Cascadia tremor spectra: Low corner frequencies and earthquake-like high-frequency falloff

Jian Zhang

Scripps Institution of Oceanography, University of California, San Diego, 9500 Gilman Drive, La Jolla, California 92093-0238, USA

Now at Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA (jjzhang@lanl.gov)

Peter Gerstoft, Peter M. Shearer, and Huajian Yao

Scripps Institution of Oceanography, University of California, San Diego, 9500 Gilman Drive, La Jolla, California 92093-0238, USA

John E. Vidale, Heidi Houston, and Abhijit Ghosh

Department of Earth and Space Sciences, University of Washington, 4000 15th Avenue NE, Seattle, Washington 98195-1310, USA

[1] The discovery of non-volcanic tremor (NVT) has opened a new window to observe major Earth plate boundaries. However, the spectral characteristics of NVT have not been well studied due to poor signal-to-noise ratio (SNR) on individual seismograms. We estimate the spectral content of Cascadia tremor between 2.5 and 20 Hz by suppressing noise using array analysis, and compute empirical path corrections using nearby small earthquakes. We demonstrate that the displacement spectra of the Cascadia tremor have corner frequencies around 3–8 Hz and fall off at $f^{-2}$ to $f^{-3}$ at higher frequencies. Our results have the following implications. (1) The high-frequency falloff of tremor agrees with the observations of regular earthquakes, suggesting that tremor can be analyzed using standard spectral models. Prior analyses that have shown a tremor spectral falloff proportional to $f^{-1}$ may reflect only the spectral behavior over a limited frequency band. (2) Tremor may be no different from a swarm of microearthquakes with abnormally small stress drops on the order of kPa, likely due to the presence of fluids. Alternatively the low corner frequencies of tremor may reflect abnormally slow ruptures. (3) Fitting a standard Brune (1970) spectral model implies a moment release rate of Cascadia tremor of $3.8 \times 10^{10}$ N·m/s assuming the tremor signals are $P$ waves (or $1.4 \times 10^{10}$ N·m/s assuming $S$-waves). This implies that a typical 20-day long tremor episode releases moment equivalent to Mw 5.1 ($P$-wave) or Mw 4.9 ($S$-wave), although these may be underestimates if the spectra deviate substantially from the Brune model at very low frequencies.

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1. Introduction

[3] The discovery of non-volcanic tremor (NVT) [Obara, 2002; Rogers and Dragert, 2003] has opened a new window for Earth scientists to model and understand plate boundary dynamics. The spectral characteristics of NVT contain important information about the physical fault processes controlling episodic tremor and slip (ETS) and shed light on the manifestations of seismic slip from slow events to megaseismic tremor records. When seismic array data are available, beamforming [e.g., Gerstoft et al., 2008] can suppress noise to isolate the coherent tremor source locations to be found where they best match the beamformer output.

[5] We assume a simple linear signal model \( v(\omega) = S(\omega) p(\omega) \Psi(\omega) + n(\omega) \), where \( S(\omega) \) is the source strength of the coherent signal, \( p(\omega) \) is the average transmission term from the source to the sensors, \( \Psi(\omega) \) represents the different arrival phases at each sensor, and \( n \) is the noise at each sensor, which is assumed Gaussian distributed with zero mean and diagonal covariance \( \sigma^2 I \), i.e., the noise is identical and independently distributed. Using this statistical model the average power spectra \( P(\omega) \) and the cross-spectral density matrix (CSDM) \( C(\omega) \) are given by:

\[
\begin{align*}
P(\omega) &= \frac{1}{N} E(v^H v) = S^2 p^2 + \sigma^2 \\
C(\omega) &= E(v^H \Psi \Psi^H) = S^2 p^2 \Psi^H \Psi + \sigma^2 I
\end{align*}
\]  

(1)

For earthquakes it is assumed that \( S^2 p^2 \gg \sigma^2 \), whereby \( \sigma^2 \) can be neglected and to estimate \( S \) we correct \( P(\omega) \) for the average propagation path \( p \). For tremor, however, \( S^2 p^2 = \sigma^2 \), and for higher frequencies noise can be dominating. This makes it difficult to estimate \( S \) directly from the \( P(\omega) \). Here we show how beamforming can be used to estimate the tremor source power.

