Where have we got to so far?

Ideal Gas Law:

$$p = \rho R T \tag{1}$$

Hydrostatic Balance:

$$\frac{dp}{dz} = -g\rho \tag{2}$$

Geopotential Height:

$$\phi = \int_0^z g dz \tag{3}$$

First Law of thermodynamics:

$$dQ = c_v dT + p dV = c_p dT - V dp \tag{4}$$

Adiabatic Lapse Rate:

$$\frac{dT}{dz} = \Gamma_d = -\frac{g}{c_p} \tag{5}$$

Potential temperature:

$$\theta = T(\frac{p}{p_0})^{\frac{R}{c_p}} \tag{6}$$

Atmospheric static stability:

$$\frac{\theta}{T}\frac{d\theta}{dz} = \Gamma - \Gamma_d \tag{7}$$

Total and partial derivatives:
Langrangian and Eulerian perspective $\psi = \psi(x, y, z, t)$

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \vec{u} \cdot \nabla\psi \tag{8}$$

Equivalence of geopotential and pressure gradients:

$$\frac{1}{\rho} \cdot \nabla p = \nabla\phi \tag{9}$$

Momentum equation:

$$\frac{d\vec{v}}{dt} = -\frac{1}{\rho}\nabla p - g\vec{k} - a\vec{v} \tag{10}$$

Coriolis acceleration:

$$\left.\frac{d\vec{v}^{fixed}}{dt}\right|^{fixed} = \left.\frac{d\vec{v}^{rot}}{dt}\right|^{rot} + \vec{\Omega} \times \vec{v}^{rot} + \vec{\Omega} \times \vec{\Omega} \times \vec{r} \tag{11}$$

Scale analysis leading to horizontal momentum equation:
N.B. $f = 2\Omega sin(\phi)$

$$\frac{d\vec{v}_h}{dt} = f\vec{k} \times \vec{v}_h - \nabla_h \phi - a\vec{v}_h \tag{12}$$

Geostrophic balance

$$\vec{v}_h = \frac{1}{f}\vec{k} \times \nabla_h \phi = \frac{1}{\rho f}\vec{k} \times \nabla_h p \tag{13}$$

Rossby number

$$R_o = \text{ratio of} \frac{\text{Acceleration term}}{\text{Corolis term}} \tag{14}$$

Cyclostrophic balance
e.g., bath tubs, hurricanes, tornadoes

$$\frac{v^2}{R} = \frac{1}{\rho}\frac{dp}{dn} \tag{15}$$

Gradient wind
Better approximation when curvature is tight

$$\frac{v^2}{R} = fv + \frac{1}{\rho}\frac{dp}{dn} \tag{16}$$

Thermal wind
Exact:

$$\frac{\partial u}{\partial ln(p)} = \frac{R}{f}\left.\frac{\partial T}{\partial y}\right)_p \tag{17}$$

Approximate:

$$\frac{\partial u}{\partial z} = -\frac{g}{fT}\left.\frac{\partial T}{\partial y}\right)_z \tag{18}$$

Vector form:

$$\frac{\partial \vec{v}_h}{\partial z} = \frac{g}{fT}\vec{k} \times \nabla_h T \tag{19}$$

Continuity equation
Version 1:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{v} = 0 \tag{20}$$

Or:

$$\frac{d\rho}{dt} + \rho \nabla \cdot \vec{v} \tag{21}$$

Or:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial \omega}{\partial p} = 0 \tag{22}$$

Or: (for horizontal streamlines)

$$\frac{1}{A}\frac{dA}{dt} + \frac{\partial \omega}{\partial p} = 0 \tag{23}$$

Thermodynamic equation

$$\frac{dT}{dt} = \frac{\kappa T}{p}\omega + \frac{\dot{Q}}{c_p} \tag{24}$$

Vorticity equation

$$\frac{d}{dt}(\zeta + f) + f(u_x + v_y) = 0 \tag{25}$$

where
$()_x = \frac{\partial ()}{\partial x}$
$\zeta = v_x - u_y$ = RELATIVE VORTICITY
$f = 2\Omega sin(\phi)$ = PLANETARY VORTICITY
$\zeta + f$ = ABSOLUTE VORTICITY

PRIMITIVE EQUATIONS

Based on conservation of momentum (Newton's 2nd law), ideal gas equation, first law of thermodynamics, and conservation of mass, we have the primitive equations which describe the motions of the atmosphere on large scales:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \omega\frac{\partial u}{\partial p} - fv = -\frac{\partial \phi}{\partial x} + F_i \tag{26}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \omega\frac{\partial v}{\partial p} + fv = -\frac{\partial \phi}{\partial y} + F_j \tag{27}$$

$$\frac{\partial \phi}{\partial p} = -\frac{RT}{p} \tag{28}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} - \omega(\frac{\kappa T}{p} - \frac{\partial T}{\partial p}) = \frac{\dot{Q}}{c_p} \tag{29}$$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial \omega}{\partial p} = 0 \tag{30}$$