Thermodynamics of air parcels

- in order to investigate how temperature changes with height, need to say something about the thermodynamics of air parcels.

1st law of thermodynamics

$$\boxed{\underset{\substack{\uparrow \\ \text{heat} \\ \text{applied}}}{dQ} = \underset{\substack{\uparrow \\ \text{internal} \\ \text{energy}}}{dU} + \underset{\substack{\uparrow \\ \text{work done}}}{dW}}$$

Also need heat capacities:

Normally from physics $\Delta Q = \underset{\substack{\uparrow \\ \text{mass}}}{m}\,\underset{\substack{\nwarrow \\ \text{spec.} \\ \text{heat} \\ \text{capacity}}}{C}\,\underset{\substack{\nwarrow \\ \text{change in temperature}}}{\Delta T}$

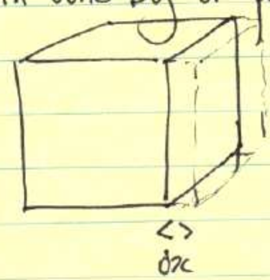
For a solid, this is good enough, but for a fluid, the heat required to raise the temperature a given amount depends on what is happening to the fluid as the heat is applied. It is useful in particular to define two particular specific heat capacities.

$$C_v = \left.\frac{dQ}{dT}\right)_v \qquad C_p = \left.\frac{dQ}{dT}\right)_p$$

C_v: constant volume C_p: constant pressure

n.b. in everything that follows, assume it is <u>per unit mass</u>, so can drop the m in dealing with the heat capacities

Work done by a parcel of air as it changes volume



$$dW = \text{force} \times \text{distance}$$
$$= p \times A \times dx$$
$$= p\,dV$$

$$\Rightarrow \quad dQ = dU + p\,dV$$

now if we hold the volume constant, gas does no work as heat is applied

$$\Rightarrow \left.\frac{dQ}{dT}\right)_v = \left.\frac{dU}{dT}\right)_v = C_v$$

now invoke Joule's Law, which says the internal energy of a gas is a function <u>only</u> of temperature (only applies for an ideal gas) which gives

$$\left.\frac{\partial U}{\partial T}\right)_V = \frac{dU}{dT} = c_v$$

$$\Rightarrow \boxed{dQ = c_v dT + p dV}$$

Now also have from ideal gas law: $pV = RT$

$$\Rightarrow p dV + V dp = R dT$$

(n.b V = specific volume)

$$\Rightarrow dQ = (c_v + R) dT - V dp$$

& now $\left.\frac{dQ}{dT}\right)_p = c_p = c_v + R$

pressure constant

$$\Rightarrow \boxed{R = c_p - c_v}$$

$c_p = 1004\ \mathrm{J\,kg^{-1}\,{}^\circ C^{-1}}$, $c_v = 717\ \mathrm{J\,kg^{-1}\,{}^\circ C^{-1}}$, $R = 287\ \mathrm{J\,kg^{-1}\,{}^\circ C^{-1}}$

~~What happens to~~

Adiabatic motion

(adiabatic = no exchange of heat $\bar{c}$ surroundings)

- What happens to the temperature of a parcel of air as it is lifted with no heating or cooling applied to it?

Starting from $dQ = c_p\,dT - V dp$

use fact that $dQ = 0$ (defⁿ of adiabatic) & $dp = -g\rho dz$

$\Rightarrow \quad V dp = -g dz \qquad (V = \text{specific volume} = 1/\rho)$

$$\Rightarrow \quad \boxed{\frac{dT}{dz} = -g/c_p}$$

DRY ADIABATIC LAPSE RATE

$$\sim -\frac{10 \ ms^{-2}}{1000 \ J\,kg^{-1}\,°C}$$

$$\boxed{\sim -10°C/km}$$

n.b. things about adiabatic lapse rate:

- if a parcel of air were lifted adiabatically its temperature would change at a rate $\Gamma' = 10°C\ km^{-1}$.

- so far this is <u>reversible</u>. parcels can move freely up and down & their temperature var^n $\bar{c}$ height will be described by Γ'.

<u>Exceptions</u>

-
- in actuality $dT/dz \sim -6.5°C\ km^{-1}$ - because of moisture in the atmosphere...

- dT/dz can sometimes be positive (is called an INVERSION) - typically when very cold surface cools the overlying air

- Stratosphere - absorption of shortwave radiation counts as diabatic heating

- Radiative cooling: timescales ~ 20 days i.e. much longer than timescale of atmospheric 'eddies'

- Conduction: only important at $z > 500\ km$, when the mean free path exceeds the scale of atmospheric motions, and in lowest ~1 mm (!) of atmosphere in the molecular boundary layer...

