Scale analysis & the Geostrophic Balance

Horizontal momentum eq^n (ie full momentum eq^n, minus terms in the vertical)

$$\overset{①}{\frac{d\underline{v}_h}{dt}} = \overset{②}{f\,\underline{v}_h \times \underline{k}} - \overset{③}{\frac{1}{\rho}\nabla p} - \overset{④}{a\,\underline{v}_h} \quad \text{where } \underline{v}_h = (u, v, 0)$$

recall: $f = 2\Omega \sin\varphi$, φ = latitude

$f\,\underline{v}_h \times \underline{k}$ always acts to the right of (& ⊥^{r} to) the direction of motion in the northern hemispher

(to the left in the southern hemisphere)

(is zero at the equator)

$$① = ② - ③ - ④$$

$\nabla p \sim \dfrac{10\,\text{mb}}{1000\,\text{km}}$

$$\sim \frac{10}{10^5} \qquad \sim 10 \cdot 10^{-4} \qquad \sim \frac{1}{1}\,\frac{10\,\text{mb} \times 100\,\text{Pa/mb}}{1000\,\text{km}} \qquad \sim 10/10^5$$

$$10^{-4} \qquad 10^{-3} \qquad 10^{-3} \qquad 10^{-4}$$

So predominantly a balance between terms ② & ③, but it is not as clear cut as for the vertical momentum equation (we will come back to this)

So, taking terms which are order 10^{-3}, we have

$$\boxed{f\,\underset{\sim}{k} \times \underset{\sim}{v}_h = -\frac{1}{\rho}\nabla p}$$

or

$$\boxed{\underset{\sim}{v}_h = \frac{1}{\rho f}\,\underset{\sim}{k} \times \nabla p}$$

This is the GEOSTROPHIC BALANCE

- this is a good first-order approximation in midlatitudes (not in the tropics - why?) for the relationship between winds and pressure gradients.

- Note that winds are perpendicular to the pressure gradients

- Also, since, as we showed last time $\frac{1}{\rho}\nabla p = \nabla \varphi$, we can write the geostrophic balance as

$$\boxed{\underset{\sim}{v}_h = \frac{1}{f}\,\underset{\sim}{k} \times \nabla \varphi}$$

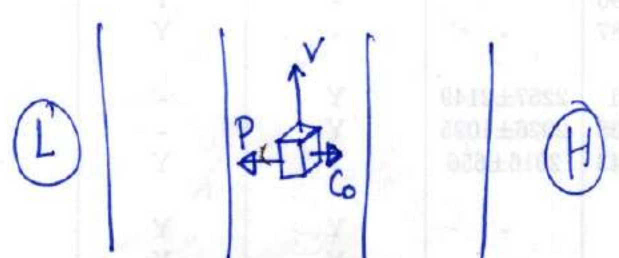
Geostrophic balance – cont^d

$$fv = \frac{1}{\rho}\frac{dp}{dx} \left(= \frac{d\varphi}{dx}\right)$$

$$fu = -\frac{1}{\rho}\frac{dp}{dy} \left(= \frac{d\varphi}{dy}\right)$$

in Cartesian coordinates

(Good to about 20% in midlatitudes)



P = pressure gradient force

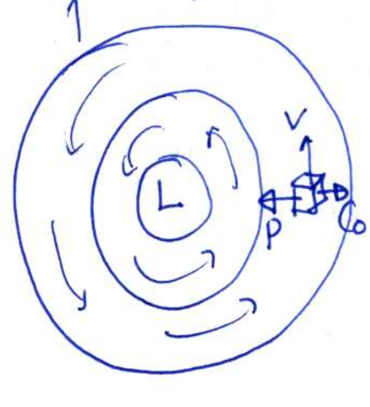
Co = Coriolis force

① if no initial velocity at time t=0, parcel would begin to move towards Low pressure

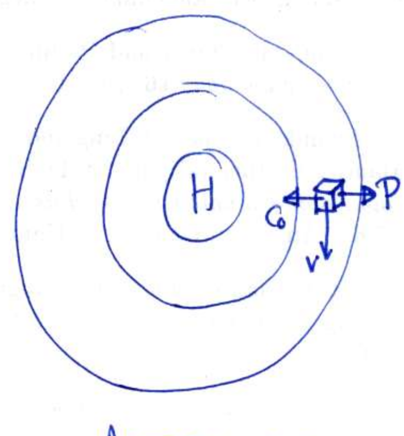
② As parcel begins to move towards Ⓛ, the Coriolis force deflects it to the right

③ Balance is only achieved when parcel moves $\perp^r$ to pressure contours

So for closed circulations:



CYCLONIC
(counterclockwise)



ANTICYCLONIC
(clockwise)

- Note in southern hemisphere, circulation is reversed!

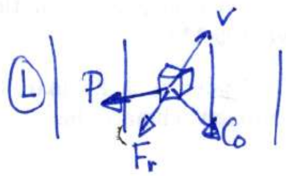
What about effects of friction?

