Today:

- Continue Bounce motion
- Longitudinal Drift
- Radiation Belt Organization:
  - Shielding layer
  - L-shell
- Field Line Equation: $r=LR_e\cos^2\lambda$
- Loss Cone
- Begin Large Scale Current

Single Particle Motion (cont'd)

- $\mu = \frac{1}{2}mv_{\perp}^2/B$ = Magnetic Moment =constant gave us gyration and drift.

Now:

- Pitch angle analysis gives us $B_m = B_o/\sin^2\alpha_o$
- Then add: Dipole field

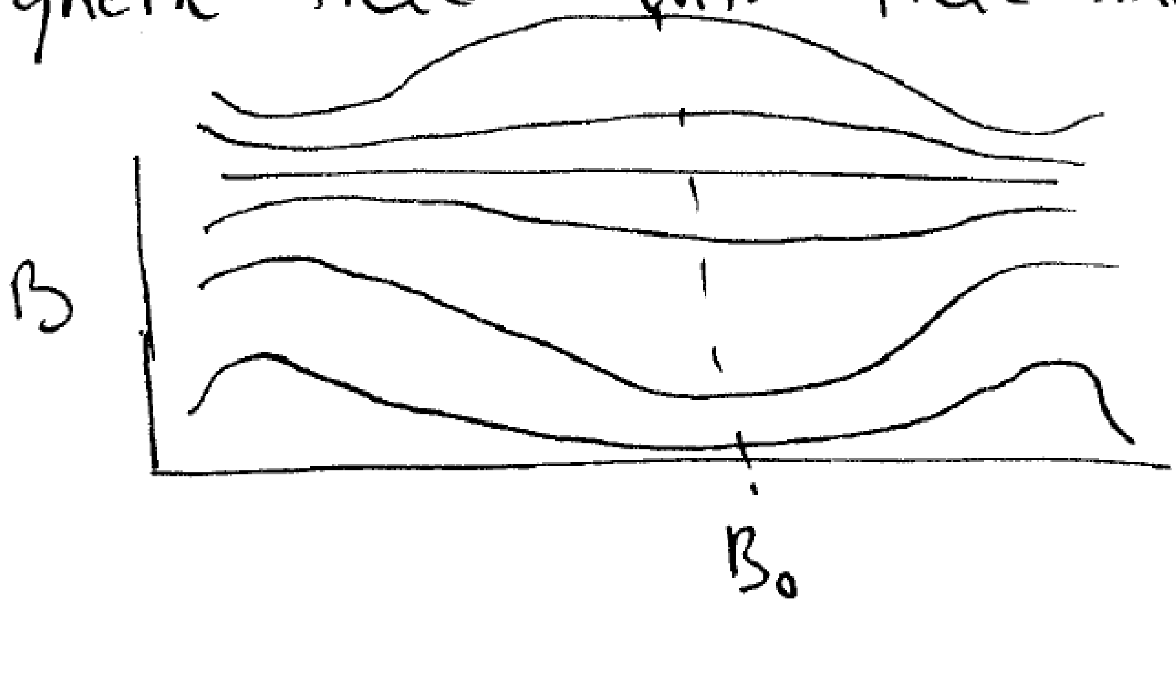
  $B(r,\lambda) = (M/r^3)*(1 + 3\sin^2\lambda)^{1/2}$ (where λ is latitude)
- We will Find :

Gyro period << Bounce period << Drift Period

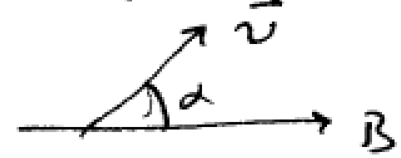
Magnetic Bottle

Magnetic Bottle

Consider a cylindrically symmetric magnetic field with field lines as



Suppose you start a particle on the axis at the place where field strength is B_0.
Suppose the velocity of the particle makes an angle α with respect to the magnetic field.
What Happens?



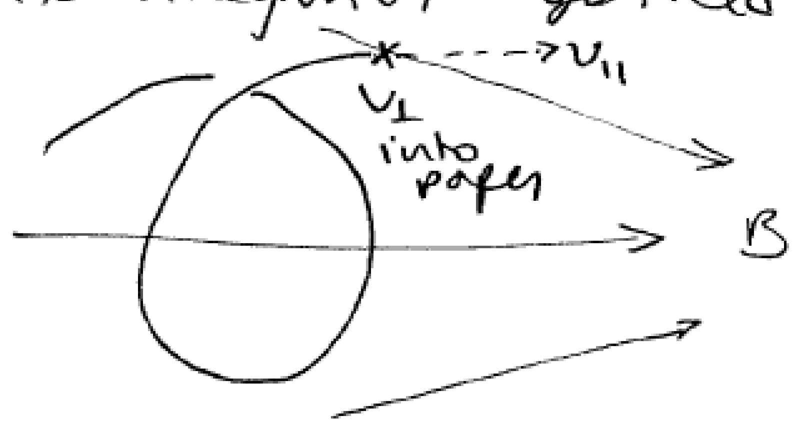
$\alpha \equiv$ pitch angle

$v_{\parallel} = v \cos\alpha$

$v_{\perp} = v \sin\alpha$

$v_{\parallel}$ carries the particle into a region of larger field strength. Some time later we have

$v_{\parallel}$ to right
$v_{\perp}$ into paper

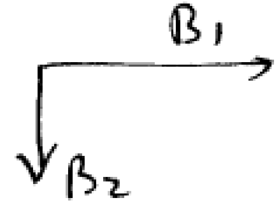


at the particle $\bar{B}$ has a component B_1 that is parallel to the guiding center field and a component B_2 that is perpendicular.

So we have

$\otimes \longrightarrow v_{\parallel}$

$v_{\perp}$ (into)



consider components of $\bar{v} \times \bar{B}$ (Lorentz Force)

$v_{\parallel} B_2$ causes force that increases $v_{\perp}$

$v_{\perp} B_2$ " " " decreases $v_{\parallel}$

Thus, $v_{\parallel}$ decreases while $v_{\perp}$ increases.

Since $v_{\parallel}^2 + v_{\perp}^2 = \text{constant}$ (energy conserved), after a while $v_{\parallel}$ will be zero. But $v_{\perp} B_2$ continues to act, so $v_{\parallel}$ changes sign

ie: Particle enters magnetic field a certain distance, stops, turns around and comes out!

This is called <u>Mirroring</u>

We want to know the point where the particle turns around, or the <u>Mirror Point</u>

Equation for $v_\perp$ is μ = constant

or $\dfrac{\frac{1}{2}mv_\perp^2}{B}$ = const.

if particle moves into region where B is larger, then its $v_\perp$ must get larger too.

<u>Question</u>: How far into the region of increasing magnetic field will a particle penetrate if it starts with pitch angle α_0 at a place where $B = B_0$?

$$\mu = \frac{\frac{1}{2}mv^2\sin^2\alpha_0}{B_0} = \frac{\frac{1}{2}mv^2\sin^2\alpha}{B}$$

($v_\perp = v\sin\alpha$ initially)

as it penetrates, B increases, so does $\sin\alpha$ until, at the mirror point $\alpha = 90°$

and $v_{\parallel} = v\cos\alpha = 0$

$v_\perp = v\sin\alpha = v$ ← all energy in $\perp$ motion

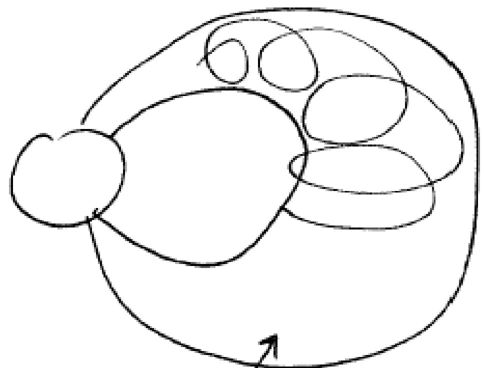
Define B at this point to be B_m

we have $\dfrac{\sin^2\alpha_0}{B_0} = \dfrac{1}{B_m}$

or $\boxed{B_m = \dfrac{B_0}{\sin^2\alpha_0}}$

All particles of <u>Any</u> energy, charge, mass will mirror at same point B_m if they start with same α_0!

Another Example of Guiding Center Motion
Particles in a Magnetic Dipole Field
<u>The Radiation Belts</u>



Bounce motion

as $v_{\parallel}$ carries particles away from the equator they enter region of increasing $\mathbf{B}$ field $\therefore$ mirror and bounce back + forth

Additionally the guiding center drifts around the earth in longitude due to gradient + curvature drifts.

I Gyration

$$\tau_{gyration} \equiv \text{gyroperiod} = \frac{2\pi}{\Omega} = \frac{2\pi m}{Be} \sim$$

10^{-2} sec protons at low alt

10^{-5} s e^- at low alt

10 sec p at $r = 10 R_e$

10^{-2} s e^- at $r = 10 R_e$

Bounce

bounce time $\equiv \tau_{bounce} = \oint \frac{ds}{v_{\parallel}}$

$$ds = \left(dr^2 + r^2 d\lambda^2\right)^{1/2} = r_e \cos\lambda \left(1 + 3\sin^2\lambda\right)^{1/2} d\lambda$$

$$v_{\parallel} = \left(v^2 - v_{\perp}^2\right)^{1/2}$$

but $\dfrac{v_{\perp}^2(r,\lambda)}{B(r,\lambda)} = \dfrac{v^2 \sin^2\alpha_e}{B_e} = \text{const}$

where α_e and B_e are equatorial pitch angle and field strength

so $v_{\parallel} = v\left(1 - \dfrac{\sin^2\alpha_e B(r,\lambda)}{B_e}\right)^{1/2}$

$B(r,\lambda) = \dfrac{M}{r^3}\left(1 + 3\sin^2\lambda\right)^{1/2}$ (Dipole field)

$= \dfrac{B_e}{\cos^6\lambda}\left(1 + 3\sin^2\lambda\right)^{1/2}$ (because $\underbrace{r = r_e \cos^2\lambda}_{\text{field line equation}}$

numerically integrating

$$\tau_{bounce} \sim 4 \frac{r_e}{v} T(\alpha_e)$$

where $T(\alpha_0) = 1.3 - .56 \sin\alpha_e$
(not a strong dependence on α_e)

For $v = c$ and $T(\alpha) = 1$

r_e	1.5 Re	3 Re	6 Re
τ_{Bounce}	.13 s	.26 s	.52 s

III Longitudinal Drift

for $J = 0 = \nabla \times B$

$$V_G = \frac{\overline{B} \times \nabla \overline{B}}{e B^3} \left(\mathcal{E}_\perp + 2 \mathcal{E}_\parallel \right)$$

integrate around drift path of 360° Longitude

gives $\tau_{drift} \cong \dfrac{44}{r_e * \mathcal{E}(\text{MeV})}$ minutes

r_e ← in earth radii

That is, $r_e \equiv L R_e \equiv L$ earth radii

for $\mathcal{E} = 0.1$ MeV and $r_e = 10 R_e$

$\tau_{drift} \cong 44$ minutes

So, in general

$$\tau_{drift} \gg \tau_{bounce} \gg \tau_{gyro}$$

Neglecting scattering and plasma instabilities particles can be trapped forever

in practice some species at some energies are trapped for 100 yrs!

to employ semi-empirical models to account for the associated adiabatic effects upon radiation-belt particles.

