A renewed argument for crystal size control of ice sheet strain rates

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Abstract. At present, it is generally believed that crystal size has no direct influence on strain rate in the ice sheets and that the fraction of strain rate enhancement there which is not ascribable to c axis fabric is due to impurity content. Here we challenge this view because it is not consistent with recent results from analyses of deformation at Meserve Glacier and instead ascribe residual enhancement in the ice sheets to variations in crystal size. We resurrect the idea that variations of crystal size can be an important part of the total shear enhancement in the ice sheets, though agree with Paterson that this effect is generally dominated by variations of crystal fabric. We propose that the enhanced shear strain rate of ice age ice in southern Greenland, as inferred from tilt of the Dye 3 borehole, can be explained as a result of combined fabric variations and crystal size variations, with these two ice properties accounting for roughly 70% and 30% of the average enhancement, respectively. Permitting a grain size dependence of ice viscosity also resolves the quandary concerning closure and tilt of the Agassiz Ice Cap borehole.

1. Introduction

1.1. General Comments

Analyses of strain rate variations within the basal layers of Meserve Glacier, Antarctica, strongly suggest that the viscosity of subfreezing polycrystalline ice can depend directly on crystal size, at stress, temperature and grain size values important for glaciologic problems [Cuffey et al., this issue]. In addition, these analyses demonstrate that rheologic effects of chemical impurities are weak (consistent with data reviewed by Budd and Jacka [1989]) and offer evidence that rock particles have negligible rheologic effects at volumetric concentrations of up to a few percent.

The concept of crystal-size-dependent rheology is not new in glaciology [Barnes et al., 1971; Koerner and Fisher, 1979; Fisher and Koerner, 1986; Dahl-Jensen and Gundestrup, 1987; Goldsby and Kohlstedt, 1997; Peltier, 1998], but it has not previously been supported by in situ measurements from which effects of impurities and crystal fabrics could both be isolated. This is important because recent claims for grain size dependence [Peltier, 1998], which are based on innovative laboratory experiments [Goldsby and Kohlstedt, 1997], rely

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Paper number 2000JB900270. 0148-0227/00/2000JB900270\$09.00 on extrapolations over a very large range of grain sizes in order to match conditions important in terrestrial glaciology. In addition, size-dependent deformation has been discounted in the most widely cited and comprehensive reviews of the subject [Duval et al., 1983; Budd and Jacka, 1989; Paterson, 1991] (see Cuffey et al. [this issue], for discussion).

In this paper we show that puzzling aspects of deformation within the ice sheets can be resolved by admitting a size-dependent rheology consistent with that inferred for Meserve Glacier (in contrast, *Peltier* [1998] addresses a different set of puzzling questions by applying extrapolations of *Goldsby and Kohlstedt*'s [1997] results). We argue specifically that variations of crystal size are a more important control on strain rate in the ice sheets than are variations of impurity content, and that crystal size variations can be an important component of the total strain rate enhancement inferred for the ice age ices.

This paper first presents methodology for applying the empirical Meserve Glacier analysis results to other locations for the purpose of exploring these concepts. We do not suggest that results from Meserve Glacier are "universal" because (1) deformation mechanisms will be a function of stress, temperature, and possibly chemistry, and (2) a broad range of empirical relations was found to be statistically consistent with measurements beneath Meserve. Second, this paper focuses on two example problems from the Northern Hemisphere ice sheets. These are the tilt of the Dye 3 borehole and deformation of the Agassiz borehole, and were chosen because they are the best characterized in terms of both strain rates and ice properties (the Dye 3 borehole being the best).

