The response of glaciers to intrinsic climate variability: observations and models of late Holocene variations in the Pacific Northwest

Gerard Roe¹ and Michael A. O’Neal²

1. Department of Earth and Space Sciences, University of Washington, Seattle, WA
2. Department of Geography, University of Delaware, Newark, DE.

Abstract

Discriminating between glacier variations due to natural climate variability and those due to true climate change is crucial for the interpretation and attribution of past glacier changes, and for expectations of future changes. We explore this issue for the well-documented glaciers of Mount Baker in the Cascades Mountains of Washington State, USA, using glacier histories, glacier modeling, weather data, and numerical weather model output. We find natural variability alone is capable of producing 2 to 3 km excursions in glacier length on multi-decadal and centennial timescales. Such changes are similar in magnitude to those attributed to a global Little Ice Age, and so our results suggest that such a climate change may not, in fact, be required in this setting. The results are also likely to apply to other Alpine glaciers, and they will therefore complicate interpretations of the relationship between glacier and climate history.
1.0 Introduction

The existence of mountain glaciers hinges on a sensitive balance between mass accumulation via snowfall and mass wastage (i.e., ablation) via melting, evaporation, sublimation, and calving. All of these processes are ultimately controlled by climate. While climate changes will obviously tend to drive glacier variations, not all glacier variations should be interpreted as being caused by climate changes. Climate is the statistics of weather, averaged over some time period of interest. The World Meteorological Organization takes this time period as thirty years, but it can be any interval relevant for the question at hand. By definition, then, a constant climate means that the statistical distributions of atmospheric variables do not change with time. Therefore variability, as characterized by the standard deviation and higher-order statistical moments, is in fact intrinsic to a constant (i.e., stationary) climate. Glaciers reflect this variability. The characteristic response time (i.e., inertia, or ‘memory’) of a glacier ranges from years to centuries (e.g., Johannesson, 1989; Harrison et al., 2001; Pelto and Hedlund, 2001; Oerlemans, 2001), and any given glacier will reflect an integrated climate history on those timescales. Thus we arrive at a key question in interpreting records of changes in glacier geometry: are the reconstruction of past glacier variations significantly different (in a statistical sense) from what would be expected as a natural response to intrinsic variability in a stationary climate? Only when this significance has been demonstrated can a recorded glacier advance or retreat be confidently interpreted as reflecting an actual change in climate.

In this paper, we adapt a linear glacier model to include an explicit and separate treatment of precipitation and melt-season temperature. We use reconstructed geometries, historical climate data and numerical model output from localities on and near Mount Baker in the Cascade Range.
of western Washington, USA (Figure 1), in order to determine what the glacier response to intrinsic climate variability in this region. Although the examples used in this study are based on the geometries of typical valley glaciers in this setting, the goal in this paper is not to simulate the evolution of any observed glacier. Instead we use the combination of observations and reconstructions of climate and glaciers to calibrate and evaluate a simple model that reproduces characteristic variations of glacier length in response to characteristic climate variations in a stationary (i.e., constant) climate. The analyses lead to some important results against which to interpret glaciers in natural settings.

Our approach mirrors that of Reichart et al. (2002), who used a down-scaled global climate model (GCM) output and a dynamical glacier model for two European glaciers (Nigardsbreen, Norway, and Rhonegletscher, Switzerland). They concluded that the present retreat exceeded natural variability, but that ‘Little Ice Age’ (LIA) advances did not. Thus a climate change (at least within their GCM/glacier model system) was not required to explain LIA-like advances. Here, our use of a linear model is a trade-off: the level of sophistication of the glacier model is less, but its simplicity allows us to derive some simple expressions that make clear the dependencies of the system response.

2.0 The Glacier Model

Glaciers are dynamic physical systems wherein ice deforms and flows in response to hydrostatic pressure gradients caused by sloping ice surfaces. There are other important factors to glacier motion among which are: ice flow is temperature dependent; glaciers can slide over their base if subglacial water pressures are sufficient; glaciers interact with their constraining side walls; and
glacier mass balance can be sensitive to complicated mountain environments (e.g., Anderson et al., 2004; Nye, 1952; Pelto and Riedel, 2001). Despite these somewhat daunting complications, a series of papers have shown that simple linear models based on basic mass balance considerations can be extremely effective in characterizing glacier response to climate change (e.g., Johannesson et al., 1989; Huybrechts et al., 1989; Oerlemans, 2001, 2004; Klok, 2003).

The model we employ includes an explicit and separate representation of melt-season temperature and annual mean precipitation in the mass balance. A schematic of the model is depicted in Figure 2, and a derivation of the model equations is presented in the Appendix. The model operates on three key assumptions, and which are typical of such models. The first assumption is that a fixed characteristic glacier depth and a fixed width of the glacier tongue can represent the glacier geometry. The second assumption is that glacier dynamics can be essentially neglected, producing instantaneous deformation. All accumulation and ablation anomalies act immediately to change the length of the glacier. The third assumption is that length variations are departures from some equilibrium value, and are small enough that the system can be linearized. These three assumptions, together with a constraint of mass conservation, allow for prescribed climate variations in the form of accumulation and temperature anomalies to be translated directly into length changes of the glacier. In Section 5.2 the validity of these assumptions is evaluated by comparing the results from the linear model with those from a nonlinear dynamical flowline model.

A schematic illustration of the model is shown in Figure 2. The model geometry of the glacier in steady state is as follows: the total glacier area is $A_{tot}$; there is a melt-zone where the melt-season
temperature is above zero, with an area $A_{T>0}$; and there is an ablation-zone where the annual melt exceeds the annual accumulation, with an area $A_{abl}$. In other words the ablation zone is the region of the glacier below the equilibrium line altitude (ELA). This definition of ablation zone is in accord with, for example, Paterson (1994), but it is important to note that the net mass loss above the ablation zone also plays a role in the glaciers dynamics, as can be seen in the derivation of the model equation in the Appendix. The glacier has a protruding tongue with a characteristic width $w$, has uniform thickness $H$, and rests on a bed with a constant slope angle, $\varphi$. The centerline length is assumed to represent the total glacier length $L$. Further refinements the simple model are possible. It would be possible to incorporate a feedback between thickness and glacier depth (e.g., Oerlemans, 2001) or a time-lag between climate anomaly and terminus response (e.g., Harrison et al., 2003), with some incremental increase in model complexity. However the agreement between the linear model and a dynamic flowline model, demonstrated in Section 5.2, is sufficient to justify the use of the current model for the question posed.

