

Homework III: Due April 28th, 2006

Question 1: Gradient wind approximation.

Starting with the horizontal momentum equation (in vector form), show that when friction is neglected and the other terms are retained, that in the presence of a pressure gradient $\frac{\partial p}{\partial r}$, and for flow with a radius of curvature r ,

$$V = -\frac{fr}{2} \pm \left[\frac{f^2 r^2}{4} + \frac{r}{\rho} \frac{\partial p}{\partial r} \right]^{\frac{1}{2}} \quad (1)$$

where V is the magnitude of the wind vector (i.e., the speed!). This is the solution for the gradient wind approximation. Note that r is measured from the center of curvature and V is considered positive when cyclonic and negative when anticyclonic.

Show that for anticyclonic curvature

$$\left| \frac{\partial p}{\partial r} \right| < \frac{\rho r f^2}{4} \quad (2)$$

and hence that for anticyclones the pressure gradient decreases towards the center. This is why the pressure gradients are small and the winds light near the center of an anticyclone.

Draw diagrams showing the balance of forces for the two cyclonic and two anticyclonic solutions. To do this, think about the different possible ways that the three forces might balance each other.

Comment on their physical realizability. This last part is horribly hard - think about the total angular momentum of the flow, which includes a contribution from the Earth's rotation and a contribution from the rotation of the parcels relative to the Earth.

Question 2: Inertial oscillations.

Starting with the terms in the gradient wind balance show that, in the absence of a pressure gradient, that the radius of curvature of the flow is $-V/f$ and is anticyclonic. What is the period of a complete oscillation? Such flow is known as *inertial* flow and is seen in both the oceans and atmosphere...

Question 3 - thermal wind

- i) If the pole is 40 K colder than the equator and the surface wind is zero what wind would you expect at the 200 mb level pressure level?
- ii) The wind at the surface is from the west. At cloud level it is from the south. Do you expect the temperature to rise or fall?

Question 4: Random physics question - buoyancy/Archimedes

A man in a boat on a pond of area A has with him a block of stone (mass M_s , density ρ_s), more dense than water, and a block of timber (mass M_t , density ρ_t), less dense than water (ρ_w). What happens to the level of water in the pond if he throws out:

- (i) the block of stone,
- (ii) the block of timber,
- (iii) both of them tied together, so that they go to the bottom of the pond?

where applicable, derive expressions for the change in the water level.