- Near the surface, friction becomes important, and geostrophic balance can no longer be maintained

- Recall friction tends to oppose the motion

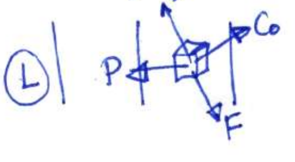
Imagine A: because of friction, parcel moves with motion towards high pressure

ie



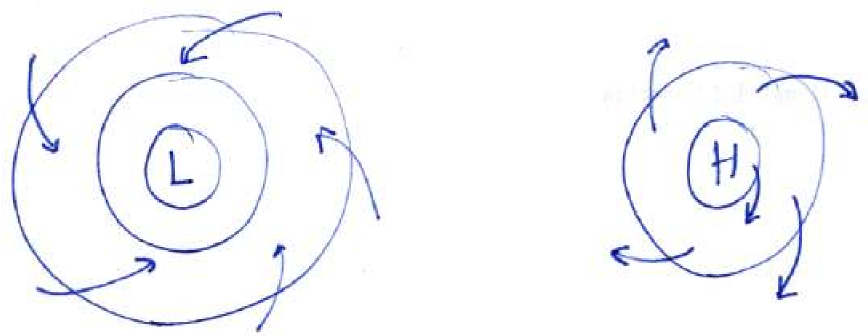
Cannot balance forces, so cannot get this

Imagine B: parcel moves towards low pressure



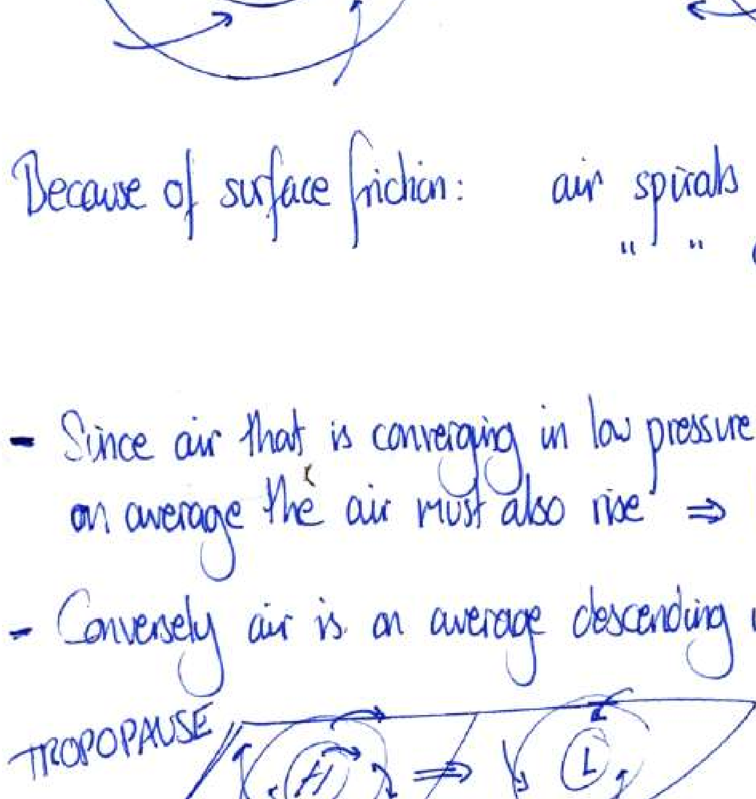
Force balance can be maintained, and so this is what happens

So, at the surface:



Because of surface friction: air spirals towards low pressure centers
" " away from high pressure centers

- Since air that is converging in low pressure has nowhere to go but up, on average the air must also rise ⇒ LOWS TEND TO BE CLOUDY
- Conversely air is on average descending within highs ⇒ CLEAR SKIES



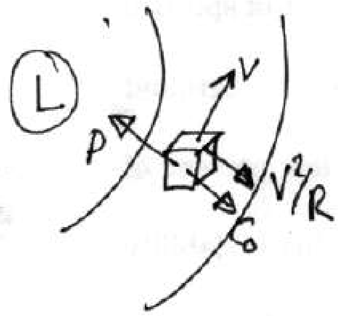
note these arrows (⇒) represent the INDIRECT circulation in the vertical that is induced by surface friction.

- At the surface, where friction is strongest, wind vectors ~~are~~ have an ~15° angle to the pressure contours
- At higher altitudes, friction is much smaller, and to a very good approximation wind vectors are // to pressure (or height) contours

In the scaling analysis, we also neglected $\frac{dV}{dt}$ compared to fV. Wh
situations when this is not correct

- A parcel of air moving in a curved arc also experiences a CENTRIF
ACCELERATION (ie acc^n due to motion in a circle)

So what if $\frac{dV}{dt} \not\ll f.V$?



Centrifugal acceleration $= V^2/R$

$\Rightarrow$ horizontal momentum equation becomes:

(assume no friction)

$$\boxed{V^2/R = -fV + 1/\rho \frac{dp}{dr}}$$

nb See
8.

Define Rossby number $= R_o =$ ratio of $\frac{\text{acceleration term}}{\text{Coriolis term}}$

$$= \frac{V^2/L}{fV} \simeq \frac{V}{Lf}$$

So small L $R_o \uparrow$

large V $R_o \uparrow$

~~small latitude $\varphi \downarrow R \uparrow$~~

~~Examples of tight curvature tornadoes~~

if $R_o \ll 1$ geostroph. bal. good

$R_o \gg 1$ " " bad - n.b. what are examples?

if $R \sim 1 \Rightarrow V^2/R = -fV - \frac{1}{\rho}\frac{dp}{dr}$ GRADIENT WIND BALANCE

ex. can explain why winds in anticyclones tend to be weaker than in cyclones. See homework question & V&H ch 8 p 374 . . . .

if $R \gg 1$, then Coriolis term doesn't matter

$$V^2/R = -\frac{1}{\rho}\frac{dp}{dr}$$ CYCLOSTROPHIC BALANCE

e.g tornadoes, hurricanes, bathtubs...