To sun ←

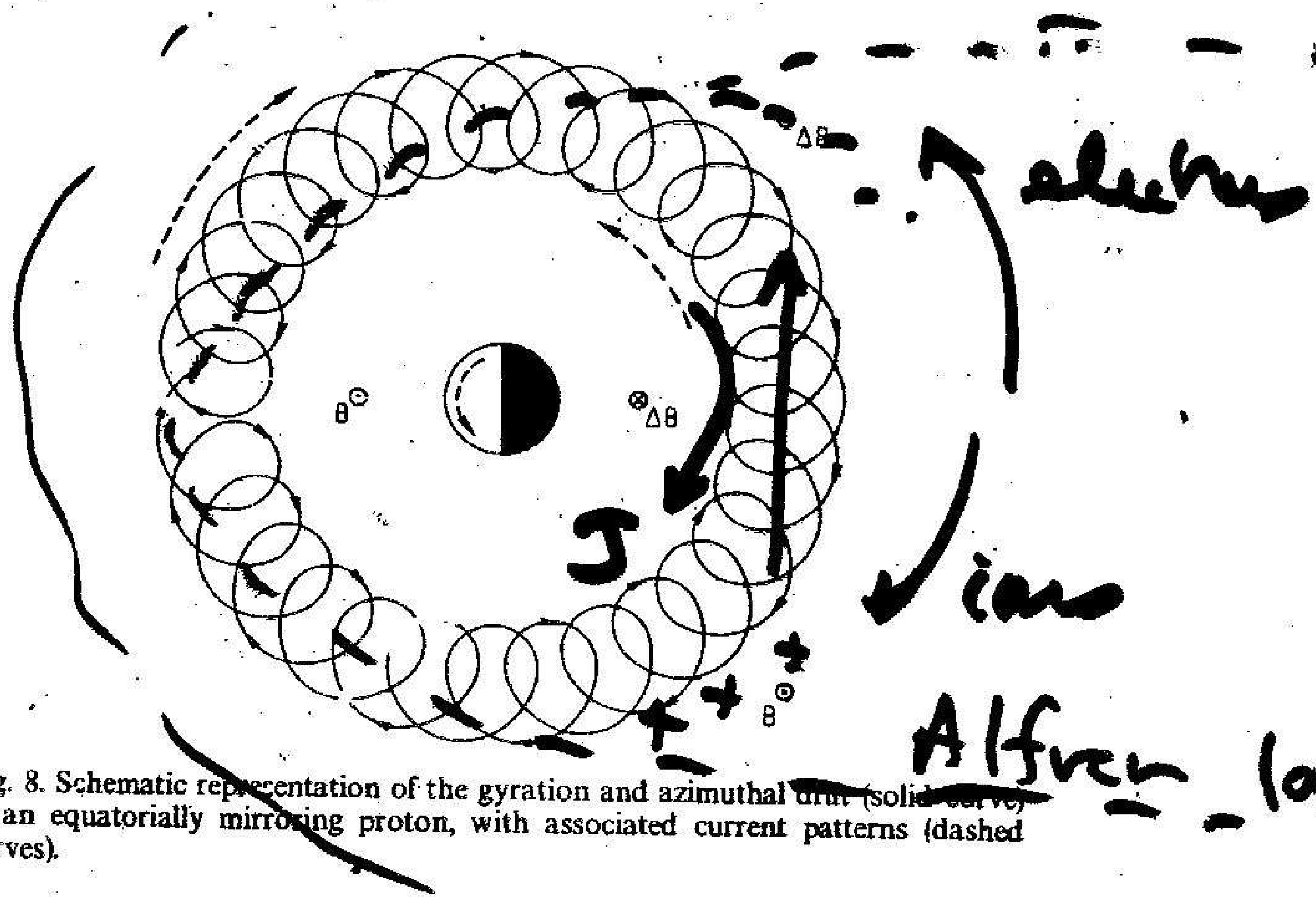


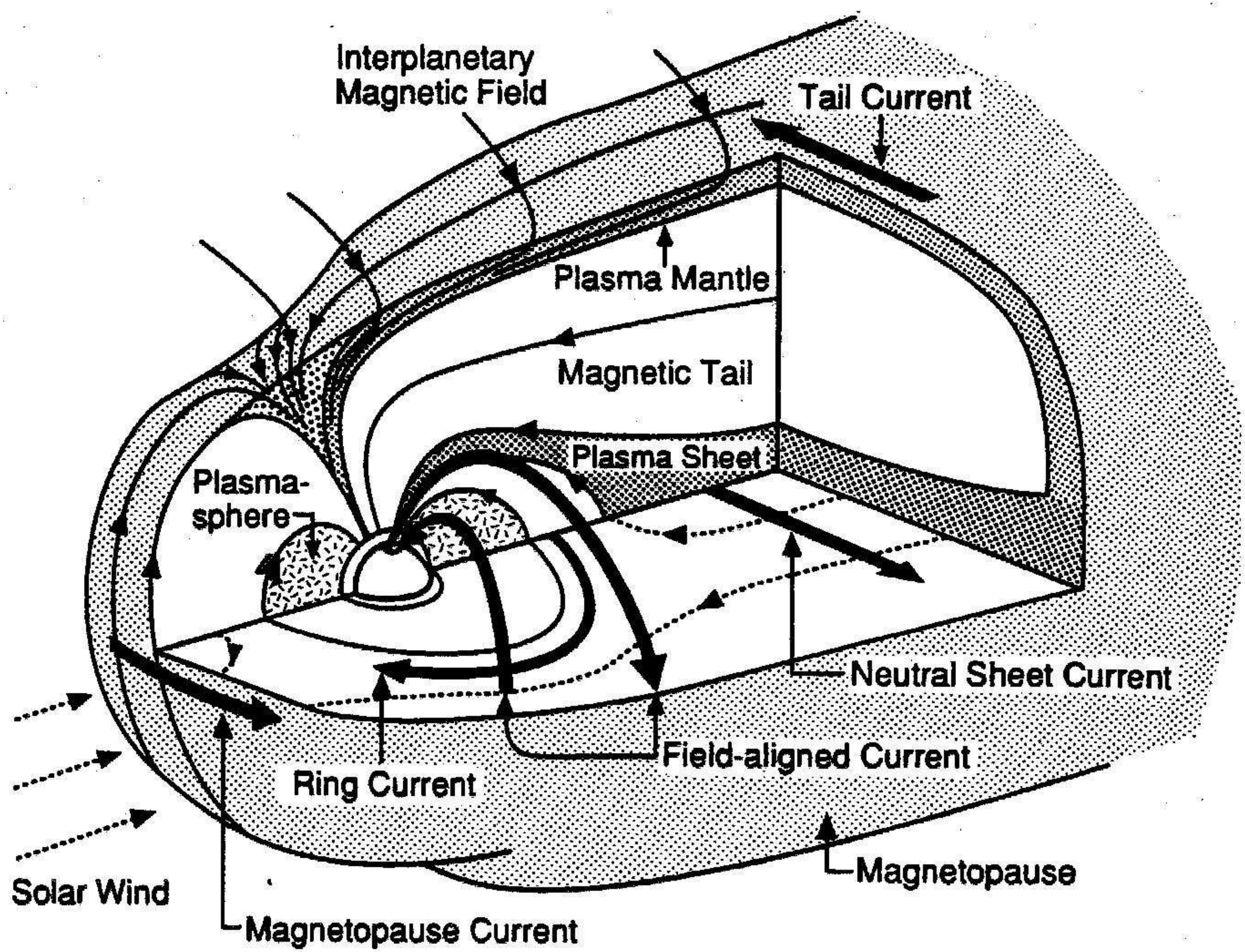
Fig. 8. Schematic representation of the gyration and azimuthal drift (solid curve) of an equatorially mirroring proton, with associated current patterns (dashed curves).

