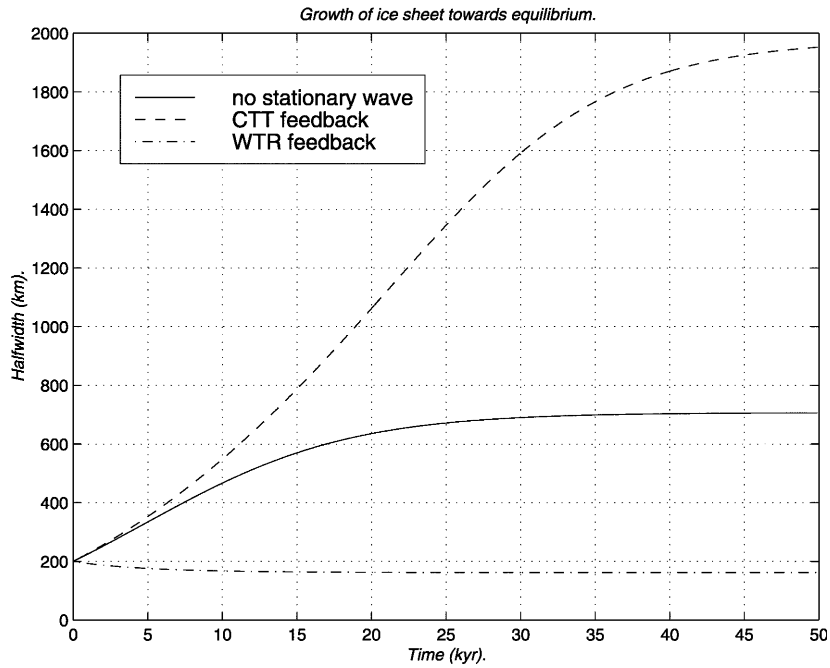

Fig. 4 Growth of ice sheets from an initial L of 200 km. Feedbacks labeled as for Fig. 2

similarly, if the growth rate is decreased, it is stabilized. Because the strength of the stationary wave feedback is $L^{1/2}$, the ability of the feedback to change the climate at the southern margin decreases as L increases. Including the feedback cannot, therefore, lead to runaway growth.

For the case of the CTT feedback, which we expect over the eastern flank of an ice sheet, the effect of the stationary wave would appear to be destabilizing. The southern margin there is maintained partly by the colder temperature caused by the stationary wave. For example, in response to a climate warming, the ice would begin to melt, thereby reducing the height of the ice sheet. This in turn reduces the amplitude of the cold temperatures due to the stationary wave, and therefore increases the amount of melting. Likewise, a climate cooling will lead to an enlargement of the ice sheet, which will increase the stationary wave amplitude, creating more cooling over the region, and further increase the ice sheet size. However, on the western slopes, we expect a WTR feedback will tend to make the ice sheet resist changes in climate: a retreat (growth) of the ice reduces (increases) the height of the ice sheet and the amplitude of the stationary wave. That implies a cooling (warming), which is in opposition to further changes in ice-sheet size. As mentioned earlier the model assumptions imply that the ice adjusts instantly over the whole ice sheet, whereas in reality this process takes several thousand years. Therefore the measure we take for the stability of the ice sheet is not the initial growth rate at the time of an imposed climate change, but the difference in halfwidth after 3 kyr (i.e., a 3 kyr integration of the growth rate). We are again neglecting the effect of the possible longitudinal redistribution of ice. We compare three runs for ice sheets starting from equilibrium and with the same initial size. In the first run, there is no stationary wave feedback. A 5 °C warming is imposed and the model integrated forward in time. The change in the halfwidth after 3 kyr is noted. In the second and third runs the stationary wave is included as a WTR feedback and a CTR feedback respectively. The climate (i.e., T_n) in these runs has to be adjusted to start from equilibrium with the same initial ice-sheet size as for the first run. Again a 5 °C warming is imposed, and the changes in the halfwidth of the ice sheet are compared after 3 kyr.

Since the feedback strength is proportional to $L^{1/2}$, the results in Fig. 5a are displayed for a range of initial ice-sheet sizes. For all initial sizes the change after 3 kyr is greater for the CTT feedback and smaller for the WTR feedback, consistent with expectations from the reasoning described. The CTT feedback has destabilized the ice sheet. For the larger initial halfwidths, the effect of the feedback is proportionately quite small; about a 50 km difference in the halfwidth over 3 kyr. The difference increases however, as the initial halfwidth decreases and the strength of the feedback becomes proportionately more powerful. For an initial halfwidth of 600 km, the ice sheet has almost completely disappeared after 3 kyr, whereas without any feedbacks the halfwidth is still around 300 km after that time.

Figure 5b shows the results for a cooling of 5 °C instead of a warming. The effect of the feedbacks are relatively small although qualitatively in the expected direction. The small difference between the different feedbacks shows that the growth of the ice sheet is basically limited by the accumulation rate.

