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COMMENT ON "A COMPARISON OF FINITE-DIFFERENCE AND FOURIER METHOD CALCULATIONS OF SYNTHETIC SEISMOGRAMS" BY C. R. DAUDT *ET AL.*

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The comparison between fourth-order and Fourier method finite-difference wave simulation schemes by Daudt *et al.* (1989), which finds Fourier methods preferable, is unfair because the fourth-order scheme they use is not as efficient as most fourth-order codes in use today. The fourth-order acoustic scheme that I have had a part in writing and using (see Vidale *et al.*, 1985; Frankel and Clayton, 1986; Vidale and Helmberger, 1988; for example) is several times more efficient than the fourth-order scheme tested by Daudt *et al.* (1989).

Our acoustic fourth-order algorithm has not been previously published, so it is described below. Virieux (1984) describes a more complicated scheme based on converting the second-order partial differential equation of motion (in pressure) into a system of two first-order equations (in stress and velocity). Levander (1988) describes the extension of Virieux's (1984) second-order scheme to fourth-order. The stress-velocity scheme is of comparable efficiency to our pressure scheme.

Our extension of the original scheme of Alterman and Karal (1968) that finite-differences the acoustic-wave equation on an equally spaced, cartesian grid to higher-order accuracy is easily done by re-writing the wave equation. This scheme extrapolates from the pressure at two points in time to the pressure at the next point in time on a cartesian grid. The acoustic equation of motion is

$$\left(\frac{1}{\rho} P_x\right)_x + \left(\frac{1}{\rho} P_z\right)_z = \frac{1}{K} P_{tt} \tag{1}$$

where P is pressure, x and z are cartesian coordinates, t is time, ρ is density, and K is bulk modulus. If one simply finite-differences the first and second derivatives in equation (1) as written, instabilities result. A simple trick allows finite-differencing to solve equation (1). Using the identity

$$(AP_x)_x = \tfrac{1}{2}[(AP)_{xx} + AP_{xx} - A_{xx}P], \tag{2}$$

equation (1) may be rewritten so that only second derivatives of x, z, and t are present. The second-order $(1, -2, 1)$ and fourth-order $(-\frac{1}{12}, \frac{4}{3}, -\frac{5}{2}, \frac{4}{3}, -\frac{1}{12})$ convolutional operators for a second derivative, as well as higher-order operators, are well-known. Plugging the second-order operator into equation (1) results in the formula of Alterman and Karal (1968), while plugging in the fourth-order operator for the two spatial derivatives and the second-order operator for the time derivative yields the solution that I and my compatriots have been using.

With such similar structure for the second- and fourth-order solutions, it is simple to compare their efficiencies. Comparing the product of the number of grid points required, the number of time steps required and the cost per grid point per time step yields the relative efficiency of the two schemes. For the same phase velocity error, I find that only one-third the number of samples per wavelength (7.6 versus 26) are needed for the fourth-order compared to the second-order operator. I have chosen an acceptable error smaller than that of Daudt *et al.* (1989) that I also use as a guide in my finite-difference applications. With a coarser grid but more restrictive stability limit (Alford *et al.*, 1974) the time step is 1.15 times larger for the fourth-order than the second-order calculation. The cost per grid point per time step reduces to applying the finite-difference operator to 11 numbers in the fourth-order case (a star five points wide in each of the two spatial dimensions, and a star three points wide in the time dimension, with the central point common to all three spokes) and applying the star to seven points in the second-order case (an operator three points wide in each direction). Each coefficient may be pre-computed and stored in a table, and therefore needs be computed only once and costs little compared to the computation in the time-stepping part of the solution.

Multiplying these three factors, the fourth-order scheme uses one-eighth of the CPU and one-twelfth of the memory compared to the second-order scheme. Since Daudt *et al.* (1989) show that the Fourier method runs four times faster than the second-order scheme, the fourth-order scheme runs in half the time of Fourier method, although it takes twice the memory. I found the same result from writing both kinds of computer programs; the acoustic fourth-order scheme runs faster than the Fourier scheme in two dimensions for a given level of accuracy. In three dimensions, however, the comparison may favor the Fourier scheme.

These comparisons are complicated by the variations in computers, with some relying on vectorization and others relying on concurrency to gain speed. Both Fourier and fourth-order methods can be vectorized, but only the fourth-order method can easily use concurrent processors. I did not use vector or concurrent machines for my comparison. The sides of a Fourier calculation are restricted to have a length that can be FFT'ed, usually 2^n, and by the nature of an FFT, the wave field is periodic. On the other hand, a truly random media requires the storage of a different differencing operator for each spatial grid point in the fourth-order scheme, which increases the demand for memory. In addition, adequacy of the free surface, the absorbing boundary, and the source region play a role in the choice of a numerical wave simulation scheme. I do not think, however, that the Fourier method has a clear advantage in the two-dimensional case, as Daudt *et al.* (1989) conclude.

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