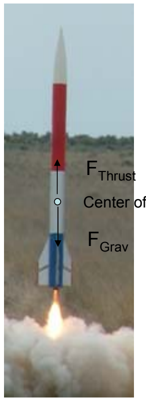
Rocket Dynamics

- Forces on the Rockets - Drag
- Rocket Stability
- Rocket Equation
- Specific Impulse
- Rocket Motors

Forces on the Rocket



Equation of Motion: $\sum F = Ma$

Need to minimize total mass M to maximize acceleration of the rocket

Forces at through the Center of Mass

(1) Gravity: $F_{Grav} = Mg$

(2) Thrust: $F_{Thrust} = \frac{d}{dt}(MV)$

The thrust force seen by the rocket is equal to the rate of change of momentum carried away in the exhaust

Thrust:

Assuming that the propellant speed is approximately constant then

$$F_{Thrust} = \dot{M}_P C_E$$

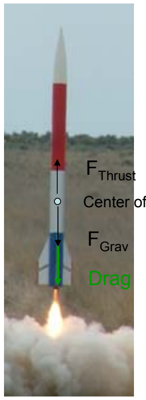
Where C_E *is the speed of the exhaust*

$\dot{M}_P$ *is the rate of change propellant mass*

In other words, how fast you go is dependent on

- *the speed of the propellant*
- *the mass available in propellant*

3rd Force: Drag



Resistance from the air to the rocket motion

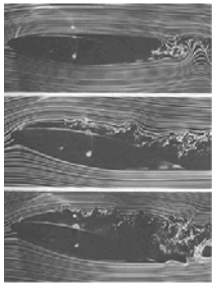
$$D = \frac{1}{2}\rho V^2 A C_D$$

- ρ is the mass density of the air (1.2 kg/m^3 @ sea level)
- V is the rocket speed
- *A is the area of the rocket perpendicular to the rocket flow*
- C_D *is the coefficient of drag*

There are three forms of drag, and their relative importance is highly dependent on the speed on the rocket relative to the sound speed, i.e. to the Mach number = V/V_s V_s~ 1050 ft/s or 331 m/s

Types of Drag

1. **Skin friction** arises from the friction of the fluid against the "skin" of the object that is moving through it. Skin friction arises from the interaction between the fluid and the skin of the body.



Rivets on skin surface add significantly to total drag at supersonic speeds

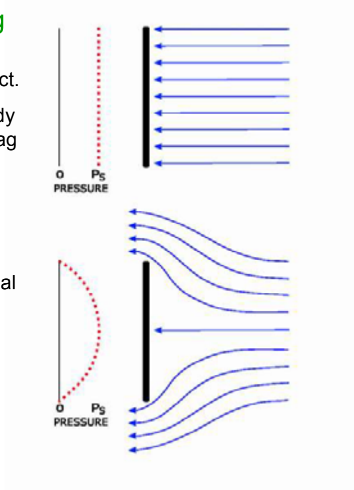
(2) Pressure Drag or Form Drag

arises because of the form of the object.

The general size and shape of the body is the most important factor in form drag

Bodies with a larger apparent cross-section will have a higher drag than thinner bodies.

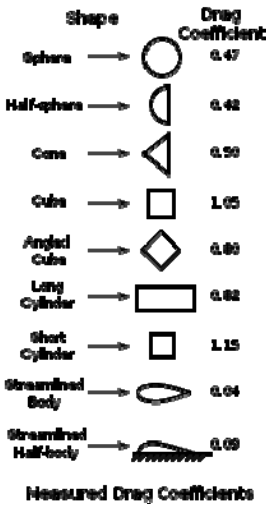
Sleek designs, or designs that are streamlined and change cross-sectional area gradually are also critical for achieving minimum form drag.



(2) Pressure Drag or Form Drag

Drag coefficients for different shapes.

Elimination of vortices in the wake reduce the drag



Wave drag

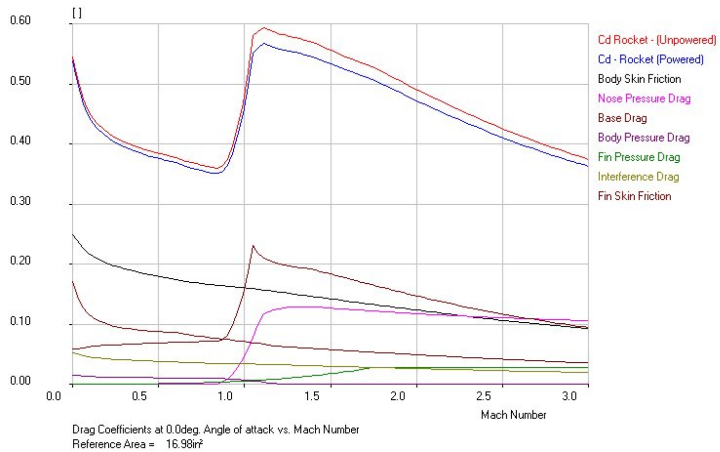
• is caused by the formation of shock waves around the aircraft which radiate away a considerable amount of energy producing enhanced drag

• Although shock waves are typically associated with supersonic flow, they can form at much lower speeds at areas on the aircraft where local airflow accelerates to supersonic speeds.

