

# Homework 1.

Due Friday, April 14th

Units, units, units....

**Question 1.** Scale height.

(i) Using the data from the handout in class calculate atmospheric scale height at the equator at  $z= 0$  km, 5 km, 10 km, and 15 km.

(ii) Show that, in a dry atmosphere,  $RTd\rho/dz = (1 - \frac{R}{c_p})dp/dz$ , and hence that to within about 30%, density has a similar characteristic height scale as pressure.

(iii) If the atmosphere had a constant density,  $\rho_s$ , and temperature,  $T_s$  (i.e., equal to the values at the surface), show that it would have a depth equal to the atmospheric scale height (i.e.,  $H = RT_s/g$ ).

**Question 2.** Geopotential height vs. geometrical height.

The geopotential height  $\mathcal{Z}$  is defined such that

$$\mathcal{Z} = \frac{\Phi}{g_0} = \frac{1}{g_0} \int_0^z g dz \quad (1)$$

where  $\Phi$  is the geopotential and where  $g_0$  has a standard value of  $9.807 \text{ ms}^{-2}$ , its mean value at the surface.

Derive an expression for the variation with height of the acceleration due to gravity (i.e., from Newton's Law of Gravity). Over a place where the value of  $g$  at the surface ( $z = 0$ ) is equal to  $g_0$ , what is the difference between  $\mathcal{Z}$  at the geometric height when  $z = 100 \text{ km}$ ?

The value of  $g$  decreases by approximately 0.5% between the equator and the poles. Again for  $z = 10 \text{ km}$  and  $z = 100 \text{ km}$ , compute the differences in geopotential height  $\mathcal{Z}$  and geometric height  $z$ , over the equator and over the poles.

### Question 3 Geopotential thickness

Suppose (as is commonly the case) that you know the sea level pressure,  $p_{msl}$ , the sea level temperature  $T_{sfc}$ , and the height of the 500 mb geopotential height,  $Z_{500}$ . Show that you can write the 1000 mb - 500 mb geopotential thickness as:

$$\Delta Z = Z_{500} - \frac{(p_{msl} - 1000 \text{ mb})}{g \times 1000 \text{ mb}} RT_{sfc} \quad (2)$$

Hence demonstrate how much difference it makes, to a lazy colleague who suggests simply using sea level pressure instead of 1000 mb pressure to calculate mean temperatures. A stupefyingly large winter storm might have a sea level pressure of 960 mb and a typical winter time surface temperature over the ocean of 10 °C, in a region with a 500 mb geopotential height of 5500 m.

### Question 4. Random physics question - ideal gas law.

A balloon filled with  $10^{-4}$  kg of hydrogen at a pressure of 1.05 atmospheres ( $1.05 \times 10^5 \text{ N m}^{-2}$ ) is allowed to rise to the ceiling of a room. What area of its surface is in contact with the ceiling? Neglect the weight of the fabric of the balloon and, if you need to, take the mean relative molecular mass of air as 28.8.

### Question 5. Energy and Work along a pathway.

Consider a dry air parcel of mass 1 kg that starts from the Earth's surface near the Equator at potential temperature  $\theta_1$  and pressure  $p_1$ . It first rises adiabatically to pressure  $p_2$  ( $p_2 < p_1$ ). It radiates, losing energy  $Q_2$  to outer space, which is at temperature  $T_{space}$ , as it moves northwards on a constant pressure surface. When its potential temperature falls to  $\theta_2$  it sinks adiabatically back to Earth (i.e., back to surface pressure  $p_1$ ), and then moves southwards along the surface to its original position and temperature. In the process it gains heat  $Q_1$  from the underlying surface which is a temperature  $T_{surface}$ .

The system just described has three parts: the parcel, outer space, and the Earth's surface. The three part system is isolated from the rest of the

world. Answer the following questions in terms of  $Q_1$ ,  $Q_2$ ,  $T_{space}$ ,  $T_{surface}$ ,  $p_1$ ,  $p_2$ ,  $\theta_1$ , and  $\theta_2$ .

(i) Find the work done by the parcel on its environment, or *vice versa*, on each of the adiabatic legs.

(ii) Find the work done on each of the isobaric (i.e., constant pressure) legs.

(iii) Show that energy is conserved during one cycle.

(iv) Compute the entropy change,  $S$ , per cycle in each part of the system (recall  $dS = dQ/T$ ). Does the entropy of the parcel increase or decrease in each part of the cycle? What about the total entropy of the system over the whole cycle?

(The key to this question is to integrate separately along each of the legs, using the fact that depending on the leg, either  $\theta$  or  $p$  does not change on that leg)

**Question 6.** Vertical motion in a dry stable atmosphere.

Consider an air parcel of unit mass, originally at equilibrium with its environment at altitude  $z = z_1$ , pressure  $p = p_1$  in the lower stratosphere, where the temperature  $T_{env} = T_1$ . The environmental temperature increases with height:  $\frac{dT_{env}}{dz} = \Gamma_{env} > 0$ . The parcel is displaced adiabatically upward by a small distance  $\Delta z$ .

(i) Find  $\theta'(\Delta z)$ , the difference between the potential temperature of the parcel and that of the environment at its new position.

(ii) What is the vertical force per unit mass on the parcel at its new position? (Think Archimedes, and assume  $\theta'/\theta_1 \ll 1$ ).

(iii) Show that the parcel will execute simple harmonic motion and find the frequency of that motion in terms of  $\Gamma_{env}$  and  $\Gamma_d$ . This frequency is called the Brunt-Vaisala frequency.

(iv) This frequency is the natural frequency of buoyancy oscillations in the

atmosphere (i.e., with gravity as the restoring force), and is very related to the frequency of mountain wave oscillations seen in the lee of mountains. Is this frequency likely to be higher on a clear, winter's day with snow cover, or on a cloudy, windy spring day (without snow)? Explain.