1.2. Definition of Enhancement and Residual Enhancement

In glacial problems a given component of strain rate $\dot{\epsilon}_{ij}$ is most often described in terms of the corresponding deviatoric stress component τ_{ij} and the second invariant of deviatoric stress τ_{II} as [Paterson, 1994]

$$\dot{\epsilon}_{ij} = E \ A(T) \ \tau_{ii}^{n-1} \ \tau_{ij}, \tag{1}$$

where A(T) is the temperature-dependent Glen's law prefactor. E is a correction factor called enhancement which is introduced to make measured strain rates match those predicted by Glen's law. There are important variations of E in the ice sheets. The ice age ices in Greenland and the Canadian Arctic ice caps are softer than Holocene ice, with average E=2.5 to 3 relative to E=1 in the Holocene ice. Paterson [1991] and Azuma and Goto-Azuma [1996] have argued convincingly that much of this enhancement results from strengthening of c axis fabrics. However, Thorsteinsson et al. [1999] have shown that at least 30% of E cannot be attributed to fabric variations in the best studied case, the tilt of the Dye 3 borehole. In addition, fabric variations do not seem to explain some aspects of borehole deformation at Agassiz Ice Cap [Fisher and Koerner, 1986] and Devon Island ice cap [Koerner and Fisher, 1979]. Thus it is necessary to consider other contributions to E, and one may decompose E into parts dependent on fabric, E_f , and a residual enhancement not dependent on fabric, E_* , as $E = E_*E_f$. The role of crystal size, chemical impurities, and solid impurities in determining E_* was the subject of the Meserve Glacier analyses of Cuffey et al. [this issue]. They write the component $E_f \tau_{II}^{n-1} \tau$ more properly as $\dot{\epsilon}^{\rm F}$, to indicate an anisotropic flow law. Here we revert to the more widely used notation.

2. Hypothesis

We propose that variations in E_* can be calculated in terms of variations in grain size D and ionic concentrations C according to

$$E_* \propto \left[(1 - \tilde{\omega}) + \tilde{\omega} D^{-m} \right] \left[1 + \Gamma(C^{\lambda}) \right], \qquad (2)$$

where m and λ are positive constants, $\tilde{\omega}$ is in the range [0,1], and Γ is a function dependent on λ and on which chemicals are used as a measure of C. Equation (2) is only useful for comparison of different ices, i.e., for understanding the ratio of E_* values for ices having different D or C.

C refers to the bulk concentration of ions in the ice crystals and grain boundaries. If there is a significant area of ice-rock contact per volume of ice, some ions will be stored in liquid films at these interfaces. Consequently, measured bulk concentrations of ions will be higher than C [Cuffey et al., 1999] and should be corrected, as Cuffey et al. [this issue] do with their parameter η . For most ices in the ice sheets, there is insufficient rock content for such corrections to be important. Thus in what follows we ignore this potential complication and set $\eta = 0$.

2.1. Using Meserve Glacier Results

To explore implications of (2) we will assume that for stress and temperature conditions comparable to those at the Meserve Glacier study site ($\tau \approx 0.5$ bar, $T \approx -17$ °C) the values for these constants may be taken directly from Meserve Glacier results. The simplest optimal relation obtained was

$$E_* \propto D^{-0.6} \left(1 + 0.019 C_{\text{so}_4} + 0.088 C_+ \right) \tag{3}$$

for D in mm and C for sulphate and acidity $(C_{so_4}$ and C_{\pm}) in μ mole L⁻¹. A initiation is that the Cuffey et al. [this issue] analysis found a broad range of acceptable values for the parameters $\tilde{\omega}$ and λ ($1 > \tilde{\omega} > 0.15$; $0.5 < \lambda < 3.5$). Thus one must explore a range of possible $(\tilde{\omega}, \lambda)$ pairs, using acceptable values for m and Γ corresponding to these pairs. The range of model behaviors can be seen by choosing three values for λ , such as $\lambda = 0.5$, $\lambda = 1.0$, and $\lambda = 1.5$ (higher values of λ do not give different results from $\lambda = 1.5$ at ion concentrations characteristic of the ice sheets), and exploring acceptable ranges of $\tilde{\omega}$ for each. These are $\tilde{\omega} \in [0.15, 1]$ if $\lambda = 1.5$, $\tilde{\omega} \in [0.2, 1]$ if $\lambda = 1.0$, and $\tilde{\omega} \in [0.45, 1]$ if $\lambda = 0.5$. For each pair of values (ω, λ) the following parameterizations give appropriate values for m according to curve fits to results presented by Cuffey et al. [this issue]:

