Scale analysis

- Consider the magnitude of terms in the full, complete equations, and neglect the terms that cannot be important for the phenomenon that one is interested in

Example the vertical component of the momentum equation.

Full momentum eqⁿ: $\frac{d\underline{V}}{dt} + 2\underline{\Omega}\times\underline{V} + \underline{\Omega}\times\underline{\Omega}\times\underline{r} = -\frac{1}{\rho}\nabla p - g - \underline{F}$

recall $\underline{V} = (u, v, w)$, & take only terms in the z dirⁿ:

$$\Rightarrow \boxed{\frac{dw}{dt} + 2\Omega u\cos\varphi + \Omega^2 r_e = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g - \underset{\substack{\uparrow \\ \text{friction coefficient}}}{a}w}$$

Now, we want to study dynamics of typical weather system:

$L \sim 1000\,km$, $H \sim 10\,km$, $\tau \sim \underset{\sim 10^5 s}{1\,day}$ typical length & timescales for weather system

Also $r_e \sim 6\times10^6 m$, $\Omega = \frac{2\pi}{\tau} \sim 10^{-4} s^{-1}$

We want to study typical weather system
i.e at large scales

$L \sim 1000\,km, \quad H \sim 10\,km, \quad \Delta t \sim 1\,day\ (10^5 s)$

$r_e \sim 10^6 m \qquad \Omega = \frac{2\pi}{\tau} \sim 10^{-4}$

$w \sim \frac{\Delta z}{\Delta t} \sim \frac{10^4}{10^5} \sim 0.1 ms^{-1}, \quad v \sim \frac{\Delta x}{\Delta t} \sim 10\ ms^{-1}$

so $\frac{\Delta p}{\Delta z} \simeq \cancel{\frac{50mb}{1000km}} \sim \frac{5\cancel{0} \times 10\cancel{0}}{10\cancel{0}\cancel{0}\cancel{0}} \quad \frac{10\cancel{0} \times 10\cancel{0}}{1\cancel{km}\cancel{10000}}$

~~$g \simeq 10 \quad \Omega^2 r \sim 10^8$~~

$$\frac{dw}{dt} + 2\Omega v \cos\varphi + \Omega^2 r_e = -\frac{1}{\rho}\nabla p + g \mp \overset{\text{friction}}{aw}$$

$\sim \frac{0.1}{10^5} \quad \sim 2 \times 10^{-4} \times 10 \quad \sim 10^{-8} \cdot 6 \times 10^6 = \sim 10 \sim 10 \quad ?$

$10^{-6} \quad 10^{-3} \quad 10^{-1} \quad 10 \quad 10$

Scale analysis contd:

So $u, v \sim \frac{\Delta x}{\Delta t} \sim \frac{L}{\tau} \sim \frac{10^6\,\text{m}}{10^5\,\text{s}} \sim 10\,\text{ms}^{-1}$

$w \sim \frac{\Delta z}{\Delta t} \sim \frac{H}{\tau} \sim \frac{10^4\,\text{m}}{10^5\,\text{s}} \sim 0.1\,\text{ms}^{-1}$ n.b this is a little high (because of atm. stability), but good enough for present purposes

back to vertical momentum equation

$$\frac{dw}{dt} + 2\Omega\cos\varphi u + \Omega^2 r_E = -\frac{1}{\rho}\frac{dp}{dz} - g - aw$$

$$\sim \frac{0.1\,\text{ms}^{-1}}{10^5\,\text{s}^{-1}} \quad 2\times10^{-4}\times10\times0.5 \quad 10^{-8}\cdot6\times10^6 \quad \frac{1}{1}\frac{100\cancel{00}\,\text{Pa}}{1000\,\text{m}} \quad \sim 10 \quad ?$$

- need to be very careful about units!

Comments on friction: $a \times w$. From dimensional analysis $a \times w$ must scale as w/τ_{fr} where τ_{fr} is a frictional timescale. It can be crudely associated with a characteristic 'spin-down' timescale. That is, (if all forcing where suddenly switched off), how long would it take for the atmosphere to stop moving? For the atmospheric boundary layer (say the lowest 1 km, this timescale is of order a couple of days. Thus in the above eqn, friction term can be neglected too.

Balance can only be achieved in the equation from terms of similar magnitude. So considering the above eq^n, the vertical momentum equation, to a very good approximation, reduces to

$$\boxed{\frac{1}{\rho}\frac{dp}{dz} = -g}$$ ie the hydrostatic equation

<u>Stoff</u>:

- We have just show that ON THESE TIMESCALES AND SPATIAL SCALE vertical acceleration, friction and centripetal forces can be neglected
- Very important to remember that other applications, such as maintain gravity waves, sound waves, atmospheric convection, these terms CANNOT BE NEGLECTED and must be included in the solution to the equations.