Basics of thermal wind

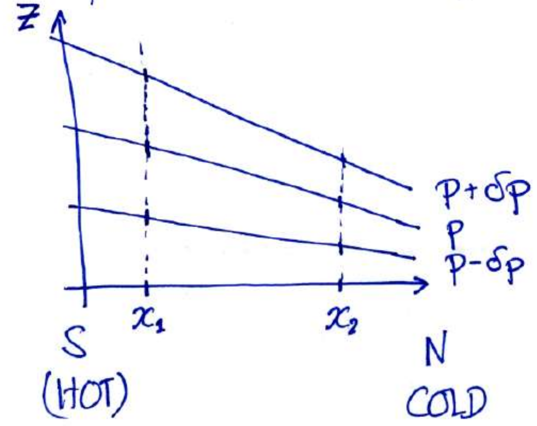
① geostrophic relationship says geostrophic winds are driven by horizontal gradients in the geopotential.

② secondly, geopotential thickness is an function of the mean temperature gradient in the layer

③ so horizontal gradients in temperature drive horizontal gradients in thickness

④ these horizontal gradients in thickness imply that the horizontal gradients in geopotential vary as a function of height. Therefore, because of ①, geostrophic winds vary with height.

Consider pressure surfaces as a fn of height & latitude



from geopotential equation $\delta z = \frac{R\bar{T}}{g_0}\delta \ln p$ where $\bar{T}$ = mean temperature within layer $\delta \ln p$

- because it is hot in the south, the thickness between pressure surfaces is greater.
- therefore, because of the increasing tilt of constant pressure surfaces as z increases, the gradient $\left.\partial\Phi/\partial x\right)_p$ increases.
- therefore the geostrophic wind must increase.

Maths of thermal wind

Geostrophic balance $u = -\frac{1}{f}\left.\frac{d\varphi}{dy}\right)_p$

$\frac{d}{dp}$: $\Rightarrow \frac{du}{dp} = -\frac{1}{f}\frac{d}{dy}\frac{d\varphi}{dp}$

now $dp = -g\rho dz = -\rho d\varphi = -\frac{p\,d\varphi}{RT}$

$\Rightarrow \frac{d\varphi}{dp} = -RT/p$

substitute $\frac{du}{dp} = +\frac{R}{fp}\left.\frac{dT}{dy}\right)_p$ [have related horizontal temperature gradient to vars of u with height]

$\Rightarrow p\frac{du}{dp} = \frac{du}{d\ln p} = \frac{R}{f}\left.\frac{dT}{dy}\right)_p$

Now, $p\frac{d}{dp} = -\frac{RT}{g}\frac{d}{dz}$ [nb note the minus sign]

$$\Rightarrow \boxed{\frac{du}{dz} = -\frac{g}{fT}\left.\frac{dT}{dy}\right)_z}$$

[nb making the approximation that $\left.\frac{dT}{dy}\right)_z = \left.\frac{dT}{dy}\right)_p$]

or in vector form

$$\frac{d\,\underline{v}_n}{dz} = \frac{g}{fT}\,\underset{\sim}{k} \times \nabla T$$

⇒ therefore in the presence of a horizontal ~~wind, the~~ temperature gradient, the geostrophic wind increases with height.

⇒ The direction of that increase is PERPENDICULAR to the temperature gradient.