$$m(\tilde{\omega}) = 0.58 \left(1 + \frac{4.3(1 - \tilde{\omega})}{1 + 4\tilde{\omega}} \right) \qquad \lambda = 1.5, \quad (4)$$

$$m(\tilde{\omega}) = 0.58 \left(1 + \frac{3.8(1 - \tilde{\omega})}{1 + 4\tilde{\omega}} \right) \qquad \lambda = 1.0, \qquad (5)$$

$$m(\tilde{\omega}) = 0.64 \left(1 + \frac{1.56(1-\tilde{\omega})}{1+2\tilde{\omega}} \right) \qquad \lambda = 0.5.$$
 (6)

For the chemistry term Γ , Meserve analyses suggest that sulphate or calcium (concentrations C_{SO_4} and C_{C_a}) are the most important ions on a per-molar basis, and that acidity may also be important. For concentrations of SO₄, Ca, and H⁺ (denoted C_{SO_4} , C_{C_a} , and C_+) we use

$$\Gamma = \tilde{\gamma}_{\lambda}^{\mathrm{SO}_4} C_{\mathrm{SO}_4}^{\lambda} + k_+ C_+ \tag{7}$$

or

$$\Gamma = \tilde{\gamma}_{\lambda}^{C_{a}} C_{C_{a}}^{\lambda} + k_{+} C_{+}.$$
(8)

The $\tilde{\gamma}$ value depends strongly on λ . A curve fit to the *Cuffey et al.* [this issue] results gives, for sulphate (and concentration in μ mole L⁻¹),

$$\tilde{\gamma}_{\lambda}^{\text{so}_4} = \exp(-4\lambda) + \exp(-6\lambda). \tag{9}$$

Table 1. Mean and Maximum Concentrations of Important Chemical Impurities in Meserve Glacier Ices^a

Ion	Mean ^b	Maximum ^b
Na ⁺ K ⁺ Mg ²⁺ Ca ²⁺	65.1	291
К+	3.98	22.3
Mg ²⁺	8.80	41.1
Ca^{2+}	22.5	81.4
NO	8.40	49.3
$\begin{array}{c} \text{NO}_3^-\\ \text{SO}_4^{2-}\\ C_T \end{array}$	17.0	51.0
C_T	202	881

^aFrom Cuffey et al. [this issue]. C_T indicates total summed ionic content.

^bunits are μ mol L⁻¹

To use another ion i (for example i = Ca) or sum of ions (for example $i = Ca + SO_4$) in place of C_{SO_4} , a sensitivity $\bar{\gamma}_i$ can be defined that is approximately consistent with the Meserve results as

$$\bar{\gamma}_{\lambda}^{i} = \tilde{\gamma}_{\lambda}^{\mathrm{SO_{4}}} \left[\frac{\bar{C}_{\mathrm{SO_{4}}}}{\bar{C}_{i}} \right]^{\lambda}, \qquad (10)$$

where the \bar{C} are average ion concentrations in the Meserve ices (Table 1). For example, if one has a data set with both SO₄ and Ca, then Γ would be $1 + \tilde{\gamma}(C_{Ca} + C_{SO_4})$, with $\tilde{\gamma} = \tilde{\gamma}^{SO_4}C_{SO_4}/(C_{SO_4} + C_{Ca}) = 0.43\tilde{\gamma}^{SO_4}$, for the case of $\lambda = 1$, and with $\tilde{\gamma}^{SO_4}$ given by (9). Although analyses suggested that SO₄ and/or Ca are the most effective softening agents at Meserve on a per molar basis, a firm conclusion was not reached and the roles of different ions remain unknown. Therefore, a range of possible combinations should be investigated. For the alternative measures of ionic content C_{Ca} , $C_{SO_4} + C_{Cl}$, C_{Cl} , and $C_{SO_4} + C_{Cl} + C_{NO_3} + C_{Na} + C_{Ca} + C_{K} + C_{Mg}$, $\tilde{\gamma}$ given by (9) should be multiplied by factors of 0.76, 0.18, 0.22, and 0.084, respectively.