and the gra

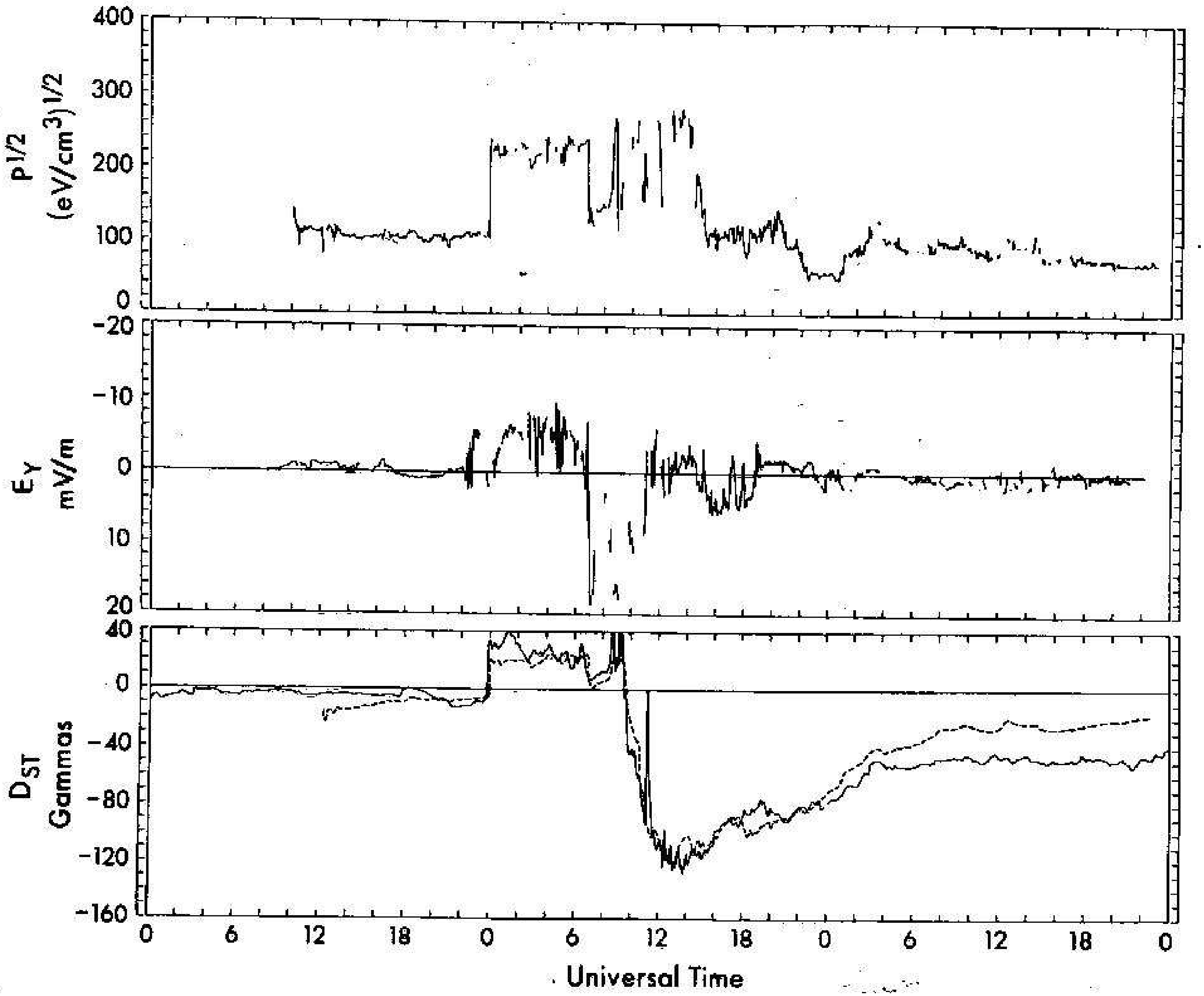
In different
to the loca
of curvatu
current de
and is give

where $\beta_{\|}$=
the pressu
field. Obse
both a
densely po
This regio
and near L
The in
in the inn
$(-\hat{\varphi})$ dire
in (1.4
is spatially
Simplif

V = ExB$_{drift}$ PLUS Grad-B$_{drift}$



Interplanetary
Magnetic Field
Tail Current
Plasma Mantle
Magnetic Tail
Plasma Sheet
Plasma-
sphere
Neutral Sheet Current
Field-aligned Current
Ring Current
Magnetopause
Solar Wind
Magnetopause Current



Equitorial Magnetic field perturbation

Radiation Belt organization

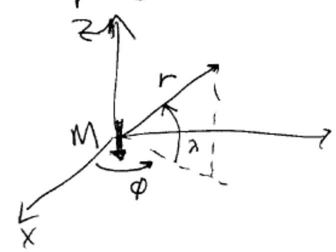
For a dipole B-field with
$\vec{M}$ = dipole moment of the earth

[$\vec{M}(\vec{r},t) = \oint \vec{\mu}\, f(\vec{r},\vec{v},t)\, d^3v$ = magnetic moment/volume

$\equiv$ Magnetization

For a dipole $\vec{B} = -\vec{\nabla}\psi$ (remind: Parks (3.10))

where $\psi = -\vec{M}\cdot\nabla\left(\frac{1}{r}\right)$ for dipole



in spherical coords:

$$B_r = \frac{\partial}{\partial r}\psi = -2M\frac{\sin\lambda}{r^3}$$

$$B_\phi = \frac{1}{r}\frac{1}{\cos\lambda}\frac{\partial}{\partial\phi}\psi = 0$$

$$B_\lambda = -\frac{1}{r}\frac{\partial}{\partial\lambda}\psi = M\frac{\cos\lambda}{r^3}$$

$$|\vec{B}| = \sqrt{B_r^2 + B_\lambda^2 + B_\phi^2} = \frac{M}{r^3}\left(1 + 3\sin^2\lambda\right)^{1/2}$$

from Parks 3.30 / 3.31 can write

$$\frac{dr}{r} = 2\frac{d(\cos\lambda)}{\cos\lambda}$$

integrate to get $\phi = \phi_0$ $\quad r = r_0\cos^2\lambda$

Now Let $L \equiv \frac{r_0}{R_e}$ marks Equatorial distance of a particular Field Line

so $\boxed{r = L R_e \cos^2\lambda}$

over →

or, to put it another way,
the Latitude at the surface is given
by $r = R_e$ so $\cos^2\lambda = \frac{1}{L}$

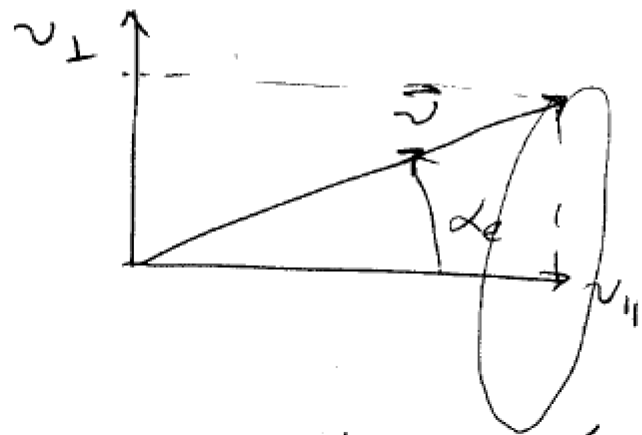
This is definition of <u>Invariant Latitude</u>

L can be defined more carefully for a distorted field.

So, for a given L shell most particles
will be undergoing 3 distinct motions –
(gyration, bounce + ∇B drift) <u>but</u> if
<u>mirror Point</u> is $< 1\,R_e$ they will be
lost due to scattering from atmosphere.

Actually, the loss altitude $\sim R_e + 100\,\text{km}$ where
probability of scattering becomes high

Which Particles are Lost ?



$d\Omega$ solid angle

Loss cone

if mirror point is $<$ $R_e + 100$ km, then particle is Lost. Find this α_e

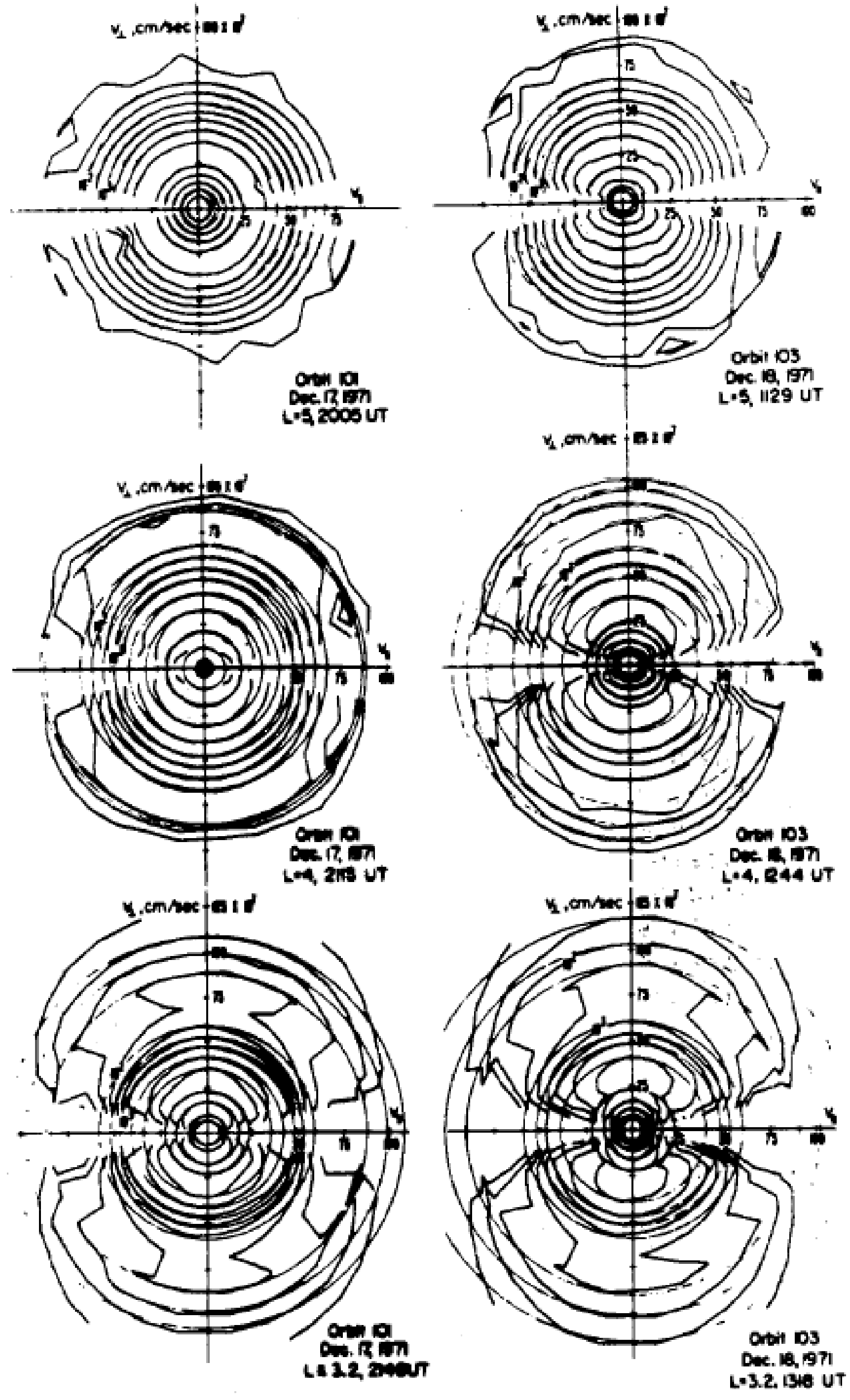
using $\mu = \text{constant} = \dfrac{v^2 \sin^2 \alpha_e}{B_e}$