Climate is prescribed in terms of a spatially-uniform accumulation rate, $P$, and a temperature-dependent ablation rate $\mu T$, where $T$ is the mean melt-season temperature and $\mu$ is an empirical coefficient, or melt factor. In effect, this ablation parameterization is a simplified form of the more frequently-used positive degree day model (e.g., Braithwaite and Oleson, 1989). Percolation of melt water and freezing of rainfall are neglected. A simplified treatment of ablation is adequate for the purpose of this paper, which is to characterize the general magnitude of the glacier response rather than to accurately capture the details,
In the Appendix it is shown that when the model is linearized, the evolution of the terminus position is governed by the following equation:

\[
\frac{dL'(t)}{dt} + \frac{\mu A_{abl} \Gamma \tan \phi}{wH} L'(t) = \frac{A_{nt}}{wH} P'(t) - \frac{\mu A_{T > 0}}{wH} T'(t),
\]

where the prime denotes perturbations from the equilibrium state and all other variables are their climatological (i.e., time mean) values. \( \Gamma \) is the atmospheric lapse rate (the decrease of temperature with elevation), and \( P' \) and \( T' \) are annual anomalies of, respectively, the average annual accumulation on the glacier, and the average melt-season temperature on the glacier’s melt-zone.

### 3.0 Discussion of Model Physics

In the absence of a climate perturbation \( (P' = T' = 0) \), equation (1) shows that the glacier relaxes back to equilibrium \( (L' = 0) \) with a characteristic time scale (or “memory”), \( \tau \), which is a function of the glacier geometry and the sensitivity of ablation to temperature:

\[
\tau = \frac{wH}{\mu \Gamma \tan \phi A_{abl}}. \tag{2}
\]

Another interpretation of \( \tau \) is that it is the timescale over which the glacier integrates the mass balance anomalies. Increasing the value of \( \mu, \Gamma, \) or \( \tan \phi \) affects the melt rate per unit distance up-glacier. Increasing \( A_{abl} \) increases the ability of the glacier terminus to accommodate an increase in the mass balance. The time scale of this response is inversely proportional to these parameters. Conversely, increasing \( H \) results in a greater amount of mass that must be removed for a given climate change. In the model of Johannesson et al. (1989), the equivalent timescale is given by \( H / \dot{b} \), where \( \dot{b} \) is the net mass balance at the terminus. The denominator in equation (2) plays the equivalent role of \( \dot{b} \) in this model.
3.1 The equilibrium response to changes in forcing.

We first consider the steady-state response of the glacier system. The second and third terms on the right hand side of equation (1) represent the climatic forcing separated into precipitation and temperature, respectively. Equation (1) can be rearranged and used to calculate the steady-state response of glacier terminus, $\Delta L$, to a change in annual accumulation, $\Delta P$, or melt-season temperature, $\Delta T$, using the fact that $dL/dt = 0$ in steady state. In response to a change in melt-season temperature, $\Delta T$, the response of the terminus is given by:

$$
\Delta L_T = \frac{A_{ad} \Delta T}{\Gamma \tan \varphi A_{abl}}.
$$

(3)

In response to a negative anomaly in temperature the glacier will advance down slope until ablation comes back into balance with the new climate. A steeper basal slope or lapse rate (i.e., a larger value $\tan \varphi$ or $\Gamma$) means the glacier will not have to advance as far to find temperature warm enough to establish mass balance. The ratio of areas in equation (3) is because as the glacier terminus advances it also expands the area that accumulation occurs over, and this partially offsets the increase melting that happens at lower elevations.

In response to a step change in annual accumulation, $\Delta P$, equation (1) can be rearranged to give

$$
\Delta L_p = -\frac{A_{ad} \Delta P}{\mu \Gamma \tan \varphi A_{abl}}.
$$

(4)

Equation (4) is analogous to equation (3): both the imposed geometry of the glacier and the melt rate at the terminus are required to account for the accumulation and the area added to the glacier tongue. Looking at the terms in equation (4), $A_{abl}$ is the area of the ablation zone, and $\Delta L_p \Gamma \tan \varphi$ is the temperature change of the terminus due to the change in length, and $\Delta P$ is the change in the
total accumulation. Equation (4) is therefore a perturbation mass balance equation – it gives the change in the length of the glacier such that the change in the total ablation rate balances the prescribed change in the total accumulation rate.

Another useful property of the linear model is that it is straightforward to evaluate the relative sensitivity of the glacier length to accumulation and melt-season temperature. Let $R$ equal the ratio of length changes due to temperature, $\Delta L_T$, and the length change due to precipitation, $\Delta L_P$.

From Equations (3) and (4):

$$R = \frac{\Delta L_T}{\Delta L_P} = \frac{A_{T>0}}{A_{\text{tot}}} \frac{\mu \Delta T}{\Delta P},$$

(5)

Thus $R$ is equal to the ratio of the ablation and melt-zone areas multiplied by the ratio of the ablation rate (i.e., $\mu \Delta T$) and accumulation rate changes.

### 3.2 The response to climate variability.

For prescribed variations in accumulation and melt-season temperature, equation (1) can be numerically-integrated forward in time to calculate the glacier response to a given climate forcing. However, to begin with, we want to characterize the length variations expected of a glacier in a constant climate, emphasizing that this means the climate has a constant mean and standard deviation. If we further suppose that the accumulation and precipitation are described by normally-distributed variations, then equation (1) is formally equivalent to a 1st-order auto-regressive process, or AR(1) (e.g., vonStorch and Zwiers, 1999). Assuming a normal distribution of accumulation cannot be, of course, strictly correct because negative precipitation is not physical, but provided the standard deviation is small compared to the mean, this approximation is still instructive to make. We further assume that accumulation and melt-season temperature are
each not correlated in time, and are also not correlated with each other. Huybers and Roe (2008) show that these assumptions are appropriate for glaciers in the Pacific Northwest. Although there is some interannual memory in precipitation, it is not very strong (e.g., Huybers and Roe, 2008), and much shorter than characteristic glacier response time scales, $\tau$. Moreover, 80\% of annual precipitation in the region falls in the winter half-year, and so correlations between annual precipitation with melt-season temperature are not significant.

Let $\sigma_T$ be the standard deviation of melt-season temperature, let $\sigma_P$ be the standard deviation of annual accumulation, and let $\nu_t$ and $\lambda_t$ be independent normally-distributed random processes. Then using finite differences to discretize the equation into time increments of $\Delta t = 1$ yr, equation (1) can be written as:

$$L'_{t+\Delta t} = L_t (1 - \frac{\Delta t}{\tau}) + A_{m,t}\Delta t \frac{\sigma_P}{wH} \nu_t + \frac{\mu A_{r,g}\Delta t}{wH} \sigma_T \lambda_t,$$

where the subscript $t$ denotes the year. We first calculate expressions for the standard deviation in glacier length due to temperature and precipitation variations separately. Let $\langle x \rangle$ represent the statistical expectation value of $x$. The following relationships hold: $\langle \nu_t \lambda_t \rangle = \langle \nu_t L_t \rangle = \langle \lambda_t L_t \rangle = 0$; $\langle L_t L_t \rangle = \langle L_{t+\Delta t}^2 \rangle$; and the expectation value of both sides of equation (7) must be the same. Firstly let $\sigma_P = 0$, in which case it follows from equation (7) that:

$$\sigma_{L_t} = \sqrt{\langle L_t^2 \rangle} = \sigma_T \frac{\mu A_{r,g} \Delta t}{wH} \sqrt{\frac{\Delta t \cdot \tau}{2}},$$

and similarly for $\sigma_{L_P}$:

$$\sigma_{L_P} = \sigma_P \frac{A_{m,t}}{wH} \sqrt{\frac{\Delta t \cdot \tau}{2}}.$$
As might be expected, the relative sensitivity of the glacier to precipitation and temperature variations is similar to that for a step-change:

\[ R = \frac{\sigma_{L_T}}{\sigma_{L_P}} = \frac{A_{T=0} \mu \sigma_T}{A \mu T \sigma_p}. \]  \( (10) \)

Note that this ratio describes the relative importance of accumulation and melt-season temperature for glacier length. It is different, of course, from the relative importance of accumulation and temperature for local mass balance, which is simply given by \( \mu \sigma_T / \sigma_p \).