The results in Fig. 5a suggest that because of the CTT feedback, smaller ice sheets ($L < 400$ km) may be less able to withstand prolonged warmings of a few thou-

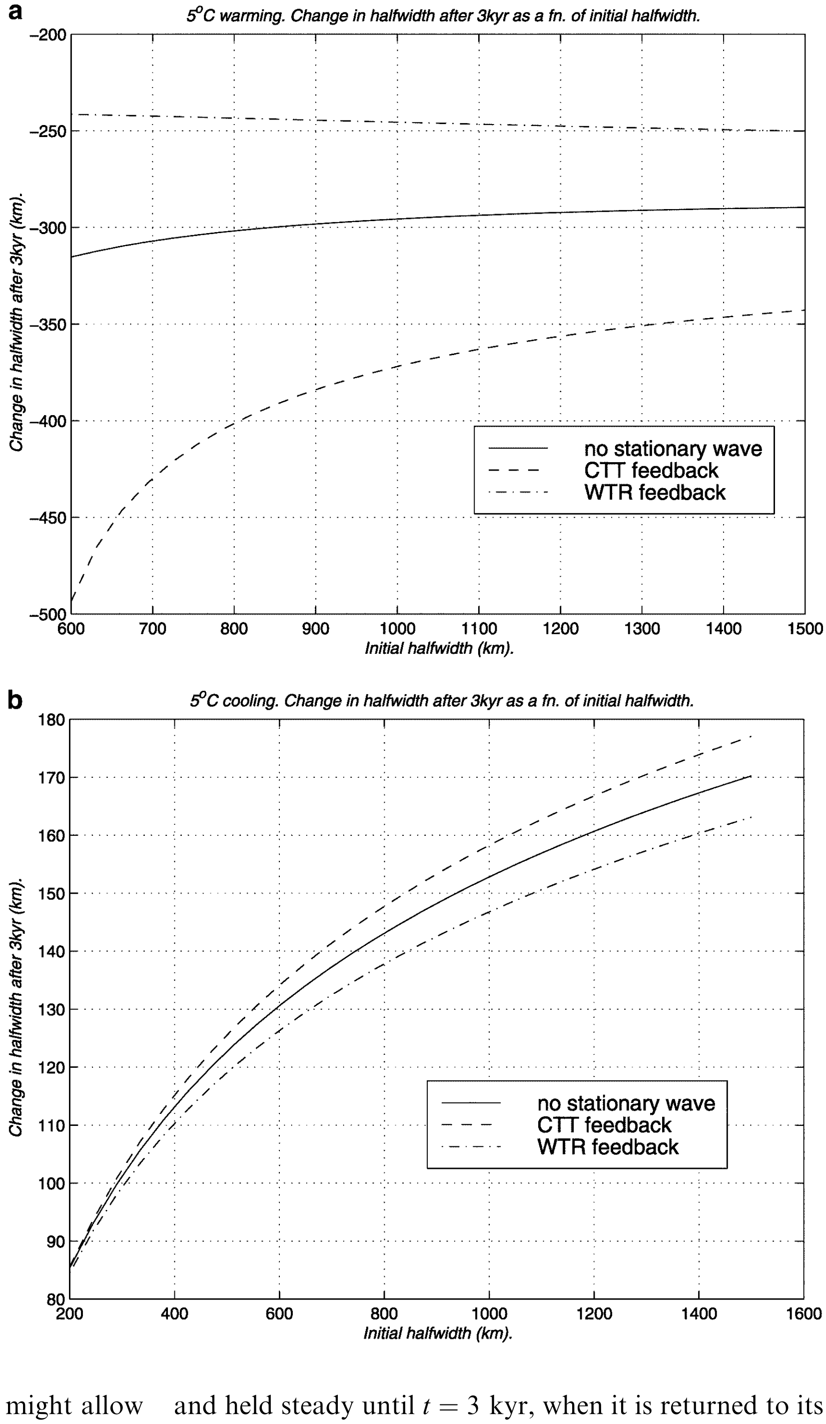

Fig. 5a, b Response of ice sheets to imposed climate change as a function of initial halfwidth. **a** A 5 °C warming, and **b** a 5 °C cooling. Curves represent change from the initial size after 3 kyr. Ice sheets start from same equilibrium size. The climate is adjusted so all ice sheets start from equilibrium with the same initial size. Feedbacks labeled as for Fig. 2

sand years. Conversely, the WTR feedback might allow an ice sheet to resist a temporary warming and survive until the temperature fell again. It is not possible to say to whether this could happen in reality without a three-dimensional model to see how the ice redistributes itself longitudinally. Nevertheless we can demonstrate this effect using the 1-D model. Figure 6 shows runs made for the three cases (no feedback, CTT feedback, WTR feedback). All three runs start with an ice sheet of halfwidth 500 km in equilibrium. As with the stability runs, an adjustment is made to T_n in the two runs with feedbacks to ensure that each ice sheet starts off in equilibrium. At $t = 1$ kyr, the climate is warmed by 5 °C and held steady until $t = 3$ kyr, when it is returned to its original value. For the no feedback case, the ice sheet undergoes strong melting, and reaches a minimum halfwidth of around 250 km. When the climate returns to its original state at 3 kyr, the ice sheet slowly grows back towards its original size. For the WTR feedback case, the ice sheet is stabilized to change: the reduction in size is less than for no feedback, and the ice sheet returns faster towards its equilibrium size. For the CTR feedback there is very strong ablation between 1 kyr and 3 kyr. After the climate returns to its previous state however, the ice sheet is now too small to be sustained. This is because the stationary wave that it creates is now