find α_e such that $\alpha = 90^\circ$ at B for $r = R_e + 100$ km

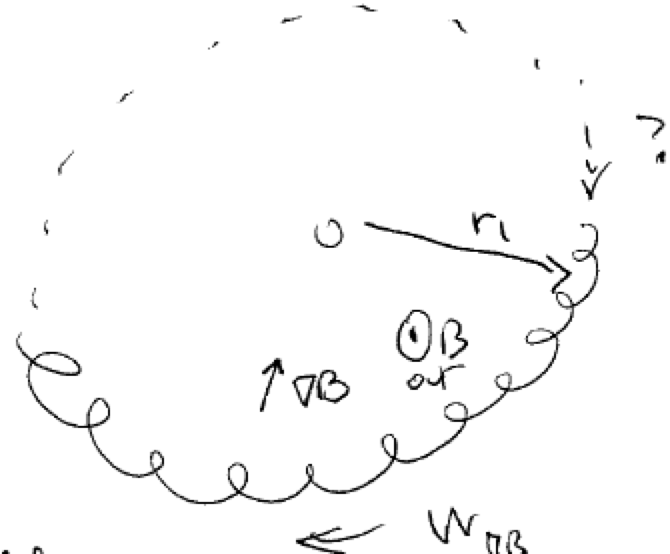
we find that $\alpha_e \sim 3^\circ$ at $L = 6$

so MOST particles are trapped.

show Pitch angle Distributions along Field Line


$v_\perp$, cm/sec
$v_\parallel$
Orbit 101
Dec. 17, 1971
L=5, 2005 UT
Orbit 103
Dec. 18, 1971
L=5, 1129 UT
Orbit 101
Dec. 17, 1971
L=4, 2113 UT
Orbit 103
Dec. 18, 1971
L=4, 1244 UT
Orbit 101
Dec. 17, 1971
Orbit 103
Dec. 18, 1971
L=3.2, 1318 UT

L-shell drift: How can you tell the drift motion always returns guiding center to starting point?



assume equatorially mirroring

Then $\mathcal{E} = \mathcal{E}_\perp$ always

Answer: if energy is conserved then B_1 with μ = constant, if start from r_1 where $B = B_1$

$$\mu = \frac{\mathcal{E}}{B_1} \text{ at } r_1$$

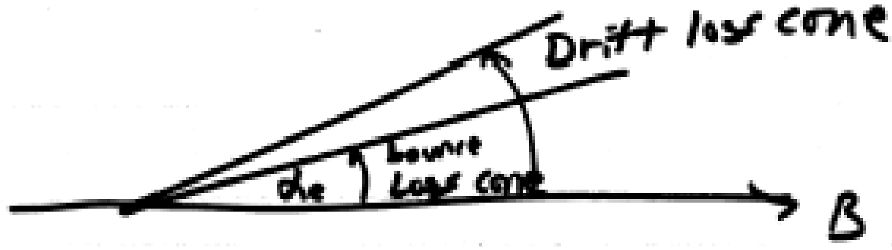
if returned at $r \neq r_1$, $B \neq B_1$

so $\mu \neq$ constant

L defines a closed shell for perfect dipole

Ah! But the field is not a perfect dipole:

Drift loss cone



Drift loss cone = largest α_e at equator which equals the bounce loss cone at some longitude.

That is: all particles in the <u>bounce</u> loss cone are lost in 1 bounce period

while

particles in the drift loss cone are lost <u>sometime</u> during their drift period.
Namely, somewhere in drift period they see the <u>Smallest</u> Earth's field so their mirror point is lowest.

waves are constantly affecting particles' pitch angles.

So, there is a random diffusion of particles from larger to smaller pitch angles, where particles get lost inside the loss cone, or drift loss cone.

Contours of constant Magnetic field Strength

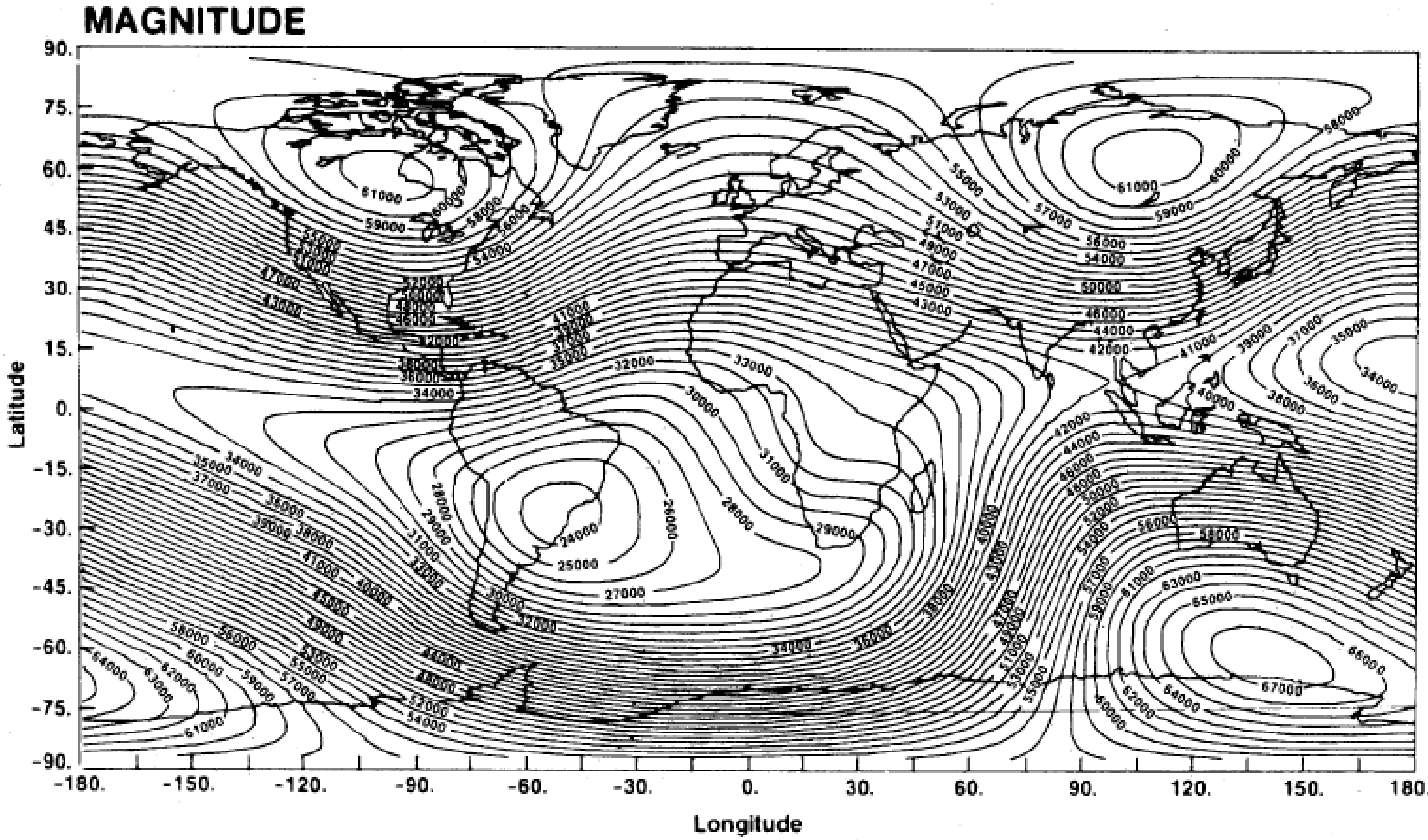


Figure 4-11. Contours of constant total field B at the surface of the earth from the model IGRF 1980.0.

