ESS 414/514
Problem Set 1
Due Friday, April 9, 2010

Problems for all students:

The first two problems illustrate that the Eulerian description of motion is usually simpler than the Lagrangian. In these problems, **r** is the current position of a fluid particle that was at **x** at time *t=0* and **u** is the velocity at **r**. Bold face denotes a vector quantity and the subscripts 1, 2 and 3 refer to components along 3 perpendicular directions.

(1) The Lagrangian description $\mathbf{r} = \mathrm{r}(\mathbf{x},t)$ of the motion of a certain lump of jello is

$$r_1 = x_1 \cos(\omega t) + x_2 \sin(\omega t)$$
$$r_2 = -x_1 \sin(\omega t) + x_2 \cos(\omega t)$$
$$r_3 = x_3$$

Find the Eulerian description $\mathbf{u}=\mathbf{u}(\mathbf{r},\mathrm{t})$. Describe the motion geometrically.

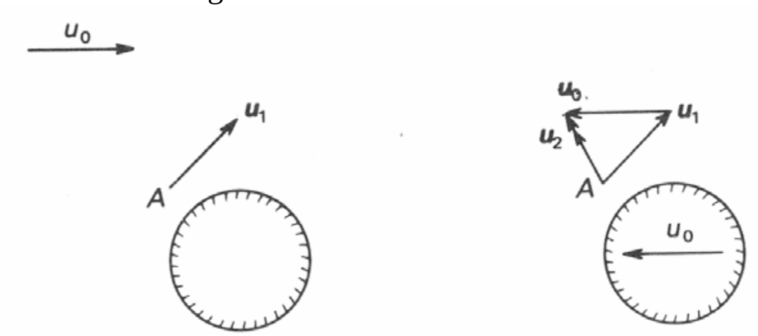
(2) The Eulerian description of the motion of another lump of jello is

$$u_1 = r_1$$
$$u_2 = -r_2$$
$$u_3 = 0$$

Derive the Lagrangian description $\mathbf{r} = \mathrm{r}(\mathbf{x},t)$. Draw a picture of the paths of the fluid particles.

Problem for 514 students only

(3) In the figure below, the cylinder on the left is stationary and the fluid velocity is $\mathbf{u}_0$ at great distance. On the right, the fluid far away is stationary and the cylinder moves to the left with velocity $\mathbf{u}_0$. Show that this change of reference frame results in changes to both $\partial \mathbf{u}/\partial t$ and $\mathbf{u}\cdot\nabla\mathbf{u}$, but leaves $D\mathbf{u}/Dt$ unchanged.



Hints: What does $D\mathbf{u}/Dt$ mean physically? What is $D\mathbf{u}_0/Dt$ equal to on the left? If $\mathbf{u}_1$ is the fluid velocity at point A on the left, what is $\mathbf{u}_2$ on the right?