Further, Cuffey et al. [this issue] found a wide range of admissible parameterizations for the effects of C_+ . Here we have incorporated this result in the simplest possible fashion. Provided that the C_+ concentration is not much greater than $1 \ \mu M$, the admissible magnitude of the C_+ effect is within the range k_+C_+ where the prefactor $k_+ \in [0, 0.6]$.

2.2. Limit of No Chemical Impurities

For ices with negligible ion present, two convenient end-members (representing large and small $\tilde{\omega}$ cases) for estimating the change in shear strain rate due purely to a change in crystal size from D_1 to D_2 (D in mm) are

$$\frac{E_1}{E_2} = \left[\frac{D_2}{D_1}\right]^{0.6}$$
(11)

$$\frac{E_1}{E_2} = \frac{1 + 0.2D_1^{-1.8}}{1 + 0.2D_2^{-1.8}}.$$
(12)

The largest grain size variations in the ice sheets are typically 1 order of magnitude, e.g., 0.5 to 5 mm at Dye 3 and 2.5 to 25 mm at Summit [Herron et al., 1985; Langway et al., 1988; Alley and Woods, 1996; Thorsteinsson et al., 1997; Tison et al., 1994]. The possible enhancement due to grain size variation alone is thus 1.7 to 4.0 at Dye 3 and 1.04 to 4.0 at Summit. A factor of 4 enhancement is very important but is smaller than the potential factor of 10 variation due to tightening of fabric from isotropic to single maximum [Budd and Jacka, 1989; Azuma and Goto-Azuma, 1996].

3. Dye 3 Borehole

The tilt of the Dye 3 deep borehole reveals an average enhancement for ice age ice of ~ 2.5 , with a maximum of 4 [Dahl-Jensen and Gundestrup, 1987]. Temperature varies from -17 to -12 °C at these depths. Recently, Thorsteinsson et al. [1999] have done a thorough analysis of the contribution of fabric variation to this enhancement and found that fabric can account for no more than 70% of the average enhancement. Their residual correlates roughly with particle content and the ratio of ions to crystal size. Dahl-Jensen and Gundestrup [1987] recognized the strong correlation between Dye 3 enhancement and dust content, crystal size, and concentrations of several ions and proposed that any of these may be dominantly controlling strain rate there. Here we further examine the residual enhancement identified by Thorsteinsson et al. [1999].

Using crystal size, sulphate, and chloride data from Herron et al. [1985], Langway et al. [1988], and Herron and Langway [1985], we calculate E_* for the bottom section of the Dye 3 borehole for various values of $\tilde{\omega}$ and λ . The strain rate and properties in the Holocene ice immediately above the climate transition are used to constrain the reference value for E_* (thus we are calculating variations in enhancement, not its absolute value). Models with $\tilde{\omega}$ in the range 0.45 to 0.75 are very strongly correlated with the residual enhancement (Figure 1 and Figure 2). For the $\lambda = 0.5$ model the contribution of ions to the residual enhancement is on average ~ 7%. For the higher λ ones it is only a fraction of a percent. Thus we suggest that crystal size variations are responsible for more than 90% of the residual enhancement at Dye 3. For the ice age ice as a whole, the 2.5 average enhancement can be explained as 70% crystal fabric, 25 to 30% crystal size, and < 5 % chemistry. In these calculations we have set $k_{+} = 0$, so that the Holocene ice is not being softened by acidity. Thus the portion of ice age residual enhancement due to chemicals may be even smaller.