Since the model is linear and the climate variations are uncorrelated, the standard deviation of glacier length when both temperature and precipitation are varying can be written:

\[ \sigma_L = \sqrt{\sigma_{L_T}^2 + \sigma_{L_P}^2}. \]  \( (11) \)

Thus, for specified glacier geometry and parameters, we can directly calculate the expected response of the glacier to random year-to-year fluctuations in the meteorological variables. In the following section we apply and evaluate this model for typical conditions of Mount Baker glaciers, and the climate of the Pacific Northwest of the United States.

4.0 Calibration of Model for Mount Baker Glaciers and Cascade Climate

*Climate parameters.* Most of the model parameters are readily determined or available from published literature. The value of \( \mu \), the melt rate at the terminus per °C of melt-season temperature, is assumed to range from 0.5 to 0.84 m °C\(^{-1}\) yr\(^{-1}\) (e.g., Patterson, 1994). We take \( \Gamma \) to be 6.5 °C km\(^{-1}\) (e.g., Wallace and Hobbs, 2004). In practice \( \Gamma \) varies somewhat as a function of location and season. These values produce vertical mass balance gradients that are consistent with profiles obtained for glaciers in the region (e.g., Meier et al., 1971).
Glacier geometry. For our rectangular, slab-shaped model glacier, the mean ablation area, $A_{abl}$, is calculated using the accumulation-area ratio (AAR) method, which assumes that the accumulation area of the glacier is a fixed portion of the total glacier area (e.g., Meier and Post, 1962; Porter, 1977). Although the method does not account for the distribution of glacier area over its altitudinal range, or hypsometry, it is appropriate for the model since we are trying to generalize to large, tabular valley glaciers with similar shapes. Porter (1977) indicates that for mid-latitude glaciers like the large valley glacier in the Cascade Range, a steady-state AAR is generally in the range of 0.6-0.8.

A range of areas for the ablation zone ($A_{abl}$) for our model is readily determined from the area of glaciers and their characteristic tongue widths using 7.5’ U.S. Geological Survey topographic maps and past glacier-geometry data from Harper (1992), Thomas (1997), Fuller (1980), Burke (1972), and O’Neal (2005). For the major glaciers on Mount Baker this information is compiled in Table 1. The large valley glaciers around Mt. Baker are all quite similar geometrically, and so we also choose a representative set of parameters, which we use for analyses in the next section (Table 1). Substituting this characteristic set of parameters into equation (2), and accounting for the range of uncertainties in $\mu$ and the AAR, $\tau$ varies between 7 and 24 years, with a mid-range value of 12 years.

Climate data. We take the melt season to be June through September (denoted JJAS). We also use annual mean precipitation as a proxy for annual mean accumulation of snow. In this region of the Pacific Northwest about 80% of the precipitation occurs during the October-to-March
winter half-year, and so we assume it to fall as snow at high elevation. This also means that annual precipitation and melt-season temperature in the region are not significantly correlated, and so can be assumed independent of each other. Since we are seeking a first-order characterization of the glacier response to climate variability, these are appropriate approximations. We are also neglecting mass input to the glacier from avalanching and snow blowing, for want of a satisfactory treatment of these processes.

The nearest long-term meteorological record is from Diablo Dam (48°30′N, 121°09′W, elev. 271 m.a.s.l.), about 60 km from Mt. Baker (48°46′N, 121°49′W, elev. 3285 m.a.s.l.), and extends back about seventy-five years. The observations of annual precipitation and melt-season temperature are shown in Figure 3a,b.

It is quite common in the glaciological literature to find decadal climate variability invoked as the cause of glacier variability on these timescales (e.g., Kovanen, 1993; Hodge et al., 1998; Nesje and Dahl, 2003; Moore and Demuth, 2001; Pederson et al., 2004; Lillquist and Walker, 2006). In particular, much is made of the Pacific Decadal Oscillation, which is the leading EOF of sea-surface temperatures in the North Pacific, and which exerts an important influence on climate patterns in the Pacific Northwest (e.g., Mantua et al., 1997). In actual fact, gridded data sets for the atmospheric variables that control glacier variability, have very little persistence: there is no significant interannual memory in melt-season temperature (Huybers and Roe, 2008), and only weak interannual memory in the annual precipitation (the one year lag autocorrelation is ~0.2 to 0.3, and so explains less than ten percent of the variance, Huybers and Roe, 2008). The interannual memory that does exist in North Pacific sea-surface temperatures comes from re-
entrainment of ocean heat anomalies into the following winter’s mixed-layer (Deser et al., 2003; Newman et al., 2003), and is consistent with longer-term proxy records for accumulation (e.g., Rupper et al., 2004). The appearance of decadal variability in time series of the PDO is often artificially exaggerated by the application of a several-year running mean through the data (e.g., Roe, 2008).

At Diablo Dam, and for this length of record, a standard statistical test for autoregression (a stepwise least-squares estimator for the significance of autocorrelations, using Schwarz’s Bayesian criterion, as implemented in ARFIT; Schneider and Neumaier, 2001; see also vonStorch and Zwiers, 1999) does not allow the conclusion that there is any statistically-significant interannual persistence in either the annual precipitation or the melt-season temperature (after linearly detrending the time series). In other words, one year bears no relation to the next. The oft-used technique of a putting a five-year running mean through the data gives a deceptive appearance of regimes lasting many years, or even decades that are wetter/drier, or warmer/colder, but that is artificial. Panels (c) and (e) of Figure 3 show random realizations of white noise with the same mean and standard deviation as the annual precipitation. Panels (d) and (f) show the same thing, but for melt-season temperature. There is a large amount of year-to-year variability, and just by chance there are intervals of a few years of above or below normal conditions. This is highly exaggerated by the application of the running mean. The similarity of the general characteristics of these random time series to the data (that can be seen by eye in Figure 3) is a reflection of the fact that the linearly-detrended annual-mean accumulation and melt-season temperature are statistically indistinguishable from normally-distributed white noise.
Thus for the atmospheric fields that are relevant for forcing glaciers, there is little to no evidence of any interannual persistence of climate regimes.