[6] Taking the form of \( C(\omega) \) in equation (1), the beamformer power is then given by

\[
b(\omega, s, \theta) = \frac{1}{N} w^H C w = \frac{1}{N} (S^2 p^2 \Psi^H \Psi + N \sigma^2)
\]

(2)

At the optimal point \( (s^{opt}, \theta^{opt}) \) from the tremor, \( w(s^{opt}, \theta^{opt}) = \Psi \), also \( \Psi^H \Psi = N \), whereby

\[
b(\omega, s^{opt}, \theta^{opt}) = \frac{1}{N} w^H C w = \frac{1}{N} S^2 p^2 \sigma^2 + \sigma^2 = S^2 p^2 N + \sigma^2 = P + S^2 p^2 (N - 1)
\]

(3)
The tremor source strength $S$ can then be estimated from the difference between the peak beamformer output $b(\omega, s^{\text{tr}}, \theta^{\text{tr}})$ and the average power $P(\omega)$, i.e.,

$$S^2 p^2 = \frac{1}{N-1} \left( b(\omega, s^{\text{tr}}, \theta^{\text{tr}}) - P(\omega) \right)$$

where the transmission loss $p^2$ can be estimated using small local earthquakes and numerical modeling. Thus, equation (4) corresponds to traditional beamforming without the diagonal of the CSDM [e.g., Brooks and Humphreys, 2006; Harmon et al., 2010].

In practice the $P(\omega)$ and the $C(\omega)$ are estimated by forming ensemble averages over $M$ snapshots, i.e.,

$$\hat{P}(\omega) = \frac{1}{NM} \sum_{m=1}^{M} v_m^H v_m$$

and

$$\hat{C}(\omega) = \frac{1}{M} \sum_{m=1}^{M} v_m v_m^H,$$

noting that $\text{tr}(\hat{C}(\omega)) = N \hat{P}(\omega)$.

### 3. Data Processing and Analysis

The Cascadia tremor activity during the main ETS event from May 5 to 21, 2008 was recorded at a dense array of $\sim 100$ short-period vertical-component sensors deployed at Big Skidder Hill, WA (Figure 1). For each sensor we first split the time domain signals (sampling frequency 100 Hz) into 3-s time snapshots, then transform the signals into the frequency domain and remove the instrument response. The beamformer output is averaged over the time window of tremor bursts. Note that while estimating tremor spectra, we retain the tremor amplitudes in the data. Previous studies using these data suggested the dominance of $P$ waves in the vertical-component records [Ghosh et al., 2009]. However, significant $S$ waves might also be present even in the vertical-component records. We thus carry out the tremor spectral analysis by considering the possibility of $S$ waves as well as $P$ waves. By shooting rays up from the plate interface, we locate the tremor using a plate interface model [McCrory et al., 2004] and a one-dimensional velocity model [Crosson, 1976]. The tremor locations generally agree with those shown in previous studies [Wech and Creager, 2008; Ghosh et al., 2009].

Seismic records from single stations (Figure 2) normally allow tremor to be observed up to about 10–15 Hz and down to $\sim 1$ Hz (Figure 3). Below 1 Hz microseismic energy dominates. Using beamforming we are able to observe tremor as coherent peak energy in beamformer outputs from 2.5 to 25 Hz (Figure 4a). The temporal variation of tremor azimuth indicates a south-to-north migration (Figure 4b). However, simultaneous tremor episodes are often observed at multiple locations. Peak energy as a function of slowness and azimuth (Figure 5) suggests that the tremor is stable and strongest within about 2.5–20 Hz, which we choose as the band for our spectral analysis. Outside of...
this range the noise dominates and so the beamforming resolution is poor.

