Notes

Rossby waves

Assume fluid of constant density & no vertical motion

From before

$$\frac{D}{Dt}(\xi+f) + (\xi+f)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) = 0$$

incompressible implies $\frac{d\rho}{dt} = 0$

$$\Rightarrow \nabla \cdot \underset{\sim}{u} = 0$$

$$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{\partial w}{\partial z}$$

but if no w $\Rightarrow \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = 0$

$$\Rightarrow \frac{D}{Dt}(\xi+f) = 0$$

Absolute vorticity is conserved following the motion...

$$\frac{D}{Dt}(\zeta + f) > 0$$

linearize: $\frac{d}{dt}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + v\frac{\partial f}{\partial y}$

$$f = 2\Omega\sin\varphi = f_0 + \frac{\partial f}{\partial y}(y - y_0) + O(y - y_0)^2 \ldots$$

$$\frac{\partial f}{\partial y} = \frac{2\Omega\cos\varphi}{a} \quad \text{since } y = a\varphi$$

$= \beta$ known as β-plane approximation..

$\frac{2 \cdot \frac{2\pi}{10^5} \times \frac{1}{\sqrt{2}}}{6.4 \times 10^6}$

$\sim 10^{-11}\,s$

$\sim 4 \times 10^{-11}\,s^{-1}m^{-1}$

do our thing $u = \bar{u} + u'$, $v = v'$

$$\Rightarrow \left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)\left(\frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y}\right) + \beta v'$$

define a stream function: ψ

$$u' = -\frac{\partial \psi}{\partial y}, \quad v' = \frac{\partial \psi}{\partial x}$$

back to linear eq^n:

$$\left(\partial/\partial t + \bar{U}\partial/\partial x\right)\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right) + \beta\frac{\partial\psi}{\partial x} = 0$$

$$\psi = \psi_0 e^{i(kx+ly-\omega t)}$$

$$(-i\omega + i\bar{U}k)(-k^2-l^2) + \beta i k = 0$$

$$\omega = \bar{U}k - \frac{\beta k}{k^2+l^2}$$

$$v_{p_x} = \frac{\omega}{k} = \bar{U} - \frac{\beta}{k^2+l^2}$$ Phase speed

- Rossby waves always propagate WESTWARD relative to the mean flow.

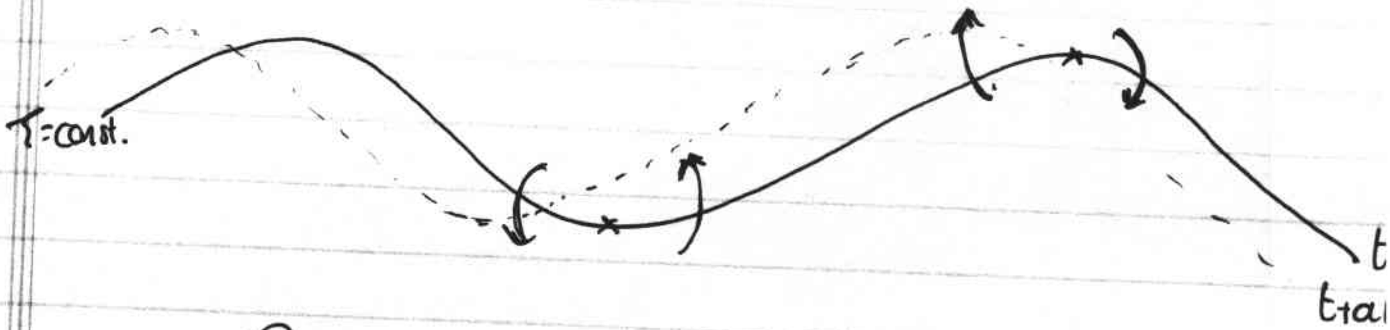
$$V_{gx} = \frac{\partial\omega}{\partial k} = \bar{U} - \left[\frac{(k^2+l^2)\beta - \beta k\cdot 2k}{(k^2+l^2)^2}\right]$$

$$= \bar{U} - \left(\frac{(l^2-k^2)\cdot\beta}{(k^2+l^2)^2}\right)$$

can be eastward or westward depending on $k \gtrless l$

Why do Rossby waves propagate westward?

Imagine a contour of constant $\xi + f$ with a disturbance on it...



Stationary Rossby waves

$$\omega = 0, \quad c = 0$$

$$\cancel{\bar{U}} =$$

$$U = \frac{\beta}{K^2}$$

$$K^2 = k^2 + l^2$$

$$K \sim \frac{2\pi}{L}$$

$$L \sim \sqrt{\bar{U}/\beta} \sim \sqrt{\frac{45}{1.4\times10^{-11}}} \qquad \bar{U} = 45\,ms^{-1}$$

$$L \sim \sqrt{3}\times10^{6}\,m$$

$$\underline{\sim 1700km}$$