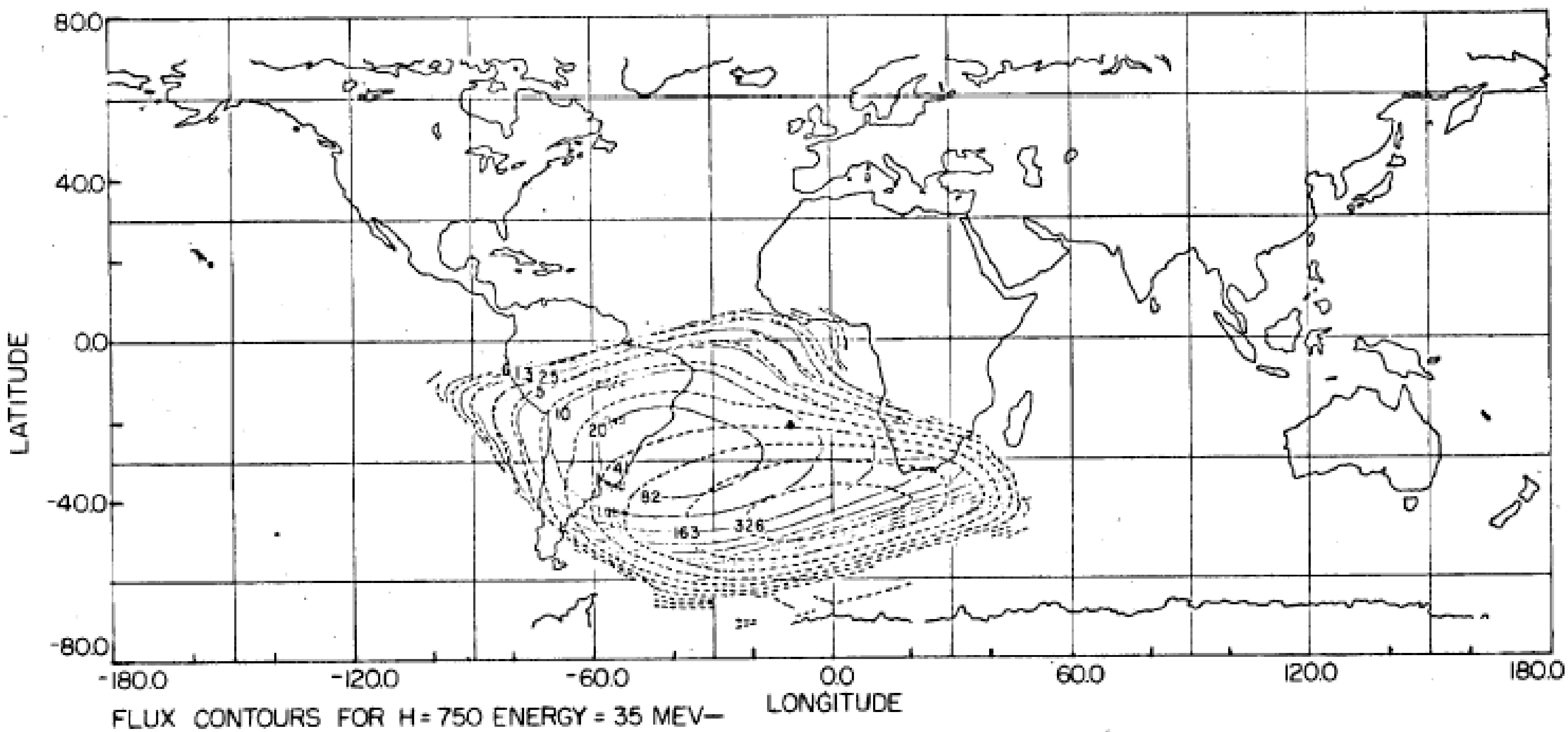


Figure 5-33. Proton isointensity flux contours as measured in the South Atlantic anomaly at an altitude of 750 km. The solid lines depict 28-45 MeV proton (ion) fluxes and the dashed lines 5-7 MeV proton fluxes. The flux units are particles/(cm^2-s-MeV).

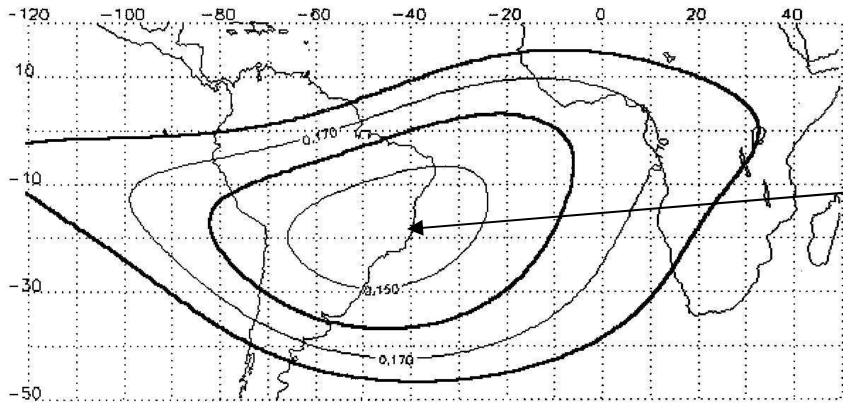


Figure 1. DGRF 65, Epoch 1970, 1336 km magnetic field contours.

Magnetic Field Minimum

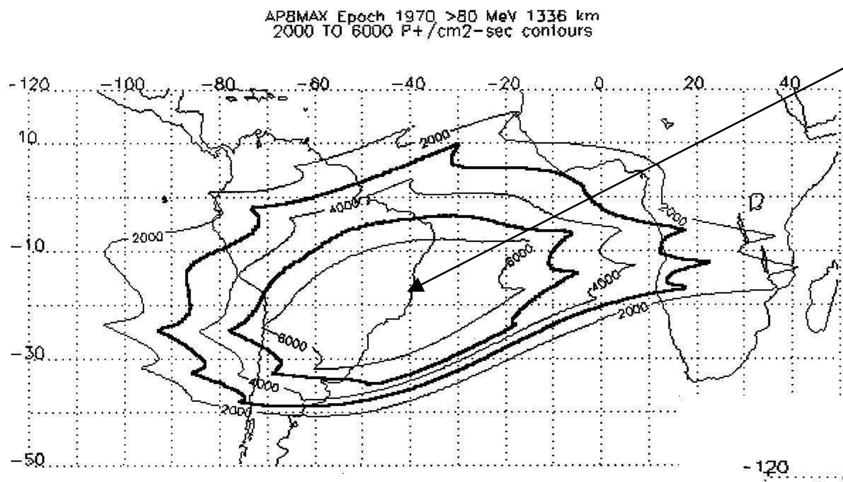


Figure 2. AP-8 MAX Epoch 1970 > 80 MeV protons at 1336 km.

Precipitating >80MeV protons

Single Event Upsets in the memory for Topex Satellite

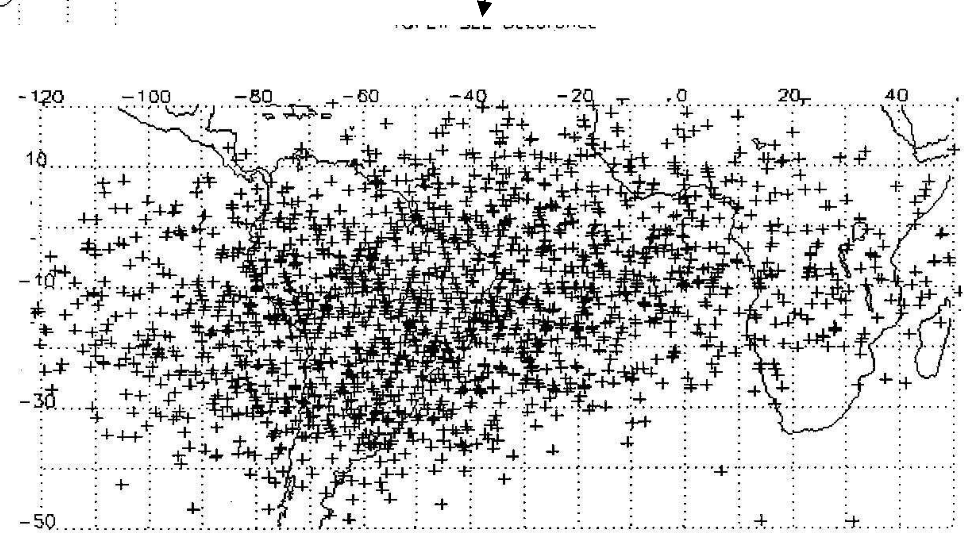


Figure 3. TOPEX SEE geographical distribution.

Advanced subject

For details see
adiabatic_invariants.pdf

Adiabatic Invariant

$$I(\bar{\zeta},\delta) = \oint d\theta\, \vec{p}(\zeta,\theta,\delta)\cdot \frac{\partial \bar{q}(\zeta,\theta,\delta)}{\partial\theta}$$

usually abreviated

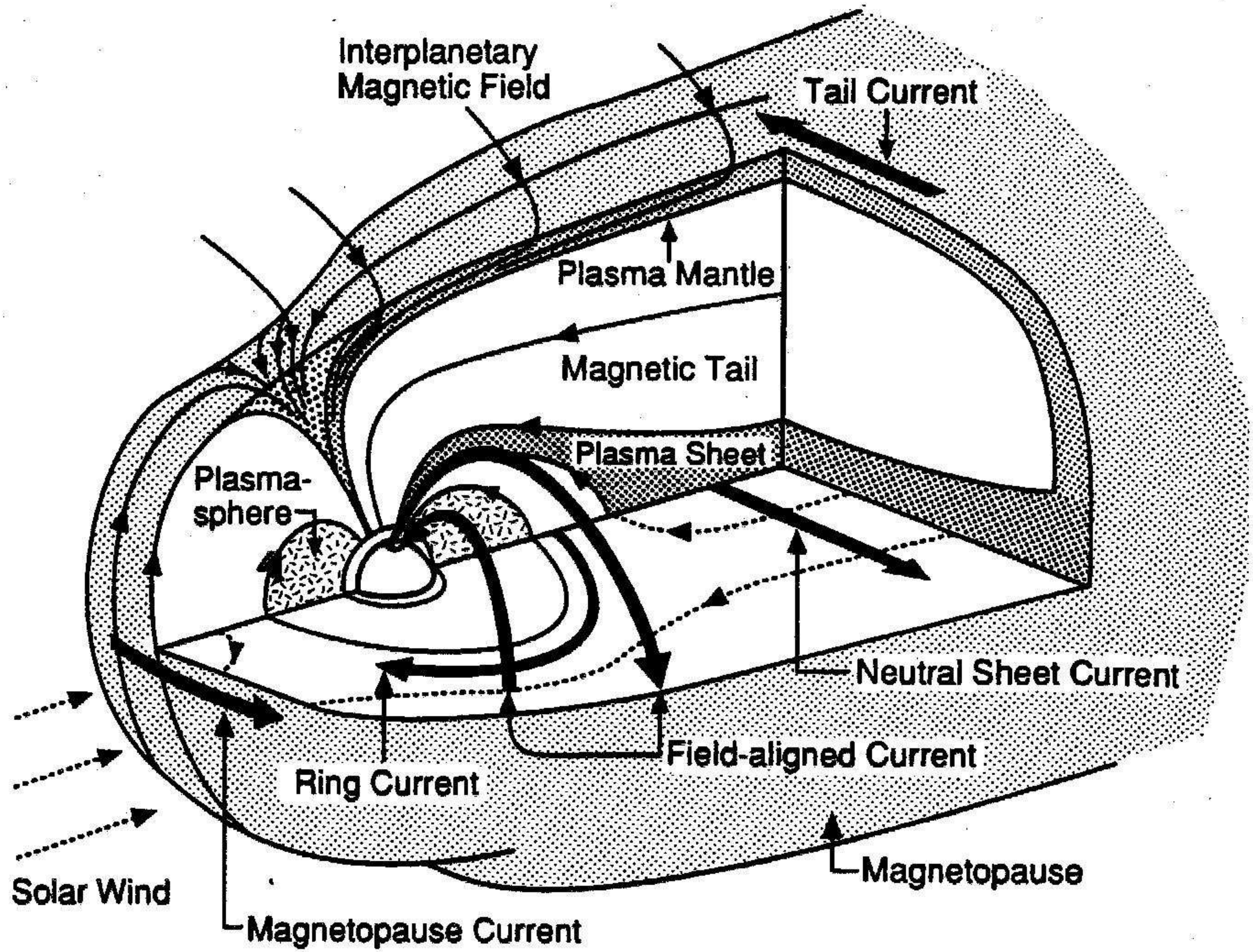
$$I = \oint_{\bar{\zeta}=\text{constant}} \bar{p}\cdot d\vec{q} \overset{\text{by Stokes theorem}}{=} \int d^3p\, d^3q$$