•The effect is typically seen at transonic speeds above about Mach 0.8, but it is possible to notice the problem at any speed over that of the critical Mach of that aircraft's wing.

•The magnitude peaks at about four times the normal subsonic drag.

•It is so powerful that it was thought for some time that engines would not be able to provide enough power to easily overcome the effect, which led to the concept of a "sound barrier".

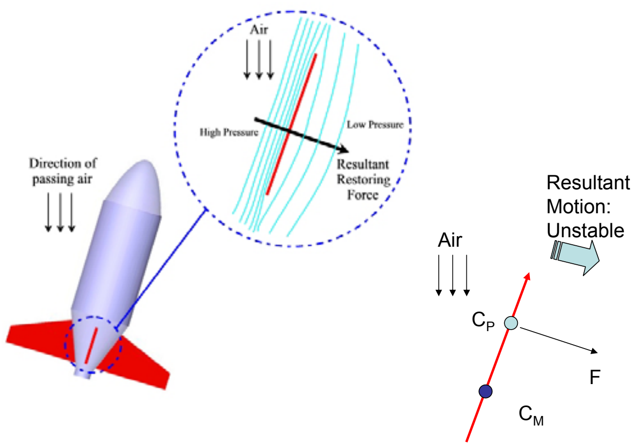


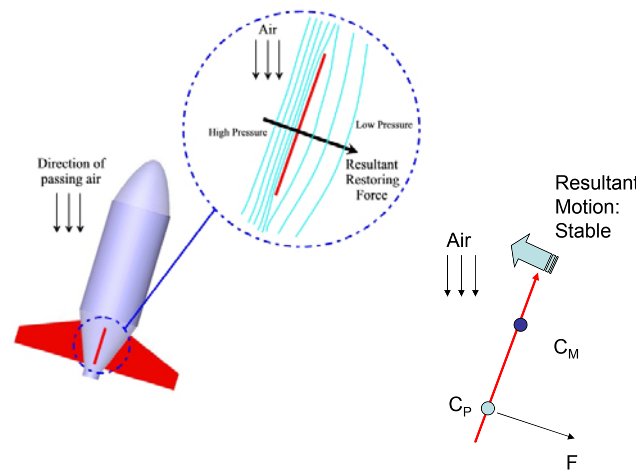
If you can get the rocket higher in altitude before going to high speed, effects from drag can be minimized

Rocket Stability

- Small perturbations will kick the rocket out of alignment
- Active control of thrust vector could be used to correct these perturbations
  - Heavy and Expensive

- Passive control system through the use of fins
- Success depends on the relative position of the *Center of Pressure* relative to the *Center of Mass*





Center of Mass has to be ABOVE Center of Pressure for Stable Flight

Rocket Equation

Neglecting drag the equation of motion for the rocket is:

$$M\frac{dV}{dt} = \frac{dM}{dt}V_E$$

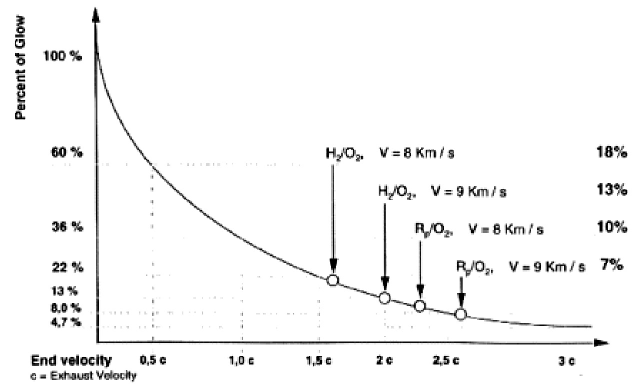
$$\int_{V_1}^{V_2} dV = V_E \int_{M_0}^{M_f} \frac{dM}{M}$$

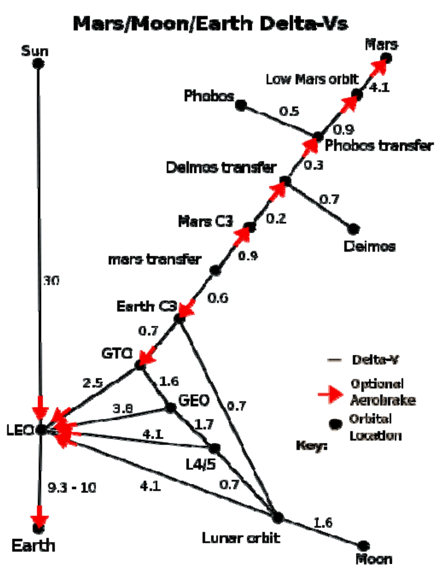
$$\Delta V = V_E \ln(\frac{M_0}{M_F})$$ OR $$\frac{M_0}{M_F} = \exp(-\frac{\Delta V}{V_E})$$

Fraction of Payload becomes exponential small when required ΔV exceeds velocity of the propellant