It is very important to stress this result. A large fraction of the glaciological literature has interpreted the recent (but pre-anthropogenic) glacier history of this region and elsewhere in terms of the persistence of decadal-scale climate regimes. For the Pacific Northwest, this is simply not supported by the available weather data. Five other long-term weather station records near Mount Baker were also analyzed: Upper Baker Dam (48°39′N, 121°42′W, elev. 210.3 m.a.s.l., 44 years); Bellingham airport (48°48′N, 122°32′W, elev. 45.4 m.a.s.l., 46 years); Clearbrook (48°58′N, 122°20′W, elev. 19.5 m.a.s.l., 76 years); Concrete (48°32′N, 121°45′W, elev. 59.4 m.a.s.l., 76 years); and Sedro Wooley (48°30′N, 122°14′W, elev. 18.3 m.a.s.l., 76 years). None of these other stations show any statistically-significant autocorrelations of annual precipitation or melt-season temperature, either. The lack of strong evidence for dominant decadal-scale regimes (i.e., significant autocorrelations on decadal time scales) of atmospheric circulation patterns is widely appreciated in the climate literature (e.g., Frankignoul and Hasselmann, 1977; Frankignoul et al., 1997; Barsugli and Battisti, 1998; Wunsch, 1999; Bretherton and Battisti, 2000; Deser et al., 2003; Newman et al., 2003). And at best, the interannual persistence that does exist for some atmospheric variables only accounts for only a small fraction of their total variance. As will be emphasized in this paper, and in the context of glacier variability, it is the inertia or memory intrinsic to the glacier itself, and not to the climate system, that drives the long time-scale variations.
For the specific climate fields used for the model, we are able to take advantage of output from a high-resolution (4 km) numerical weather prediction model, the Pennsylvania State University–National Center for Atmospheric Research Mesoscale Model version 5, or MM5 (Grell et al., 1995). MM5 has been in operational use over the region for the past eight years at the University of Washington. It provides a unique opportunity to get information about small-scale patterns of atmospheric variables in mountainous terrain that begins to extend towards climatological time scales: a series of studies in the region has shown persistent patterns in orographic precipitation at scales of a few kilometers (Colle et al., 2000; Garvert et al., 2006; Anders et al., 2007; Minder et al., 2008).

Although eight years is a short interval for obtaining robust statistics, the output from MM5 at the grid point nearest Diablo Dam agrees quite well with the observations there. From seventy-five years of observations at Diablo Dam the mean annual accumulation is 1.89±0.36 m yr\(^{-1}\) (±1σ hereafter, unless stated otherwise). By comparison the output from MM5 at the nearest grid point to Diablo Dam is 2.3±0.41 m yr\(^{-1}\). For melt-season temperatures the values in observations and MM5 are 16.8±0.78 °C and 12.7±0.93 °C, respectively. The nearest meteorological observation to Mt. Baker comes from the Elbow Lake SNOTEL site\(^1\) (48°41′N, -121°54′W, elev. 985 m.a.s.l.), about 15 km away. For eleven years of data, the observed annual accumulation is 3.7±0.77 m yr\(^{-1}\), compared with 4.7±0.80 m yr\(^{-1}\) in the MM5 output. For melt-season temperatures the numbers are 13.3±1.2 °C, and 11.7±0.8 °C, respectively. It is the standard deviations that matter for driving glacier variations, so we consider this agreement sufficient to

\(^1\) http://www.wcc.nrcs.usda.gov/gis/index.html
proceed with using the MM5 output. For Mt. Baker MM5 output gives an annual accumulation of $5.5\pm1.0$ m yr$^{-1}$, and a melt season temperature of $9.3\pm0.8$ °C.

Spatial correlations in the interannual variability of mean annual precipitation and melt-season temperature in the vicinity of Mt. Baker are high ($>0.8$, O’Neal, 2005; Pelto, 2006; Huybers and Roe, 2008). Therefore, when the glacier model is evaluated against the glacier history of the past 75 years, we use the time series of observations at Diablo Dam scaled to match the standard deviation of the MM5 output at Mt. Baker.

**Parameter and data uncertainties.** The combined uncertainty in AAR and $\mu$ is nearly a factor of four. These dominate over other sources of uncertainty, and so we therefore focus on their effects in the analyses that follow. Both of these factors have their biggest proportional effects on the ablation side of the mass balance (for the melt factor, exclusively so). Thus, as we find for Mount Baker glaciers, while it may be that a glacier is most responsive to accumulation variations, the uncertainty in that responsiveness is dominated by uncertainty in the factors controlling ablation. In this paper we are after a general picture of glacier response to climate, so we explore this full range of uncertainty. However, for a specific glacier of interest, it is possible to better constrain both the AAR and the melt factor by careful measurements. At which point, it may be that other sources of uncertainty need to be more carefully accounted for.

5.0 Results

We first use the parameters of the typical Mount Baker glacier (Table 2) and use equations (8) and (9) to calculate $\sigma_{t_j}$ and $\sigma_{t_p}$, the variations in the model glacier’s terminus to characteristic
melt-season temperature, $\sigma_T$, and precipitation variability, $\sigma_P$, at Mount Baker. The range in $\sigma_T$ is from 74 to 308 m, with a mid-range value of 150 m. The magnitude of $\sigma_P$ is significantly larger, from 299 to 554 m, with a mid-range value of 391 m. Assuming the melt-season temperature and annual precipitation are uncorrelated, the combination of the two forcings can be calculated from the square root of the sum of the squares, which gives a range of 308 m to 634 m, and a mid range estimate of 419 m, and is obviously dominated by precipitation variability.

The ratio of the relative importance of precipitation and temperature variations on the glacier terminus confirms the dominance of precipitation variability in driving glacier terminus variations. Using equation (10), the ratio $R$ varies from 0.25 to 0.56, with a mid-range value of 0.38. In other words, the model suggests that, taking the characteristic local climate variability into account, the average Mount Baker glacier is between 2 and 4 times more sensitive to precipitation than to temperature variations. This is due to the very large interannual variability in precipitation. Thus we conclude that variability in Mount Baker glaciers are predominantly driven by precipitation variability.

As noted above, the relative importance of precipitation and melt-season temperature for the local mass balance is different from $R$, and is equal to $\mu \sigma_T / \sigma_P$. For Mt. Baker then, the model suggests that precipitation is more important than melt-season temperature by between 1.5 and 2.5, depending on the value of $\mu$ chosen. This range is consistent with, for example, the conclusions of Bitz and Battisti (1999) as to the cause of mass balance variations for several glaciers in this region.
A key point to appreciate about equation (10) is that the relative importance of precipitation and melt-season temperature for a glacier depends on the characteristic magnitude of the climate variability and so depends on location, as well as glacier geometry. Huybers and Roe (2008) use regional data sets of climate variability to explore how $R$ varies around the Pacific Northwest. Maritimes climates tend to have high precipitation rates and high precipitation variability, but muted temperature variability. The reverse is the case farther inland where, in more continental climates, and temperature variability becomes more important for driving glacier variations.