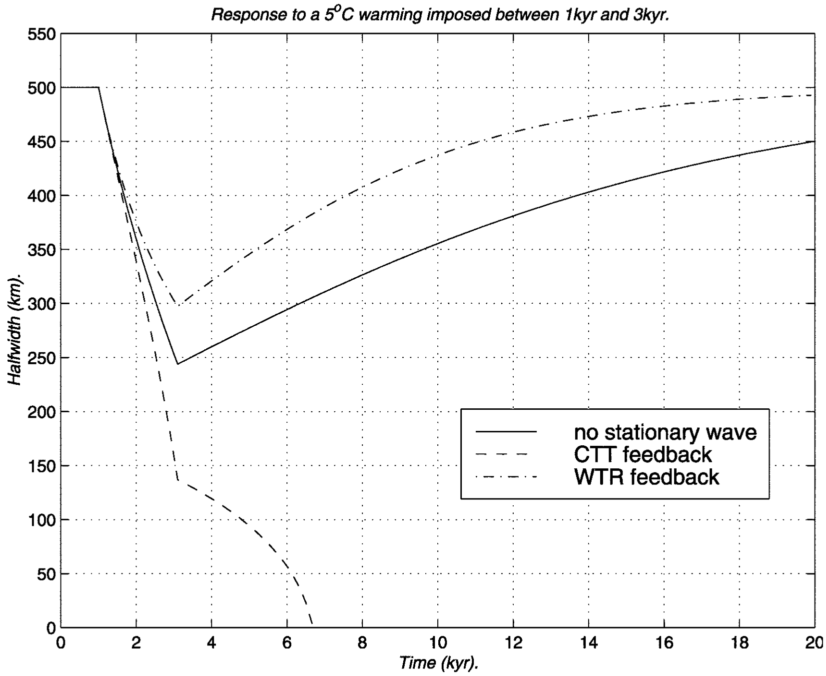

Fig. 6 Response of ice sheets to temporary warming. A 5 °C warming is imposed between 1 kyr and 3 kyr. The climate is adjusted so all ice sheets start from equilibrium with the same initial size. Feedbacks labeled as for Fig. 2

no longer large enough to support sufficiently cold temperatures for the ice sheet to exist. Therefore the ice sheet continues to collapse down towards zero. In this CTT case, a further cooling of around 7 °C is necessary to initiate a new ice sheet. This would lead to a sort of hysterisis loop in the ice age cycle with a large cooling required to get the ice sheet to initiate, and a smaller temperature increase needed to bring about a collapse.

5 Discussion

Using a simple parametrization for the ice sheet – stationary wave feedback, we have demonstrated the significant effect it has on an ice-sheet's size, growth and decay. We suggest therefore that the evolution of the Pleistocene ice sheets cannot be understood without accounting for the changes that they induced in the atmospheric circulation.

The simple model used here has not included any variation in the precipitation and the behavior of the model (especially the time dependent behavior) is dependent to some degree on the functional choice for the feedback. In a companion paper (Roe and Lindzen 2000), a 3-D coupled ice sheet – stationary wave model is presented which can account for some of the present model's short-comings.

Consistent with the CTT feedback presented here, Roe (1999) and Vettoretti et al. (2000) have suggested that the stationary wave circulation pattern might account for the inability of energy balance models (which do not represent atmospheric dynamics) to reproduce the penetration of the Laurentide ice sheet at the LGM deep into New England. Further, it is possible that the WTR feedback might explain the absence of LGM ice in Alaska. As mentioned in the introduction, diagnosing these feedbacks from GCM integrations is problematic. Moreover the climates produced by the current suite of GCM simulations of the LGM, assessed as part of the Paleoclimate Model Intercomparison Project, are not even consistent with the known configuration of the ice sheets (Pollard et al. 2000).

We have focused on the consequences of the topographic forcing due to the ice sheet because of Cook and Held's (1988) results showing that it dominated the atmospheric response, at least in winter. The ice sheet is however also a source of thermal forcing on the atmosphere. If this forcing were large enough to induce significant circulation changes then similar feedbacks to those presented here would operate, with the induced perturbations perhaps being proportional to the area of the ice sheet rather than its height. The results of this study establish that the nature and magnitude of the feedbacks associated with the ice sheet-forced changes in circulation are such as to be significant in the evolution of the ice sheet itself.

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