Potential temperature

- turns out to be a very useful property of the atmosphere
- defined as the temperature a parcel of air would have if it is moved adiabatically and reversibly to a standard reference pressure p_0 (usually taken to be 1000 mb). $\Rightarrow$ take a parcel of air at pressure p & temperature T. Bring it down to the ground. What temperature would it then have?

Start with $dQ = c_p dT - V dp$

use $dQ = 0$, & $V = RT/p$

$$\Rightarrow c_p dT = RT \frac{dp}{p}$$

$$\Rightarrow \int_\theta^T \frac{dT}{T} = \frac{R}{c_p} \int_{p_0}^{p} \frac{dp}{p}$$

$$\Rightarrow \ln T/\theta = R/c_p \ln p/p_0 = \ln (p/p_0)^{R/c_p}$$

$$\Rightarrow \boxed{\theta = T (p/p_0)^{R/c_p}} \quad \text{POTENTIAL TEMPERATURE}$$

n.b. as long as $dQ = 0$, θ is conserved as air moves around

$\Rightarrow$ θ is an ATMOSPHERIC TRACER

Other reasons to care about potential temperature

1, It is a measure of the entropy of the air

$$dS = \frac{dQ)_{rev}}{T}$$

can show (e.g. W&H) $\boxed{S = c_p \ln \theta + \text{const.}}$

$\Rightarrow$ if θ changes, S changes, which indicates that some irreversible process has taken place

$\Rightarrow$ if θ does not change, motion is called ISENTROPIC

2, It acts as a tracer – 'flags' a parcel of air as it moves around, since many processes in the atmosphere are close to isentropic.

3, Can reveal the stability of the atmosphere (i.e what happens if I lift a parcel of air. Does it continue to rise or does it fall back down)?

Begin with $$\theta = T\left(\frac{P_0}{P}\right)^{R/c_p}$$

$$\Rightarrow \ln\theta = \ln T + \frac{R}{c_p}\ln \frac{P_0}{P}$$

further manipulation

take $\frac{d}{dz}$ $\Rightarrow \frac{1}{\theta}\frac{d\theta}{dz} = \frac{1}{T}\frac{dT}{dz} + \frac{R}{c_p}\cdot\frac{p}{p_0}\cdot -\frac{p_0}{p^2}\cdot\frac{dp}{dz}$ ①

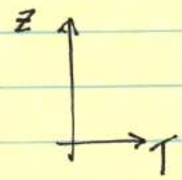
now $\frac{dp}{dz} = -g\rho = -\frac{gp}{RT} \Rightarrow \frac{1}{p}\frac{dp}{dz} = -\frac{g}{RT}$

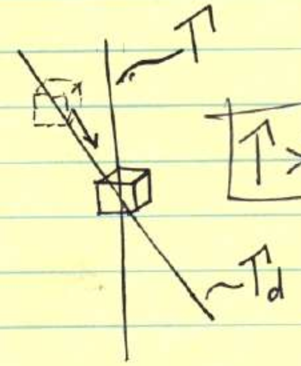
subs. into ① $\Rightarrow \frac{1}{\theta}\frac{d\theta}{dz} = \frac{1}{T}\frac{dT}{dz} + \frac{1}{T}\frac{g}{c_p}$

$$\Rightarrow \frac{T}{\theta}\frac{d\theta}{dz} = \left(\frac{dT}{dz} + \frac{g}{c_p}\right)$$

$$= \underset{\substack{\uparrow\\ \text{actual}\\ \text{lapse rate}}}{\Gamma} - \underset{\substack{\uparrow\\ \text{dry adiabatic}\\ \text{lapse rate}}}{\Gamma_d}$$

Example:

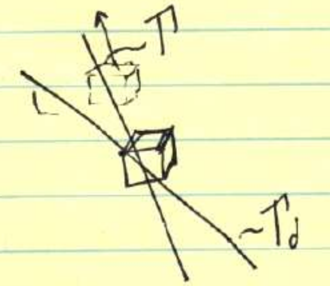




$\boxed{\Gamma > \Gamma_d}$

when parcel rises it follows Γ_d. diagram shows it will then be colder (& therefore heavier) than its surroundings ⇒ it will sink.
⇒ STABLE

$\boxed{\Gamma < \Gamma_d}$



when parcel is lifted → hotter, less dense → continues to rise
⇒ UNSTABLE

So depend on whether $\frac{d\theta}{dz} \lessgtr 0$, can tell whether atmosphere will be stable or unstable to perturbations...