4. Agassiz Ice Cap

Although fabric information here is too sparse for a complete analysis as at Dye 3, the Agassiz Ice Cap basal layers provide the next most complete data set [Fisher

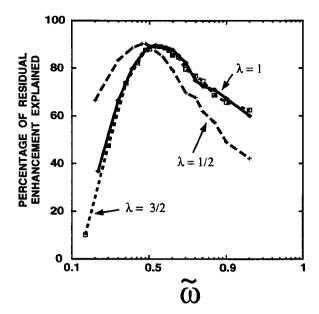


Figure 1. Percentage of variance of the non fabric residual enhancement inferred from tilt of the Dye 3 borehole [*Thorsteinsson et al.*, 1999] that can be accounted for with various models which were deemed acceptable in the Meserve strain rate analysis.

and Koerner, 1986], and have been the subject of insightful analyses [Fisher and Koerner, 1986; Paterson, 1991]. Fisher and Koerner derived enhancement factors for both borehole tilt and borehole closure, and found that both correlate best with calcium content and microparticle content, both of which are also fairly strongly correlated with crystal size. They argued that the inferred enhancements could not result from fabric variations because enhancements for closure and tilt are positively correlated and similar in magnitude, despite the fact that these deformations must utilize shear planes of completely different orientation.

Paterson [1991] challenged this interpretation and argued for primary control by fabric. He points out that the enhancement of Wisconsin relative to Holocene ice decreased through time from 2.3 in the first year to only 1.3 several years later. Thus all but 1.3 or less of the enhancement contrast cannot result from a fixed property of the ice such as impurity content. Paterson argues that recrystallization occurred rapidly in the fine-grained impurity-rich ice and more slowly in the coarse-grained clean ice, thus reducing the E contrast over time.

While this argument is almost certainly correct regarding the decrease of E contrast over time, it seems unlikely that the initial contrast of 2.3 can be explained this way. The impure ices had not yet attained tertiary creep in the first year [Fisher and Koerner, 1986]. Therefore it is difficult to envision that the impure ices started the year stiffer to closure than the pure ices and made enough progress toward tertiary creep to be softer than the pure ice by a factor of 2.3, which is by coincidence nearly identical to the E contrast with respect to tilt.

Crystal size-dependent deformation appears to provide a solution to this puzzle. Fine grains can account for the initial enhancement and the positive correlation between closure and tilt enhancements. In addition, crystal size will change as recrystallization proceeds and thus also can explain the reduction through time, as coarse-grained ices become finer in response to the deformations associated with borehole closure.

Application of Meserve numbers supports the plausibility of this argument. Calcium concentrations are high enough in the Agassiz ice age ice (5 μ mol/L) that they may be important, accounting for between 0 and $1 + \tilde{\gamma}_{1/2}(0.76C_{c_*})^{1/2} = 1.36$ leaving a factor of 1.69 to 2.3 to be explained. The crystal sizes are roughly 4.5 and 1.5 mm for the hard and soft ices, respectively. For $\tilde{\omega} = 0.5$ and $\tilde{\omega} = 1$ the corresponding enhancement contrast would be 1.36 and 1.93, respectively. These numbers are in the right range. Note that again such variations in strain rate are small compared to the po-

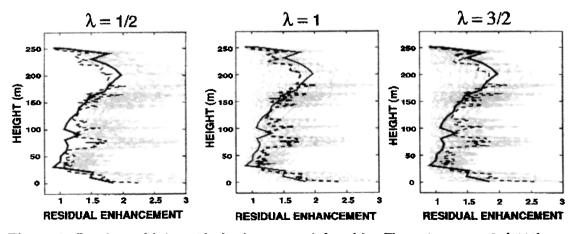


Figure 2. Dye 3 non fabric residual enhancement inferred by *Thorsteinsson et al.* [1999] as a function of height above the bed (solid line). Dashed lines show our model results for various values of $\tilde{\omega}$. The best model is shown as a heavy dashed line.

tential factor of 10 variations associated with a range of c axis fabrics from isotropic to single maximum [Azuma, 1994]. However, a large range of c axis fabrics is not observed in the deep ices here or at Dye 3 [Fisher and Koerner, 1986; Thorsteinsson et al., 1999].