5.1 Historical Fluctuations of Mount Baker Glaciers

Historical maps, photos, and reports of Mount Baker glaciers indicate that they were retreating rapidly from 1931 to 1940, paused, and then began re-advancing between 1947 and 1952 (e.g., Long, 1955; Fuller, 1980; Harper, 1992). This advance continued until approximately 1980 when these glaciers again began to retreat. Although Rainbow and Deming glaciers began to advance about 1947, earlier than the other Mount Baker glaciers, the terminal movement between 1947 and 1980 is between 600 and 700 meters for each Mount Baker glacier, underscoring the similar length responses of these glaciers over this period.

We use the 75 year long record from the Diablo Dam weather station data, scaled to have the variance equal to the MM5 output at Mount Baker, and integrate equation (1) for the typical Mount Baker glaciers for period from 1931 to 2006, and for the range of model uncertainties given in Table 2 (Figure 4). The initial condition for the glacier model terminus is a free parameter. Choosing $L' = 600$ m produces the best agreement with the observed record.
Maximum changes in glacier length are on the order of 1000 meters, similar to the observed data for this period and approximately 50% of the observed magnitude of the glacier-length changes over the last two hundred years. There are some discrepancies between the model and the historical record – the model appears to respond a little quicker that the actual glaciers, probably due to the neglect of glacier dynamics in the model. However, we emphasize that the point is not to have the model be a simulation of the historical record, correct in all details. Rather, the point is to establish that the characteristic magnitude and the approximate timescale of glacier variations is reasonably captured by the model.

5.2 Glacier variations over longer timescales.

The success at simulating glacier length variations using historical climate data for the last 75 years suggests the model provides a credible means for estimating characteristic length-scale variations on longer timescales. Table 2 gives the range of estimates for the standard deviation of glacier fluctuations in response to this natural variability. By definition of the standard deviation of a normally-distributed process, the glacier will spend ~30% of the time outside of ±1σ ~5% of its time outside of ±2σ, and ~0.3% outside of ±3σ. Thus the statistical expectation is that, for three years out of every thousand, the maximum length of the glacier and minimum length during that time will be separated by at least 6σ. Table 2 shows the range of parameter uncertainty give 6σ varies between 1850 m and 3800 m, with a mid-range estimate of 2500 m. We emphasize this millennial-scale variability must be expected of a glacier even in a constant climate, as a direct result of the simple integrative physics of a glacier’s inertia, or memory.
To convey a sense of what this means in practice, Figure 5a shows a 5000 yr integration of the linear model, with geometry parameters equal to our typical Mount Baker glacier. The glacier model was driven by normally-distributed random temperature and precipitation variations with standard deviations given by the MM5 output for Mount Baker. By eye, it can be seen that there is substantial centennial variability, with an amplitude of 2 to 3 km. Also shown on Figure 4a are maximum terminus advances that are not subsequently overridden. Therefore these suggest occasions when moraines might be left preserved on the landscape (though the precise mechanisms of moraine deposition and conditions for preservation remain uncertain). Just by the statistics of chance, the further back in time you go, the more widely separated in time moraines become (e.g., Gibbons et al., 1984). Again we emphasize that none of the centennial and millennial variability in our modeled glacier terminus arises because of a climate change. To infer a true climate change from a single glacier reconstruction, the glacier change must exceed, at some statistical level of confidence, the variability expected in a constant climate.

Figure 5b shows the power spectral estimate of the terminus variations in Figure 4a, together with the theoretical spectrum for a statistical process given by equation (7) (e.g., Jenkins and Watts, 1969; von Storch and Zwiers, 1999). It can be shown that half of the variance in the power spectrum occurs at periods which are at least 2\(\tau\) times longer than the physical timescale of the system, in this case, \(\tau = 12\) years (e.g., Roe, 2008). Thus there is centennial, and even millennial variability in the spectrum, all fundamentally driven by the simple integrative physics of a process with a perhaps-surprisingly short timescale, and forced by simple stochastic year-to-year variations in climate.
Comparison with a dynamic glacier model

We next briefly evaluate the behavior of the linear glacier model compared to that of a dynamic flowline model (e.g., Paterson, 1994). The model incorporates the dynamical response of a single glacier flowline, based on the shallow ice approximation (e.g., Paterson, 1994). Assuming a no-slip lower boundary condition, the flowline obeys a nonlinear diffusion equation governed by Glenn’s flow law (e.g., Paterson, 1994):

\[
\frac{\partial h(x)}{\partial t} + \frac{\partial}{\partial x} \left[ 2A(\rho g)^n h(x)^{n+2} \left( \frac{dz_s(x)}{dx} \right)^n \right] = \dot{b}(x)
\]  

(12)

where \( x \) is the horizontal co-ordinate; \( h(x) \) is the thickness of the glacier; \( z_s \) is the surface slope; \( \dot{b}(x) \) is the net mass balance at a point on the glacier; \( \rho \) is ice density; and \( g \) is gravitational acceleration. \( n \) is the exponent relating applied stress to strain rates, set equal to 3, and \( A \) is a flow factor taken to be \( 5.0 \times 10^{-24} \text{ Pa}^{-3} \text{ s}^{-1} \). The flowline model is a reasonable representation of ice flow, and also includes the nonlinearities of the mass balance perturbation. Thus a comparison between models is a useful evaluation of the validity of the assumptions made in deriving the linear model.

Equation (12) can be solved using standard numerical methods. Figure 6 shows an integration of the dynamical flowline model for climatic and geometrical conditions identical to those applying for the linear model calculations shown in Figure 5, and for average values of parameters shown in Table 1. The dynamical model produces glacier length with a standard deviation of 360 m, compared with 420 m for the linear model. Thus the magnitudes of variations produced by the two models are similar, though there is a suggestion that the linear model may overestimate the glacier response by about 15%. Obviously a much fuller exploration of the glacier dynamics is possible: no glacier sliding has been included, nor has there been any accounting for the
confining lateral stresses of the side-walls. Computational constraints limit the model resolution to 75m, and the pattern of the climatic perturbations has been assumed uniform. Such an exploration is important to undertake and is the subject of ongoing work. However the purpose in the present study is to establish that the linear model produces a reasonable magnitude of glacier variability compared to a model incorporating ice flow dynamics and without a linearization of the mass balance.

6.0 Summary

A simple linear model has been presented for estimating the response of typical midlatitude glaciers to climate forcing, with a particular focus on the interannual variability in accumulation and melt-season temperature that is inherent even in a constant climate. With one free matching parameter allowed, and otherwise using standard physical, geometrical and climatic parameters pertaining to the region, the model produces a reasonable simulation of the observed variations of glaciers on Mt. Baker in the Cascade Range of Washington State over the past seventy-five years. The magnitude of variability in the simple model also approximates that seen in a more complicated model incorporating ice flow dynamics.

Mount Baker glacier lengths are more sensitive to accumulation than to melt-season temperature, by a factor of between two and four. The maritime climate and mountainous terrain of the region produces large interannual accumulation variability (~1 m yr⁻¹), and muted melt-season temperature variability. By contrast, calculations using the same model for glaciers in continental climates show the reverse sensitivity (Huybers and Roe, 2008). The expression given in equation (10) is a simple and robust indicator of the relative importance of melt-season
temperature and accumulation variability for a glacier. The factor of two uncertainty is principally due to uncertainty in the melt factor and the AAR. Both of these can be much more tightly constrained for specific glaciers by careful observations.