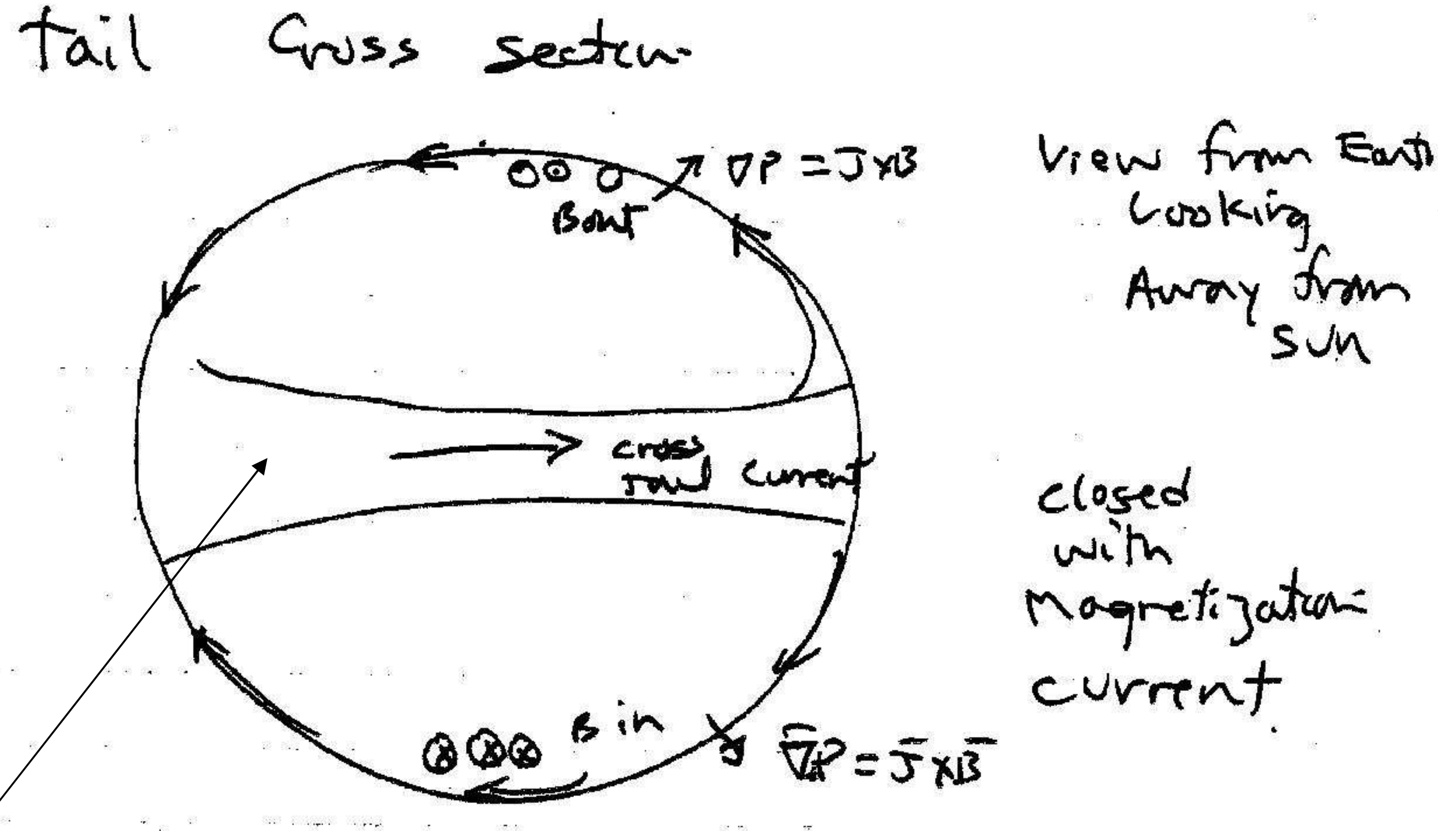
but $\int d^3p\, d^3q$ has same value for any canonical $\bar{p}, \bar{q}$

$\therefore$ since transformation ~~from~~ from unperturbed to perturbed $\vec{p}, \bar{q}$ is canonical

$$\underbrace{\int d^3p\, d^3q}_{\text{actual motion}} = \underbrace{\int d^3p\, d^3q}_{\text{unperturbed motion}} = \underbrace{\oint_{\zeta=\text{constant}} \bar{p}\cdot d\bar{q}}_{\text{unperturbed motion}}$$

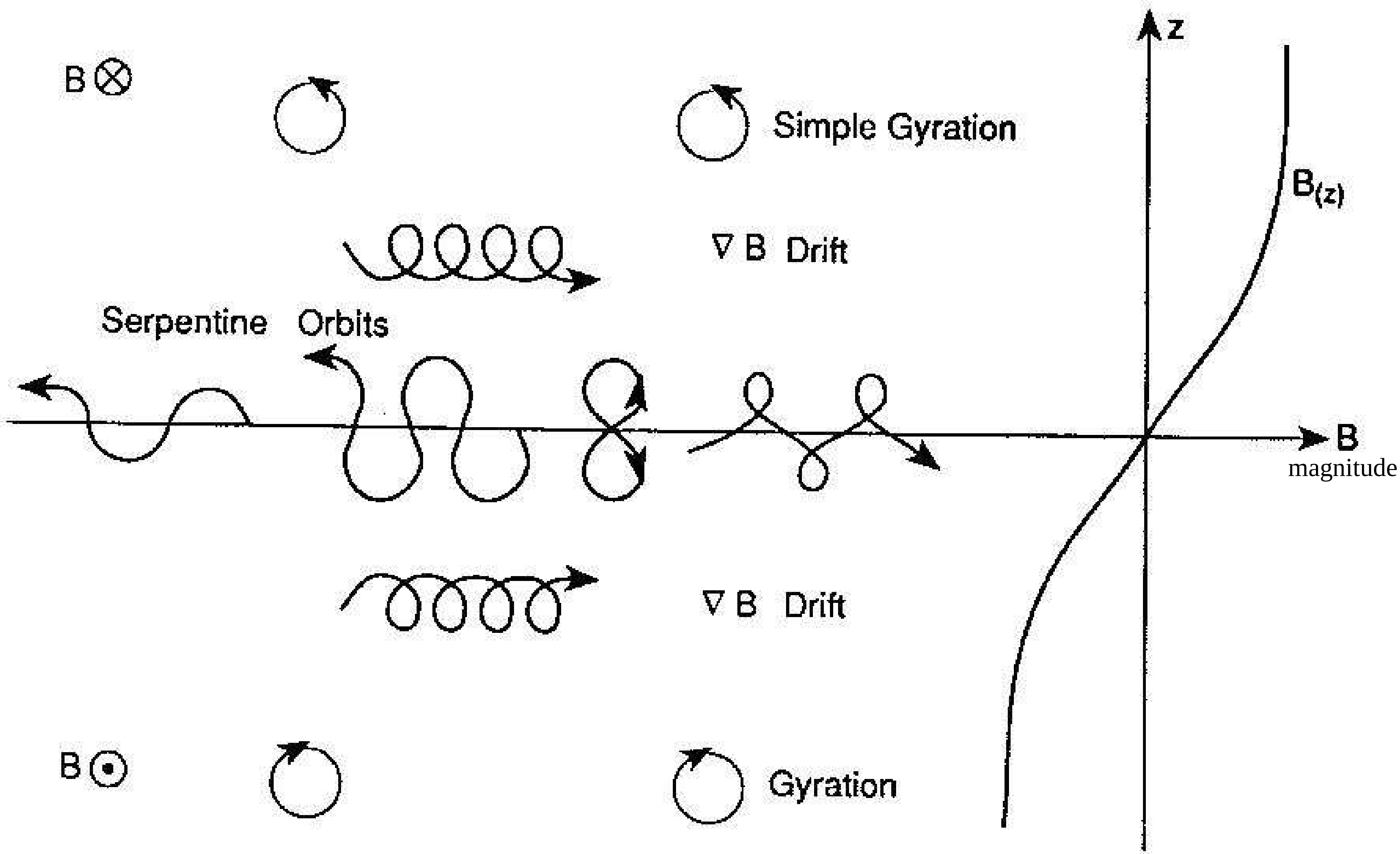
- Now, lets look at magnetotail Tail curents
- Then combine cold and hot plasma drifts
- Cold:
  - Sunward convection on closed field lines
  - Plasmasphere co-rotatation
- Hot
  - Ring current
  - Partial ring current/Alfven layer
- Then: Aurora and ionosphere