5. Discussion

Similar analyses could be conducted for other locations if the fabric data were available. Koerner and Fisher [1979] report that the Devon Island Ice Cap borehole also has enhanced strain rates of ice age ices that do not seem to correlate with fabric changes but do correlate with both crystal size and impurity content. Here, as at Agassiz, the enhancement occurs for borehole closure.

We are claiming that crystal size variations are an important control on shear enhancement in the ice age ices via direct dependence of ice deformation mechanisms on crystal size. An important question then is what are the causes of the crystal size variations? Alley et al. [1986] have made a strong case that chemical impurities slow grain growth by preventing grain boundary migration, which causes impure ices to be fine grained. Thus an important ultimate cause of much of the grain size variations and hence strain rate variations may very likely be the chemical impurity content, even if there is no direct mechanical connection. The suggestion of Fisher and Koerner [1986] that impure layers within the ice sheets should generally be modeled as having lower viscosity than clean layers remains valid. Paterson [1991] has argued in addition that chemical impurities are also the root cause of some fabric variations, because the impurities cause finer grains, which more readily develop a strong fabric. This is an interesting idea which needs more research. At this point, all we can argue is that for a given chemical content, there will be interdependent evolution of grain size, fabric, and strain rate toward a characteristic value. In basal layers such as those of Meserve Glacier, where active entrainment of bed material [Cuffey et al., 2000] and folding associated with inhomogeneities and bed topography can occur, this characteristic value may never be reached, and correlation of grain size, impurities, and fabric may be weak. In the ice age ices of the ice sheets the correlation is much stronger.

A general prediction that we can offer, which would support results presented here and by Cuffey et al. [this issue] if shown to be true, is that a stronger grain size dependence should be found at higher temperatures and/or lower stresses. Conversely, dislocation climb is favored at high stresses and low temperatures [Duval et al., 1983; Goldsby and Kohlstedt, 1997] and will more completely dominate other mechanisms under these conditions.

We wish to emphasize that analyses presented here and by *Cuffey et al.* [this issue] do not provide evidence for or against the specific deformation mechanism and rheologic relation proposed by Goldsby and Kohlstedt [1997] and used by Peltier [1998]. There are several possible mechanisms that will produce a grain size dependence: volume diffusion, grain boundary diffusion, and grain boundary sliding [Duval et al., 1983; Goldsby and Kohlstedt, 1997]. We are approaching this problem differently and hope to establish in the future whether the two converge. A strength of our approach is that it does not rely on extrapolations of laboratory experimental results over a large range of grain sizes and temperature. A primary weakness is that our results lack quantitative precision.

6. Conclusion

On the basis of results obtained at Meserve Glacier, we think it likely that grain size variations contribute importantly to enhanced shear of ice age ices in the Northern Hemisphere ice caps (an idea considered possible by Koerner and Fisher [1979], Fisher and Koerner [1986], and Dahl-Jensen and Gundestrup [1987]). This hypothesis can explain the residual enhancement inferred from tilt of the Dye 3 borehole and may resolve the controversy concerning the Agassiz Ice Cap borehole. However, a definitive test of this hypothesis has not been made, and we fully acknowledge that the analyses presented here are no substitute for predictive physical models, which need to be developed. Finally, we note that with respect to enhancement variations in the ice sheets, variations of crystal fabric can be at least as important, and probably dominant, in agreement with Paterson's [1991] review. In contrast to Paterson, we attribute residual enhancement variations within ice age ices primarily to variations of crystal size, not to those of ionic content.

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