Within the bounds of the observed natural variability in climate expressed by instrumental observations between 1931 and 1990, and the range of model parameters that we consider to be reasonable, the 1.3- to 2.5-km length fluctuations on Mount Baker attributed to the LIA can be accounted for by the model without recourse to changes in climate. A variety of external climate forcings are commonly invoked to explain glacier-length fluctuations on centennial to millennial scales: changes in the strength of the atmospheric circulation (e.g., O’Brien et al., 1995); atmospheric dust from volcanic eruptions (e.g., Robock and Free, 1996); and variations in sunspot activity (e.g., Soon and Baliunas, 2003). However the model results indicate kilometer-scale fluctuations of the glacier terminus do not require a substantial change in temperature or precipitation and should be expected simply from natural year-to-year variability in weather.

To attribute regional or global glacier changes as having been driven by a climate change, we must first falsify the null hypothesis that there was no climate change. In particular, to attribute the nested sequences of late Holocene moraines on Mount Baker to a distinct climate change, we require that changes were larger, or of longer duration, than that expected from the observed climatic variability over the past 75 years, a condition that is not required by the model predictions. Furthermore, any systematic regional or global climate change that does take place will always be superimposed on top of this natural climate variability. This complicates the
identification of any such global climate signal, and requires an even greater magnitude of change before it can be recognized unequivocally.

7.0 Discussion

Model framework
The linear glacier model required several important assumptions about ice dynamics and also neglects nonlinearities in the glacier mass balance. We discuss here what consequences of having done this might be. Comparing with a nonlinear dynamical flow-line glacier model, we find that the linear model produces variability about 15% larger than the dynamical model. This is sufficient agreement: we emphasize that for the purpose of this paper the magnitude of glacier variability only needs to be reasonably represented, and therefore that the linear model is an adequate tool for the question posed. Indeed, in view of other uncertainties in the problem such as the details of the glacier mass balance, it is unclear what value would be added by employing a more complex glacier model. Nonetheless, further exploration of the reasons for the differences between the two glacier models would be interesting and enlightening.

We note also that we focused on a single, characteristic Mount Baker glacier, but one should expect some sizeable differences in the magnitude of glacier variability, even among glaciers so close together as those around Mount Baker, because of differences in geometry. For example, $A_{tot}$ has a considerable influence on glacier variability, as we see, for example, from equations (8) or (9), and $A_{tot}$ varies by a factor of two among Mount Baker glaciers (Table 1).
Our approach to the relationship between climate and glacier mass-balance was crude. A distinction between snow and rain might be more carefully made. Based on the fraction of annual precipitation that falls in winter, we estimate this might have perhaps a 20% effect on our answers. Secondly, we assumed a simple proportionality between ablation and the temperature of a melt-season temperature of fixed length. A treatment based on positive degree days could easily be substituted (e.g., Braithwaite and Oleson, 1989). However it is not temperature per se that causes ablation, but rather heat. A full surface energy balance model is necessary to account for the separate influence of radiative and turbulent fluxes, albedo variations, cloudiness, and aspect ratio of the glacier surface on steep and shaded mountain sides (e.g., Rupper and Roe, 2008a,b). It is hard to single out any one of these effects as more important than any other. To pursue all of them in a self-consistent framework would require a detailed surface energy balance and snow pack model, including the infiltration, percolation and re-freeze of melt-water. The resulting system would be complicated, and it is not clear that, with all its attendant uncertainties, it would produce a higher quality or more meaningful answer than our first-order approach.

Several other factors that we have not incorporated probably act to enhance glacier variability over and above what we have calculated. We have neglected mass sources due to avalanching and wind-blown snow, both of which increase the effective area over which a glacier captures precipitation. We have assumed that the glacier surface slope is linear. The characteristically convex-up profile of a real glacier acts to enhance the glacier sensitivity, since the ablation area as well and the ablation rate increases with increasing melt-season temperature (e.g., Roe and Lindzen, 2001). Finally while no statistically-significant interannual memory in annual precipitation could be demonstrated from the available weather station records near Mt. Baker,
other gridded atmospheric reanalysis datasets do suggest some weak interannual persistence. Although it varies spatially within the region, Huybers and Roe (2008) find some one-year lag correlations in annual mean precipitation anomalies of around 0.2 to 0.3. This small autocorrelation makes it slightly more likely that the next year’s precipitation anomaly will have the same sign as this year’s and so act to reinforce it. Huybers and Roe (2008) show that a one-year autocorrelation of 0.3 is enough to amplify the glacier variability by 35%, similar to that found by Reichart et al. (2002). On the basis of all the arguments given above, we have every reason to think that our estimate of the glacier response to natural climate variability errs on the conservative side – it may well be larger in reality.

**Implications**

One lesson from our analyses is that small-scale patterns in climate forcing, inevitable in mountainous terrain, are tremendously important for adequately capturing the glacier response. Had we used nearest long-term record from the weather station at Diablo Dam we would have underestimated the glacier variability by 65%. The lapse rate that the glacier surface experiences during the melt season has a important effect on the glacier response, as can be seen from equations (2), (8), and (9). The relevant lapse rate is likely not simply a typical free-air value assumed here, but has some complicated dependence on local setting and mountain meteorology. The archive of high-resolution MM5 output provides an invaluable resource for the investigation of such effects and will be the focus of future investigations.

It is also possible to take advantage of spatial patterns of glacier variability in interpreting climate. Huybers and Roe (2008) show that spatial patterns of melt-season temperature and
annual precipitation are coherent across large tracts of western North America, though not always of the same sign – there is an anti-correlation of precipitation between Alaska and the Pacific Northwest, for example. On spatial scales for which patterns of natural climate variability are coherent, coherent glacier variability must be expected also – tightly clustered glaciers provide only one independent piece of information about climate. Huybers and Roe (2008) use equation (1) to evaluate how patterns of melt-season temperature and annual accumulation are convolved by glacier dynamics into regional-scale patterns of glacier response.

Patterns of climate variability that are both spatially coherent and also account for a large fraction of the local variance are at most regional in scale, and so the current world-wide retreat is a powerful suggestion of global climate change (e.g., Oerlemans, 2004). However a formal attribution of significance requires an accounting of the relative importance of melt-season temperature and precipitation in different climate settings, of how much independent information is actually represented by clustered glacier records, and of whether the trend rises above the background variability. We anticipate that the global glacier record would probably pass such a significance test, but performing it would add greatly to the credibility of the claim. The model presented here provides a tool for such a test.