Interplanetary
Magnetic Field
Tail Current
Plasma Mantle
Magnetic Tail
Plasma Sheet
Plasma-
sphere
Neutral Sheet Current
Field-aligned Current
Ring Current
Solar Wind
Magnetopause
Magnetopause Current



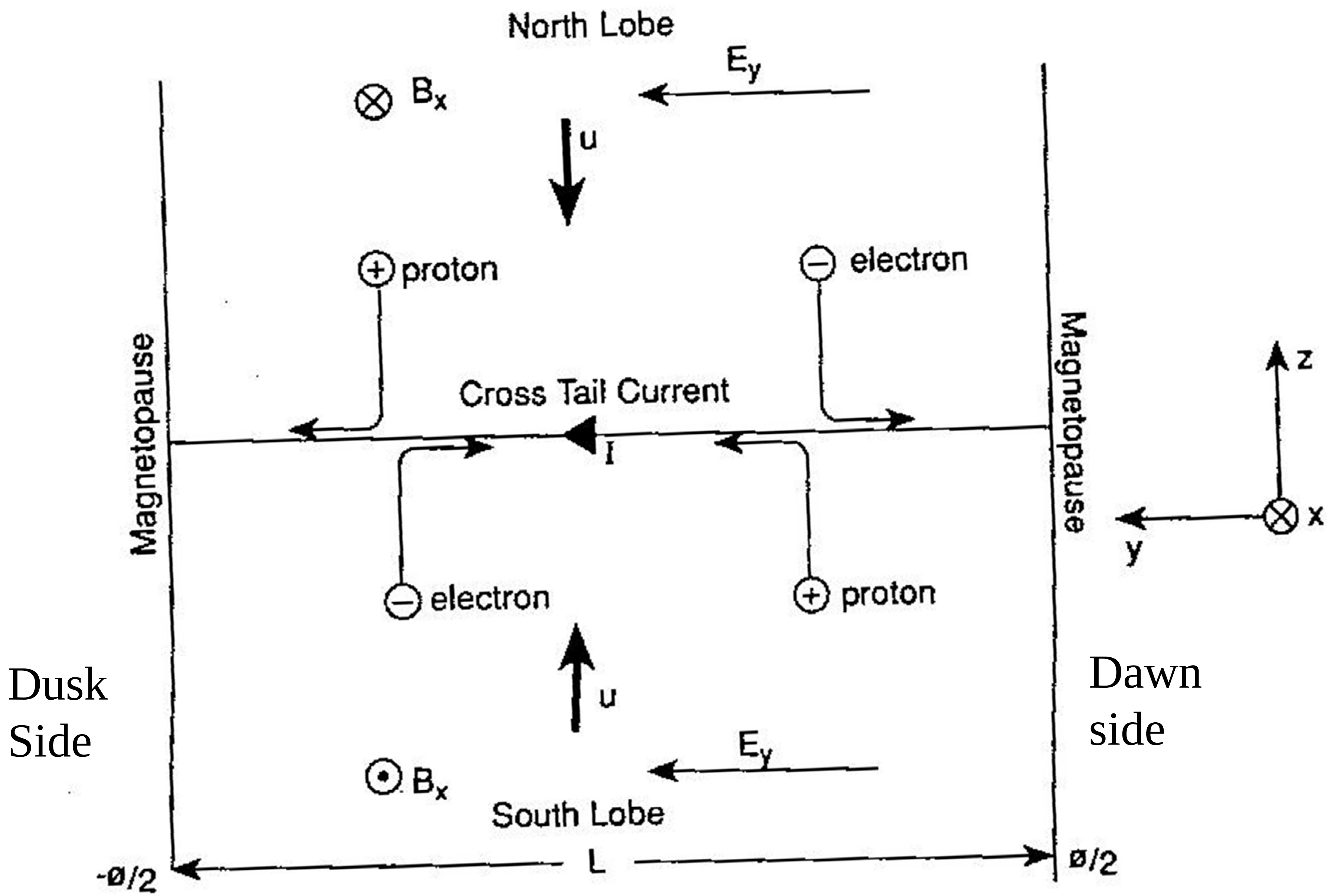
How can there be a current

Like this: charge moving ACROSS the B –field?