[10] Although the beamformer peaks represent the strongest tremor phase assuming a point source and ray theory through a layered homogeneous Earth, peak locations (azimuth and slowness) are variable with frequency (Figure 6), likely due to distributed sources, scattering, multipathing, and the possible mix of $P$ and $S$ wave energy. We have not attempted to resolve how much of this variation is caused by a distributed source versus scattering and other path effects. However, synthetic tests of a distributed source indicate that our spectra estimates from the peak beamformer outputs at each frequency are robust (Figure 7).

[11] Single-station spectra calculations overestimate the signal power at high frequencies where the noise power is comparable to or greater than the signal, and beamforming provides an efficient way to suppress noise. Consider plots of signal and noise strength, as shown in Figure 8a. Using 72 elements of the array we extract both the median of the single-station amplitude estimates and the beam amplitude $S$. At low frequencies the signal dominates and both methods work well, but for higher frequencies where the noise dominates, only the
The tremor amplitude spectrum (displacement, $E$) then assumes that the displacement amplitude $M_c$ for earthquakes in Northern Cascadia.

We then focus on three tremor bursts (Figures 1a and 9) that are each close to one of three local earthquakes (May 08 h = 8.4 km $M_c$ = 1.4; May 17 h = 52 km $M_c$ = 1.8; May 16 h = 53 km $M_c$ = 1.6, where h is depth; $M_c$ is coda magnitude). Small earthquakes close to the tremor locations can be used to estimate the empirical path term $p^2$ in equation (4) (Figure 10). In doing so, we take advantage of the study by Atkinson [1995], who analyzed earthquake source spectra in this region and solved for a $Q$ model as well as earthquake stress drops. We first model the source spectra of each small earthquake by using the corner frequency $f_c$. For earthquakes, coda magnitudes are approximately equivalent to local magnitudes, i.e., $M_c \approx M_L$. Kao et al. [2010] has found that $M_L \approx M_W$ for earthquakes in Northern Cascadia. Thus the earthquake moment can be estimated using $M_W = \frac{2}{3} \log_{10} (M_0 - 9.1)$ [Kanamori, 1977]. $f_c$ can then be estimated using the Madariaga [1976] dynamic rupture model: $\Delta \sigma = \frac{7}{16} \left( \frac{f_c}{k \beta} \right)^3 M_0$, where $\beta$ is shear-velocity, $k$ is 0.32 for the P-wave spectrum, and 0.21 for S waves, given a rupture velocity of 0.9$\beta$, and we assume a stress drop of $\Delta \sigma = 3$ MPa [Atkinson, 1995].

We then assume that the displacement amplitude spectrum of the small earthquakes is given by a standard Brune [1970] source model $A(f) = \frac{1}{\Omega_0^2 + (f/f_c)}$, where $\Omega_0$ is the low-frequency spectral level. This allows an estimate of the empirical path term $p^2 = E(\omega)/A(\omega)$, where $E(\omega)$ is the observed earthquake spectrum measured from a 3-s window of the $P$- or $S$-wave train (Figure 10a). Corrections due to the difference in hypocentral distance ($R$) of the tremor and earthquake are considered by assuming $R^{-1}$ geometric spreading [Atkinson, 1995]. In addition we correct for the depth difference between shallow crustal earthquakes and tremor (Figure 10c) using a $Q$ model [Atkinson, 1995] that accounts for the difference between events in the shallow crust and those at the plate boundary ($Q = 174 f_0^{0.58}$ for $S$ waves of crustal events and $Q = 263 f_0^{0.49}$ for $S$ waves of plate boundary events; $Q$ values for $P$ waves are a factor of 1.4 higher). Although we assume the earthquakes have a typical stress drop of 3 MPa and that their source spectra follow the Madariaga model, our results do not depend very much on these assumptions, as we find that changing the assumed stress drops to values from 0.3 to 30 MPa has little effect on the resulting path-effect corrected source spectra (Figure 10d).