Percentage Payload to Orbit Using

(1) H^2/O_2 and (2) R_p (highly refined kerosene)/O_2





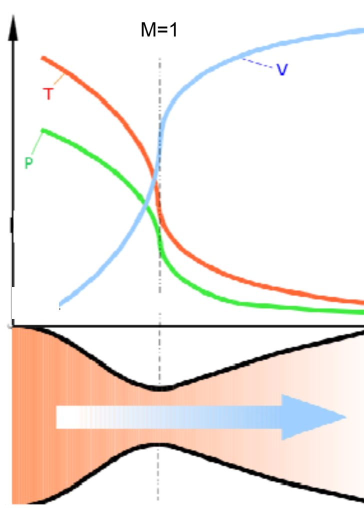
Specific Impulse (Isp)

- is a way to describe the efficiency of rocket and jet engines.
- It represents the impulse (change in momentum) per unit of propellant.
- The higher the specific impulse, the less propellant is needed to gain a given amount of momentum.

$$Isp = \frac{\int F dt}{g\int \dot{m} dt} = \frac{V_E \int \dot{m} dt}{g\int \dot{m} dt} = \frac{V_E}{g}$$

Specific impulse of various propulsion technologies

Engine	"Ve" exhaust velocity (m/s)	Specific impulse (s)	Energy per kg of exhaust (MJ/kg)
Turbofan jet engine	2,900	3,000	~0.05
Solid rocket	2,500	250	3
Bipropellant liquid rocket	4,400	450	9.7
Ion thruster	29,000	3,000	430



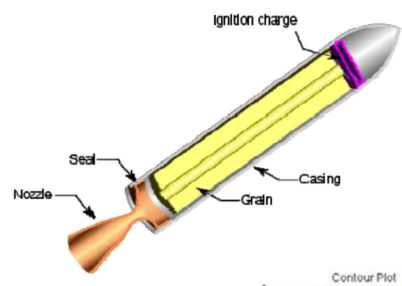
Rocket nozzles

the hot gas produced in the combustion chamber is permitted to escape from the combustion chamber through an opening (the "throat"), within a high expansion-ratio 'de Laval nozzle'.

Provided sufficient pressure is provided to the nozzle (about 2.5-3x above ambient pressure) the nozzle *chokes* and a supersonic jet is formed, dramatically accelerating the gas, converting most of the thermal energy into kinetic energy.

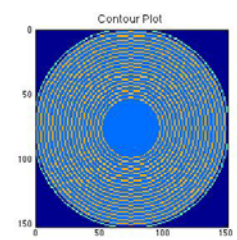
Type	Description	Advantages	Disadvantages
Solid rocket	Ignitable, self sustaining solid fuel/oxidiser mixture ("grain") with central hole and nozzle	Simple, often no moving parts, reasonably good mass fraction, reasonable I_{sp}. A thrust schedule can be designed into the grain.	Once lit, extinguishing it is difficult although often possible, cannot be throttled in real time; handling issues from ignitable mixture, lower performance than liquid rockets, if grain cracks it can block nozzle with disastrous results, cracks burn and widen during burn.
Hybrid rocket	Separate oxidiser/fuel, typically oxidiser is liquid and kept in a tank, the other solid with central hole	Quite simple, solid fuel is essentially inert without oxidiser, safer; cracks do not escalate, throttleable and easy to switch off.	Some oxidisers are monopropellants, can explode in own right; mechanical failure of solid propellant can block nozzle (very rare with rubberised propellant), central hole widens over burn and negatively affects mixture ratio.
Monopropellant rocket	Propellant such as Hydrazine, Hydrogen Peroxide or Nitrous Oxide, flows over catalyst and exothermically decomposes	Simple in concept, throttleable, low temperatures in combustion chamber	catalysts can be easily contaminated, monopropellants can detonate if contaminated or provoked, I_{sp} is perhaps 1/3 of best liquids
Liquid Bipropellant rocket	Two fluid (typically liquid) propellants are introduced through injectors into combustion chamber and burnt	Up to ~99% efficient combustion with excellent mixture control, throttleable,	Pumps needed for high performance are expensive to design, huge thermal fluxes across combustion chamber wall can impact reuse, failure modes include major explosions, a lot of plumbing is needed.

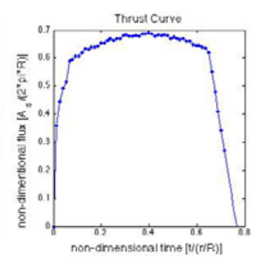
Solid Rocket Motors



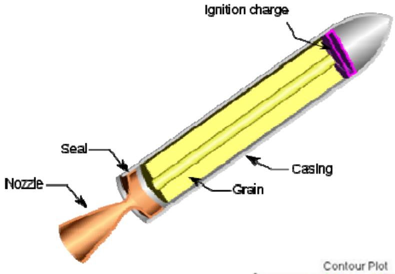
Single Grain Motor: Thrust increase with time due to increased surface area as the grain burns away.

Use of Multiple grains so that burning between grains leads to more uniform burn





Solid Rocket Motors



Alternately, use 5-point Finocyl form a more uniform burn