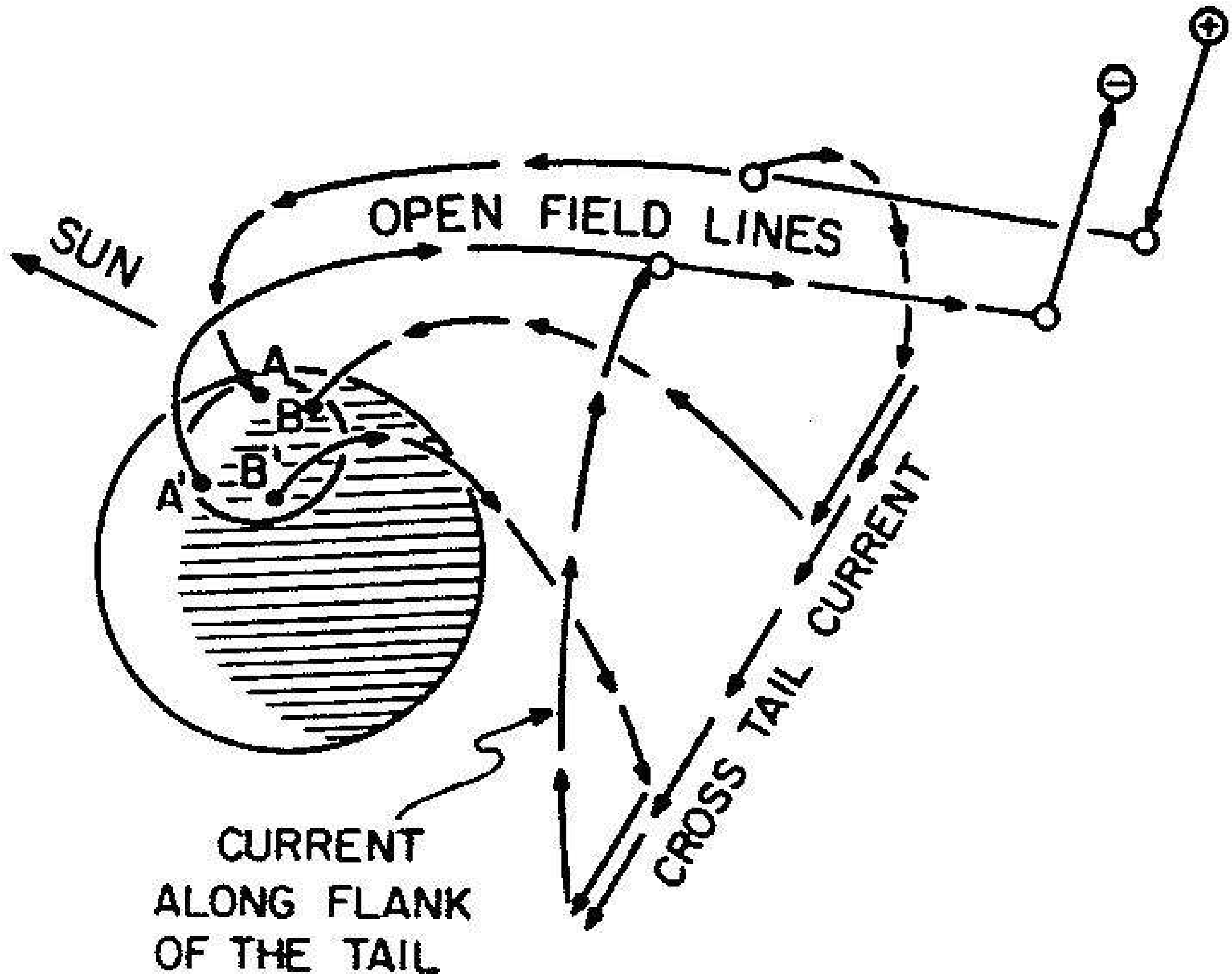


View from the Tail looking back at Earth

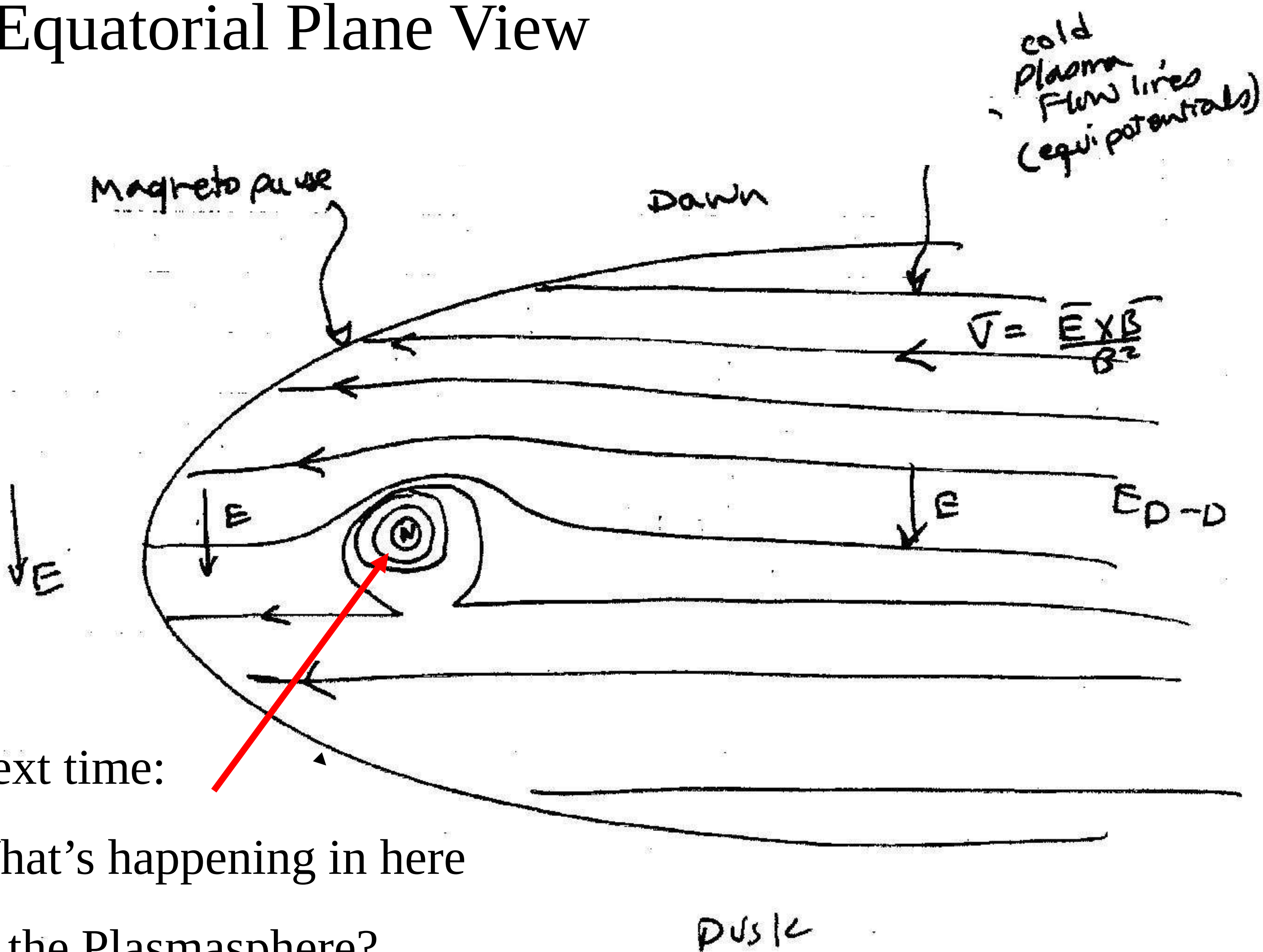
Dusk ← dawn



View from Tail towards Earth


SUN
OPEN FIELD LINES
A
B
B'
A'
CROSS TAIL CURRENT
CURRENT
ALONG FLANK
OF THE TAIL

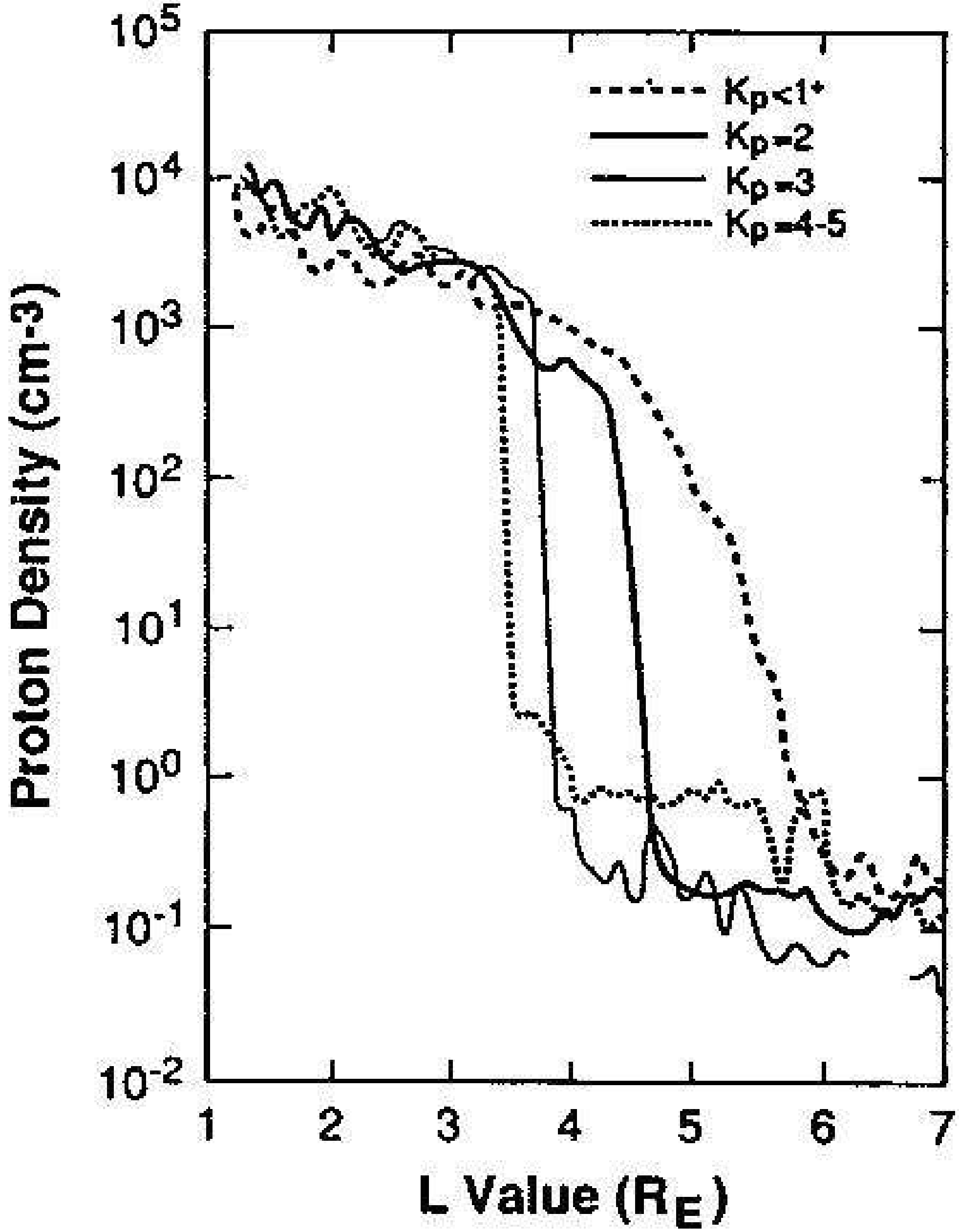
Equatorial Plane View



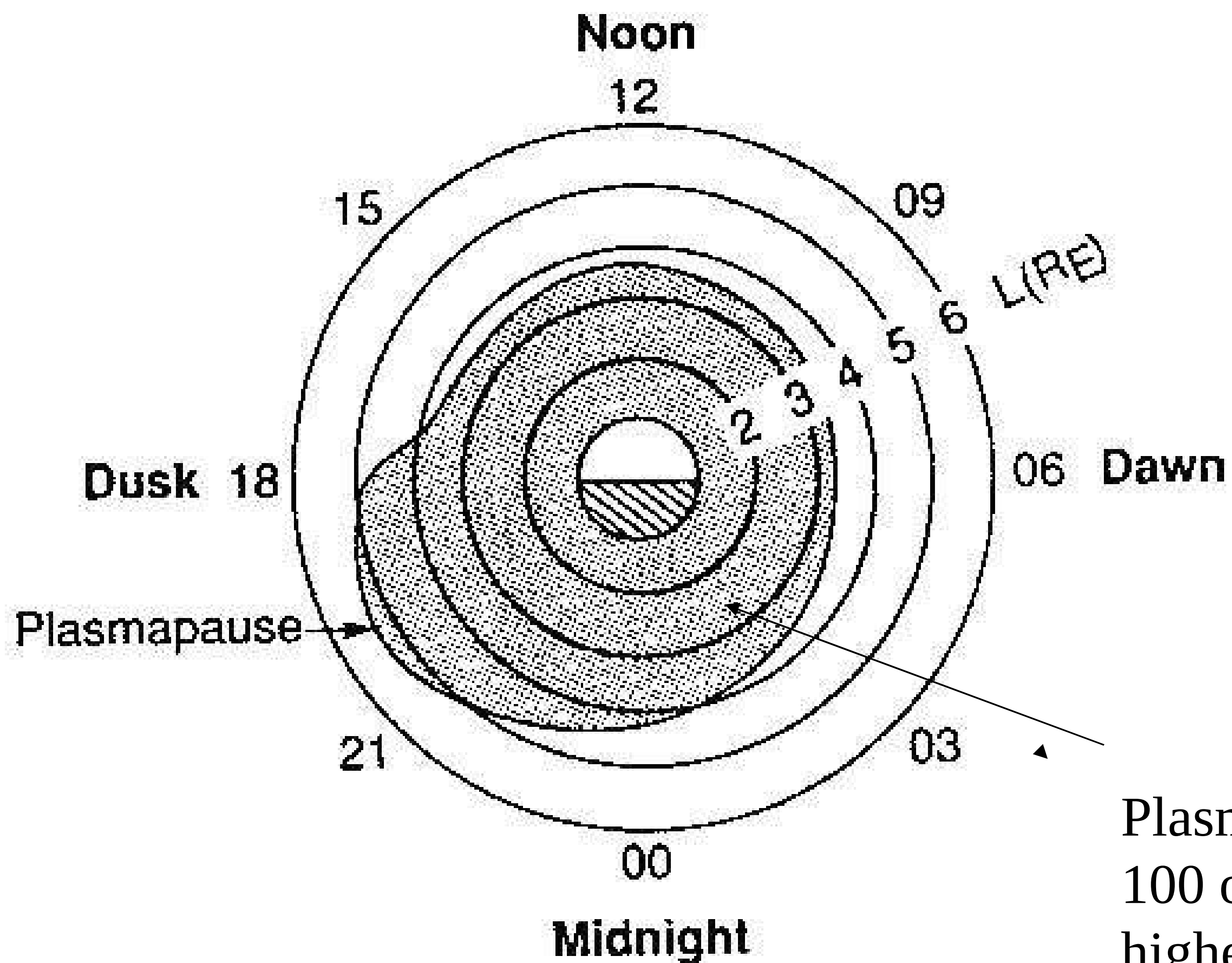
Next time:

What’s happening in here

In the Plasmasphere?



Magnetic activity index



Plasma density 100 or 1000 times higher than outer magnetosphere