4. Empirical Correction for Path and Attenuation Using Small Nearby Earthquakes

We focus on three tremor bursts (Figures 1a and 9) that are each close to one of three local earthquakes (May 08 h = 8.4 km $M_c$ = 1.4; May 17 h = 52 km $M_c$ = 1.8; May 16 h = 53 km $M_c$ = 1.6, where h is depth; $M_c$ is coda magnitude). Small earthquakes close to the tremor locations can be used to estimate the empirical path term $p^2$ in equation (4) (Figure 10). In doing so, we take advantage of the study by Atkinson [1995], who analyzed earthquake source spectra in this region and solved for a $Q$ model as well as earthquake stress drops. We first model the source spectra of each small earthquake by using the corner frequency $f_c$. For earthquakes, coda magnitudes are approximately equivalent to local magnitudes, i.e., $M_c \approx M_L$. Kao et al. [2010] has found that $M_L \approx M_W$ for earthquakes in Northern Cascadia. Thus the earthquake moment can be estimated using $M_W = \frac{2}{3} \log_{10} (M_0 - 9.1)$ [Kanamori, 1977]. $f_c$ can then be estimated using the Madariaga [1976] dynamic rupture model: $\Delta \sigma = \frac{7}{16} \left( \frac{f_c}{k \beta} \right)^3 M_0$, where $\beta$ is shear-velocity, $k$ is 0.32 for the P-wave spectrum, and 0.21 for S waves, given a rupture velocity of 0.9$\beta$, and we assume a stress drop of $\Delta \sigma = 3$ MPa [Atkinson, 1995].

We then assume that the displacement amplitude spectrum of the small earthquakes is given by a standard Brune [1970] source model $A(f) = \frac{1}{\Omega_0^2 + (f/f_c)}$, where $\Omega_0$ is the low-frequency spectral level. This allows an estimate of the empirical path term $p^2 = E(\omega)/A(\omega)$, where $E(\omega)$ is the observed earthquake spectrum measured from a 3-s window of the $P$- or $S$-wave train (Figure 10a). Corrections due to the difference in hypocentral distance ($R$) of the tremor and earthquake are considered by assuming $R^{-1}$ geometric spreading [Atkinson, 1995]. In addition we correct for the depth difference between shallow crustal earthquakes and tremor (Figure 10c) using a $Q$ model [Atkinson, 1995] that accounts for the difference between events in the shallow crust and those at the plate boundary ($Q = 174 f_0^{0.58}$ for $S$ waves of crustal events and $Q = 263 f_0^{0.49}$ for $S$ waves of plate boundary events; $Q$ values for $P$ waves are a factor of 1.4 higher). Although we assume the earthquakes have a typical stress drop of 3 MPa and that their source spectra follow the Madariaga model, our results do not depend very much on these assumptions, as we find that changing the assumed stress drops to values from 0.3 to 30 MPa has little effect on the resulting path-effect corrected source spectra (Figure 10d).

5. Tremor Source Spectra

Applying beamforming and empirical path corrections from the nearby earthquakes (Figure 10), we obtain noise-suppressed source spectra at 2.5–20 Hz for the typical tremor bursts, assuming the dominant signals in the records are $P$ waves (Figure 11). Corner frequencies and high-frequency falloff rates are estimated by fitting a standard Brune
Figure 4. Beamformer outputs. (a) Peak power, and (b) peak azimuth variations (color coded) of the tremor bursts during May 6–19, 2008, as a function of frequency and time. The strong energy from mid May 15 to mid May 16 is rain noise.
Figure 5. Slowness-azimuth beamformer outputs (0.6-Hz bandwidth for each frequency) for the tremor bursts during May 9 03:00–04:00. (a) Beam of tremor at sampled frequencies. The stable and strongest tremor is band-limited within about 2.5–20 Hz. Resolution is poor for low frequencies and noise dominates for the frequencies outside the 2.5–20 Hz range. (b) Microseism energy dominates the spectrum below 2.5 Hz. Color is coded for power (dB) but each subplot has a different scale.
Figure 6. Peak phases (slowness and azimuth respectively) versus frequencies, from the beamformer results for the three tremor bursts in Figure 1. Red lines indicate the medians.

