~~Ideal~~ Ideal Gas Law:

- Below ~100km, to a good approximation the atmosphere behaves as as ideal gas
- Mean free path: $\lambda \sim 10^{-7}m$ @ $z=0km$, $\lambda \sim 1m$ @ $z=100km$

So: $pV = nR^*T$

p = pressure, V = volume, n = no. of moles
R^* = universal gas constant = $8.31\ J\,kg^{-1}\,°C^{-1}$
T = absolute temperature

- For the atmosphere, it is easier to deal with densities, so gas law is gas specific:

$$R = \frac{R^*}{M_d}$$

M_d = molecular weight of dry air

$$\simeq \underset{\substack{\uparrow \\ N_2}}{80\% \times 28} + \underset{\substack{\uparrow \\ O_2}}{20\% \times 32}$$

$$\simeq 29\ g/mol$$

$$\Rightarrow R = \frac{8.31}{29/1000}$$

$$\simeq 287\ J\,kg^{-1}\,°C^{-1}$$

$$\Rightarrow \quad pV = \underbrace{n \times M_d}_{\text{mass}} \times R \times T$$

$$\Rightarrow \quad \boxed{p = \rho R T}$$

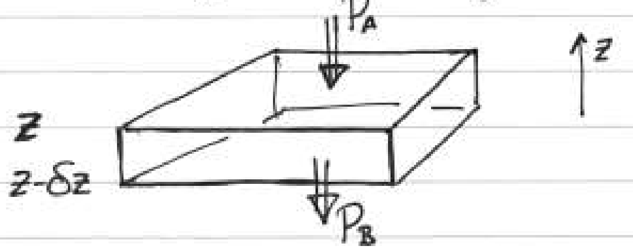
density

Equation of State for Atmosphere

Also, sometimes see: $\alpha = 1/\rho =$ "specific volume"

<u>Hydrostatic Eq^n</u>:

- ideal gas law immediately allows us to see how pressure varies with altitude



imagine slab of the atmosphere. The force on face B equals force on face A plus the weight of the slab

$$\boxed{\Rightarrow \quad p_B \times A = p_A \times A + g \cdot \rho \cdot A \cdot \delta z} \quad (1)$$

A = area of face A, B

Since $p = p(z)$, can write

$$p_B = p_A - \frac{dp}{dz}\delta z \qquad \left(1^{st} \text{ order Taylor expansion}\right)$$

① becomes:

$$\Rightarrow \left(p_A - \frac{dp}{dz}\delta z\right)A = p_A \cdot A + Ag\rho\,\delta z$$

$$\Rightarrow \boxed{\frac{dp}{dz} = -g\rho} \qquad \text{Hydrostatic equation}$$

→ Using ideal gas law $p = \rho RT$

$$\frac{dp}{dz} = -\frac{gp}{RT}$$

$$\Rightarrow \frac{dp}{p} = -\frac{g}{RT}\,dz$$

Note RT/g has dimensions [m]

Let $H = RT/g$ = ATMOSPHERIC SCALE HEIGHT ~ constant (why

$$\sim \frac{287 \times 300}{9.8}$$

~ **9 km** in tropics (7.5 km at poles)

So $\frac{dp}{p} \simeq -\frac{dz}{H}$ H const.

integrate from surface $z=0$, $p=p_s$ to altitude z, pressure p

$$\int_{p_s}^{p} \frac{dp}{p} = -\int_{0}^{z_s} \frac{dz}{H}$$

$$\ln p/p_s = -z_s/H$$

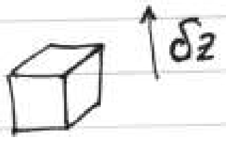
$$\Rightarrow \boxed{p = p_s e^{-z_s/H}}$$

So pressure decays exponentially away from the surface with a characteristic scale, H.

<u>Geopotential</u> - turns out to be a useful way of representing height

ϕ = work done against gravity in lifting 1 kg parcel of air from surface to altitude z.

= geopotential



wk done = force x distance

$$\phi = \int_0^z \cancel{m} g \, dz$$

n.b. g not constant
why?? 2 reasons

$$\phi = \int_{p_0}^{p} \frac{dp}{\rho}$$

(using ~~the ideal gas law~~) hydrostatic equation)

n.b. geopotential depends only on the elevation the parcel is at, and <u>not</u> on how it got there.

Can define geopotential height:

$$Z = \phi(z)/g_0 \qquad g_0 = 9.8 \text{ ms}^{-2}$$

Value of geopotential height:

back to hydrostatic eq^n:

$$\frac{dp}{dz} = -g\rho$$

from def^n of geopotential $\quad d\varphi = g\,dz$

n.b. looks a lot like hydrostatic eq^n, but have got rid of density

$$\Rightarrow \quad d\varphi = -RT\,\frac{dp}{p}$$

$$\Rightarrow \quad \varphi_2 - \varphi_1 = -R\int_{p_1}^{p_2} T\,\frac{dp}{p}$$

$$Z_2 - Z_1 = \frac{R\bar{T}}{g_0}\ln p_1/p_2 \qquad \bar{T} = \text{average temperature}$$

$\Rightarrow$ the mean temperature in a layer of the atmosphere is proportional to the GEOPOTENTIAL HEIGHT THICKNESS

Example:

What is the geopotential height thickness between 1000 mb & 500 mb which would mean the average temperature in that layer was freezing?

$$\Delta z = \frac{287 \times 273}{9.8} \ln \frac{1000}{500}$$

$$\simeq 5500 \text{ m}$$

Forecasters use the 1000 mb-500 mb thickness as an indicator for ~~detecting the~~ determining the rain-snow line in winter...