The long-term, kilometer-scale fluctuations predicted by the model provide the opportunity to suggest alternative interpretations or scenarios for moraine ages that are often attributed to poorly dated glacier advances from the 12th to 20th centuries. Many moraines at Mount Baker and in other Cascade glacier forelands with similar physiographic settings and glacier geometries have been dated by dendrochronology using tree species that are at the limit of their lifespan.
The range of glacier fluctuations produced by the model, combined with these poor constraints in the actual landform ages, suggest that these moraines may be products of even earlier advances, not necessarily synchronous with each other and certainly not necessarily part of a global pattern of climate fluctuations. Random climatic fluctuations over the past 1000 years may have been ample enough to produce large changes in glacier length, and until quantitative dating techniques can used to reliably correlate widely separated advances from this interval, these advances cannot be used as the main evidence for a synchronous signal of regional or global climate change.

The primary purpose of this paper was to explore the idea that substantial long-timescale glacier variability occurs even in a constant climate. We conclude that the effect of such variations cannot and must not be ruled out as a factor in the interpretation of glacier histories. The results also raise the possibility that cause of variations recorded in many glacier histories may have been misattributed to climate change. Although glacier records form the primary descriptor of climate history in many parts of the world, those records are in general fragmentary, and provide only a filtered glimpse of the magnitude of individual glacier advances and retreats, and of the regional or global extent of the coherent patterns of glacier variations.

The formal evaluation of whether the magnitude or regional coherence of glacier variability does, or does not, exceed that expected in a constant climate is a detailed and complicated exercise. At a minimum, it involves knowing: small-scale patterns of climate forcing and their variability; the relationship between those variables and the glacier mass balance; and finally, that the glacier dynamics are being adequately captured. Regional- or global-scale patterns of
past glacier variability are also useful, but suffer from difficulties in accurately cross-dating the histories. Our results demonstrate, however, that such an evaluation must be performed before glacier changes can be confidently ascribed to climate changes. Given the very few examples where this has been done at the necessary level of detail, a substantial reevaluation of the late Holocene glacier record may be called for.

**Acknowledgements:** The authors thank Kathleen Huybers, Summer Rupper, Eric Steig, and Brian Hansen, and Carl Wunsch for insightful conversations and comments, and are enormously grateful to Justin Minder and Neal Johnson for their heroic efforts to compile the MM5 output from archives. GHR acknowledges support from NSF continental dynamics grant #0409884.
Appendix A: Derivation of linear glacier model equations

Here we derive the equations used in the linear glacier model, using the geometry shown in Figure 2. Following Johannesson et al., (1989), the model considers only conservation of mass.

The rate of change of glacier volume, $V$, can be written as

$$\frac{dV}{dt} = \text{accumulation} - \text{ablation}. \quad (A1)$$

We assume that the ablation rate is $\mu T$, where $T$ is the melt-season temperature and $\mu$ is the melt factor and all melting ends up as run-off. A constant might be added to the ablation rate as in Pollard (1982) or Ohmura and Wild (1998). However the model equations will be linearized about the equilibrium state, the constant would not enter into the first-order terms.

Let the temperature at any point on the glacier be given by

$$T = T_p + x\Gamma\tan\varphi, \quad (A2)$$

where $x$ is the distance from the head of the glacier, $T_p$ is the melt-season temperature at the head of the glacier, $\Gamma$ is the atmospheric lapse rate, and $\varphi$ is the slope of the glacier surface (assumed parallel to the bed). The temperature at the toe of the glacier of length $L$ is then equal to

$$T(x = L) = T_p + L\Gamma\tan\varphi. \quad (A3)$$
There is some melting wherever the melt-season temperature exceeds 0 °C, and the area of the melting zone is given by $A_{T>0} = w(L - x_{T=0})$. Since temperature decreases linearly with elevation and the local melt rate is proportional to temperature, the total melting that occurs is equal to one half multiplied by the total melt area, multiplied by the melt rate at the toe of the glacier (this is simply the area under the triangle of the melt rate – distance graph), or in other words:

$$\text{ablation} = \frac{1}{2} \cdot A_{T>0} \cdot \mu_{T=L}.$$  \hspace{1cm} (A4)

Note that melting above the ELA is included in this calculation. The total accumulation is just the product of the precipitation rate, $P$ (assumed uniform over the glacier), and the total glacier area, $A_{\text{tot}}$.

We now linearize the glacier system, about some equilibrium climate state denoted by an overbar: $P \rightarrow \bar{P} + P'$, $T_p \rightarrow \bar{T}_p + T'$, and $L \rightarrow \bar{L} + L'$, where the climate anomalies are uniform over the glacier. Similar to previous studies, $H$ and $w$ are assumed constant (e.g., Johanneson et al., 1989, though this assumption can be relaxed, e.g., Oerlemans, 2001), which means $V' = wHL'$. We also note that the melt-season freezing line, $x_{T=0}$, can be found from (A3):

$$\bar{T}_p + T' = -x_{T=0}\Gamma\tan\varphi.$$  \hspace{1cm} (A5)

So in the perturbed state and using (A2), (A4) becomes:
\[ \text{ablation} = \frac{1}{2} \cdot w \left[ (\bar{L} + L') - x_{T=0} \right] \cdot \mu \left[ \bar{T}_p + T' + (\bar{L} + L') \Gamma \tan \varphi \right]. \quad (A6) \]

Substituting \( x_{T=0} \) from (A5), and rearranging a little gives

\[ \text{ablation} = \frac{\mu w}{2 \pi} \left[ (\bar{L} + L') \Gamma \tan \varphi + \bar{T}_p + T' \right]^2, \quad (A7) \]

which using (A5) can be written as

\[ \text{ablation} = \frac{\mu w}{2 \pi} \left[ (\bar{L} - x_{T=0}) \Gamma \tan \varphi + L' \Gamma \tan \varphi + T' \right]^2. \quad (A8) \]

Note that \( w(\bar{L} - x_{T=0}) = \bar{A}_{T>0} \) is the area over which melting occurs. For accumulation we can simply write

\[ \text{accumulation} = (\bar{P} + P')(\bar{A}_{w0} + wL'). \quad (A9) \]

Taking only first order terms, and combining (A1), (A8), and (A9), we get

\[ wH \frac{dL'}{dt} = (\bar{P}_w - \mu \bar{A}_{T>0} \Gamma \tan \varphi)L' + \bar{A}_{w0} P' - \mu \bar{A}_{T>0} T'. \quad (A10) \]

One last simplification is possible. At the climatological equilibrium line altitude (ELA)
\[ \bar{P} = \mu \bar{T}_{ela}, \text{ and so } \bar{wP} = \mu \bar{wT}_{ela} = \mu \bar{w}(\bar{T}_p + \bar{x}_{ela} \Gamma \tan \varphi) = \mu \bar{w} \Gamma \tan \varphi (\bar{x}_{ela} - \bar{x}_{T=0}), \quad (A11) \]

where (A2) and (A5) have been used. Defining the ablation-zone in the sense of, for example, Paterson (1994), as the region below the ELA, we can write. Finally then, (A10) becomes:

\[
\frac{dL'}{dt} + \frac{\mu A_{abl} \Gamma \tan \varphi}{wH} L' = \frac{A_{tot}}{wH} P' - \frac{\mu A_{T=0}}{wH} T', \quad (A12).
\]

where the overbars have been dropped for convenience. (A12) is the governing equation of the glacier model used in this study.