Figure 7. Synthetic test of the point-source assumption. (a) Five point sources (stars) and their plane wave beam at 2.5, 10, 20 Hz; (b) Source spectra of the five point sources; (c) Summed spectra (black) of the five point sources compared with the beam spectrum (red) obtained by assuming a single point source.
Figure 8. Signal extraction from records with strong high-frequency noise. (a) Simulated signal (red) and noise (black) spectra at each station. (b) Signal strength estimated using beamforming (red), and the median of single-station amplitudes (green). Beamforming estimate (path-effect uncorrected) of typical displacement spectra (average of 3-s intervals) of the Cascadia tremor during (c) May 9 03:00–04:00; (d) May 8 03:39–03:52; and (e) May 12 11:50–12:50 (red) is compared with the median of single-station estimates of the tremor (green), and the median noise amplitude (black) estimated from tremor-free data on March 11, 2008 (same daily hour as the tremor so the cultural noise is similar).
[1970] model using least squares, with the RMS error less than 0.05 in log[Amplitude]. We then repeat the above analysis by assuming the dominance of $S$ waves in the array records. Locations of the three tremor bursts are generally closer to the array than those under the $P$-wave assumption (Figure 12a). Note for the path correction that the third tremor burst is paired with the second earthquake, given that its $S$-wave location is closer to the second earthquake than to the third. Compared with the estimates under the $P$-wave assumption, fitting a Brune [1970] model provides similar high-frequency falloff rates and slightly higher corner frequencies (Figure 12b). Here we discard the corner frequency estimate of the third tremor case ($f_c = 14.4$ Hz) because of the uncertainty due to its step-like spectral shape.

[16] Regardless of wave-type ($P$ or $S$), the Cascadia tremor spectra show high-frequency falloff of $f^{-2}$ to $f^{-3}$ that agrees with regular earthquakes. This suggests that tremor can be modeled using standard earthquake source models. However, the Cascadia tremor has much lower corner frequencies ($f_c \sim 3$–8 Hz) than would be seen for regular earthquakes producing comparable seismic amplitudes, consistent with tremor being composed of a nearly continuous sequence of low-frequency microearthquakes (LFEs) [Shelly et al., 2006; Beroza and Ide, 2011].

6. Discussion

[17] Using the amplitude difference between the tremor and the nearby earthquake (Figures 11 and 12) and assuming $M_c \approx M_L \approx M_W$ ($M_c$: coda duration magnitude; $M_L$: local magnitude; $M_W$: moment magnitude), we can roughly calibrate the tremor moment with the earthquake magnitude. Again the earthquake moment $M_0^{eq}$ can be estimated using $M_W = \frac{2}{3}(\log_{10} M_0 - 9.1)$. For the simple Haskell fault model [e.g., Shearer, 2009], the far-field amplitude spectrum for low frequencies is related to the moment as $\log A(\omega) = \log M_0 + G$, where $G$ is a scaling term that includes geometrical spreading, which is approximately the same if the earthquake and tremor are close. This gives $\log[A^{tre}(\omega)] - \log[A^{eq}(\omega)] = \log M_0^{tre} - \log M_0^{eq}$.