Lastly, using (A11) the relationship between the melt-zone area and the ablation-zone area is given by

\[
A_{T=0} = A_{abl} + w(x_{ela} - x_{T=0}) = A_{abl} + \frac{P_w}{\mu \Gamma \tan \varphi}. \quad (A13)
\]

For the standard model parameters given in Table 1, \( A_{T=0} \) adds another 1.67 km\(^2\) to \( A_{abl} \).

**References**


Soon, W., and S. Baliunas, 2003: Proxy climate and environmental changes of the past 1,000 years. Climate Research, 23, 89-110.


Tables and Figures

<table>
<thead>
<tr>
<th></th>
<th>Boulder</th>
<th>Deming</th>
<th>Coleman</th>
<th>Easton</th>
<th>Rainbow</th>
<th>‘typical’</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{tot}$ (km$^2$)</td>
<td>4.30</td>
<td>5.40</td>
<td>2.15</td>
<td>3.60</td>
<td>2.70</td>
<td>4.0</td>
</tr>
<tr>
<td>$A_{abl,0.8}$ (km$^2$)</td>
<td>0.86</td>
<td>1.08</td>
<td>0.43</td>
<td>0.72</td>
<td>0.54</td>
<td>0.80</td>
</tr>
<tr>
<td>$A_{abl,0.7}$ (km$^2$)</td>
<td>1.29</td>
<td>1.62</td>
<td>0.64</td>
<td>1.08</td>
<td>0.81</td>
<td>1.20</td>
</tr>
<tr>
<td>$A_{abl,0.6}$ (km$^2$)</td>
<td>1.72</td>
<td>2.16</td>
<td>0.86</td>
<td>1.44</td>
<td>1.08</td>
<td>1.60</td>
</tr>
<tr>
<td>$\tan\varphi$</td>
<td>0.47</td>
<td>0.36</td>
<td>0.47</td>
<td>0.34</td>
<td>0.32</td>
<td>0.40</td>
</tr>
<tr>
<td>$w$ (m)</td>
<td>550</td>
<td>450</td>
<td>650</td>
<td>550</td>
<td>300</td>
<td>500</td>
</tr>
<tr>
<td>$H$ (m)</td>
<td>50</td>
<td>50</td>
<td>39</td>
<td>51</td>
<td>47</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 1: Parameters for the major Mount Baker glaciers, obtained from a variety of sources. See Figure 2 for a schematic illustration of the model parameters, and the text for details. $A_{abl}$ is shown for several different choice of the AAR. Also given in the table are a choice of a set of typical parameters, used in the glacier model.
Table 2: Minimum, mean, and maximum estimates of standard deviations in glacier lengths for various glacier properties for the typical Mt. Baker glacier defined in Table 1, and driven by climate variability determined from the MM5 model output. See text for more details. The range of values here is generated from the range of uncertainties in the melt factor and the accumulation area ratio.

<table>
<thead>
<tr>
<th></th>
<th>Min</th>
<th>Mid</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$ (years)</td>
<td>7</td>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td>$\sigma_{L_T}$ (m)</td>
<td>74</td>
<td>150</td>
<td>308</td>
</tr>
<tr>
<td>$\sigma_{L_P}$ (m)</td>
<td>299</td>
<td>391</td>
<td>554</td>
</tr>
<tr>
<td>$\sigma_L = \sqrt{\sigma_{L_T}^2 + \sigma_{L_P}^2}$ (m)</td>
<td>308</td>
<td>419</td>
<td>634</td>
</tr>
<tr>
<td>Sens ratio; $R = \frac{\sigma_{L_P}}{\sigma_{L_T}}$</td>
<td>0.25</td>
<td>0.38</td>
<td>0.56</td>
</tr>
<tr>
<td>$6\sigma_L$ (m)</td>
<td>1848</td>
<td>2514</td>
<td>3804</td>
</tr>
</tbody>
</table>
Figure 1: Major Mount Baker glaciers superposed on a contour map (c.i. = 250 m). Glaciers are shown at their ‘Little ice age’ maxima, 1930, and present positions.
Figure 2: Idealized geometry of the linear glacier model, based on Johanneson et. al. (1989). Precipitation falls over the entire surface of the glacier, $A_{\text{tot}}$, while melt occurs only on the melt-zone area, $A_{T>0}$. The ablation zone, $A_{\text{abl}}$, is the region below the ELA. Melt is linearly proportional to the temperature, which, in turn, decreases linearly as the tongue of the glacier recedes up the linear slope, $\tan \phi$, and increases as the glacier advances down slope. The height $H$ of the glacier, and the width of the ablation area, $w$, remain constant. Figure courtesy of K. Huybers.
Figure 3. (a) Annual mean precipitation recorded at Diablo Dam near Mt Baker, over the last seventy-five years, equal to $1.89 \pm 0.36(1\sigma)$ m yr$^{-1}$; (b) melt-season (JJAS) temperature at the same site, equal to $16.8 \pm 0.78(1\sigma)$ °C; these atmospheric variables at this site are statistically uncorrelated and both are indistinguishable from normally-distributed white noise with the same mean and variance. The application of a five-year running mean imparts the artificial appearance of multi-year regimes. Random realization of white noise are shown for annual-mean accumulation (panels (c) and (e)); and for melt-season temperature (panels (d) and (f)). Note the general visual similarity of the random realizations and the observations.
Figure 4: Model glacier-length variations from 1931 to 2006, using the time series of annual precipitation and melt-season temperature from Diablo Dam, scaled by the MM5 output for Mount Baker. The grey shading shows zone for the range of model uncertainties given in Table 2. Also shown is the historical glacier fluctuation record from Harper (1992) and O’Neal (2005), and Pelto (2006). Negative numbers mean retreat. The initial perturbation length at 1931 for the glacier model is a free parameter and was chosen to be 600 m, and was chosen to produce the best fit with the historical record.
Figure 5: a) 5000 year integration of the linear glacier model, using parameters similar to the typical Mount Baker glacier, and driven by random realizations of interannual melt-season temperature and precipitation variations consistent with statistics for Mount Baker from the MM5 model output. The gray shading shows the range of results for the uncertainties in parameters given in Table 2. The black line shows the time series for the mid-range estimate of parameters. The green line is a 100-year running average. The dots denote maximum terminus advances that are not subsequently over-ridden, and so are possible times for moraine formation; b) the black line is the power spectral estimate of the mid-range time series generated using the mid-range parameters. The green line is the theoretical red-noise spectrum (solid), together with its 95% confidence band (dashed). Spectrum was calculated using a periodogram with a 1000-year Hanning window. The arrow show the frequency corresponding to $1/\tau$, and so the spectrum emphasizes that much of the variability in the glacier time series occurs at periods which are much longer than the physical response time of the glacier.
Figure 6. As for Figure 4, but using the dynamical flowline glacier model described in the text. Note the slightly reduced length variations compared to the linear model. The frequency range of the spectrum here is less than in Figure 4 because of the discretized output from the numerical model.