[18] We thus can use the low-frequency amplitude difference between the tremor and earthquakes (Figures 11 and 12) to estimate the tremor moment $M_0^{tre}$, which amounts to an average seismic moment release rate of $3.8 \times 10^{10}$ N·m/s under the $P$-wave assumption, or $1.4 \times 10^{10}$ N·m/s under the $S$-wave assumption. This is 3 orders of magnitude smaller than the moment release rate calculated by Aguiar et al. [2009] from geodetic constraints on the slow slip events that accompany the tremor. Thus while tremor occurs during the slow slip episodes, the tremor process constitutes only a small portion of the total moment release during an ETS event. Assuming tremor moment release is proportional to time, a typical 20-day long tremor episode releases moment equivalent to an Mw = 5.1 event under the $P$-wave assumption, or Mw = 4.9 under the $S$-wave assumption. Note that these may be underestimates if the spectra deviate substantially from the Brune [1970] model at very low frequencies.
Figure 10. The processing steps for computing the empirical path-effect correction using a small nearby earthquake ($M = 1.4$, depth = 8.4 km) on May 8, for the tremor bursts during May 8 03:39–03:52 (Figure 9a). (a) Example time series of the earthquake. (b) Using the empirical path-correction (green) from the difference between the smoothed earthquake spectrum (solid blue) and the modeled earthquake source spectrum (dashed blue), the raw tremor spectrum (dashed red) is corrected to the tremor source spectrum (solid red); (c) correction for the difference of attenuation $Q$ for the sources at different depths (tremor: depth = 40 km, assuming it is located at plate interface, and earthquake: depth = 8.4 km); (d) tremor source spectrum with path-effect correction, assuming earthquake stress drop $\Delta \sigma$ of 3 MPa (red), compared with the estimates for $\Delta \sigma$ of 0.3 MPa (magenta) and 30 MPa (blue).
Using the circular crack relation for stress drop as a function of crack radius $r$ [Eshelby, 1957], i.e., $\Delta \sigma = \frac{7M_0}{16r^3}$, and assuming each LFE lasts $\sim 1\ s$, typical earthquake stress drops of 0.3–30 MPa and our estimate of the tremor moment release would result in spectral corner frequencies of 31 to 161 Hz according to the Madariaga [1976] dynamic rupture model, i.e., $r = \frac{k\beta}{f_c}$. The much lower corner frequencies of 3–8 Hz that we observe for tremor imply either very low stress drops of 0.2–5 kPa (and $r$ of 105–428 m), or a much slower rupture process than typical earthquakes (the Madariaga [1976] model assumes the rupture velocity is 90% of the shear wave velocity). The low tremor stress drops on the order of kPa also agree with the stress drops suggested by Cascadia slow slip models [Liu and Rice, 2007; Rubin, 2008].

Thus although the tremor spectral falloff implies similarities to the radiated pulse shapes generated by discrete slip events, the physics of the slip process for these tremor events is likely very different from that of regular earthquakes. Based on rate-and-state friction models [e.g., Liu and Rice, 2007; Rubin, 2008], a low stress drop is suggestive of low effective stress (normal stress minus pore pressure) due to large volumes of hydrous fluids, and thus favored by the evidence of high pore pressures at the plate interface beneath Cascadia [Peacock et al., 2011]. Furthermore, dilatancy stabilization may play an important role controlling the behavior of tremor and/or slow slip [Segall et al., 2010; Liu and Rubin, 2010]. This view of tremor as the result of local fine-scale weak faults is also favored by the fact that tremor can be triggered by the small stress changes due to tides and the passage of teleseismic surface waves [Rubinstein et al., 2008, 2009].

7. Conclusions

Spectral characteristics of non-volcanic tremor (NVT) have proven difficult to analyze due to their poor signal-to-noise ratio (SNR). If array data are available, beamforming can be used to suppress noise and allow accurate tremor spectral analysis. By studying the Cascadia tremor recorded at a dense array, we conclude that the displacement spectra of the Cascadia tremor have corner frequencies around 3–8 Hz and fall off at $f^{-2}$ to $f^{-3}$ at higher frequencies.
The earthquake-like high-frequency falloff suggests that tremor can be studied using standard spectral models for regular earthquakes. Low corner frequencies imply either abnormally small stress drops due to possible high pore pressures (large amount of fluids), or slow ruptures. Fitting a standard Brune [1970] spectral model, we infer that a lower bound of moment release rate of Cascadia tremor is $3.8 \times 10^{10}$ N·m/s assuming the tremor signals we observed are P waves, or $1.4 \times 10^{10}$ N·m/s assuming S waves.

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