ESS 414/514
Problem Set 3
Due Monday, April 26, 2010

Problems for all students

1. Assuming that the Earth's mantle is an incompressible, Newtonian fluid with kinematic viscosity 10^{16} m^2/s.
   (a) ***Estimate*** how long it would take for flow in the mantle to adjust if the relative motion between India and Asia instantaneously changed due to their collision.
   (b) This time scale is not reasonable for the real mantle. Why not?

2. A lava flow is 10 meters thick when it comes to rest on the flank of Kiluea Volcano in Hawaii. Assume that it cools very quickly to a nearly uniform temperature of 100° C by the boiling of rainwater water in cracks that form as the flow cools and contracts.
   (a) Estimate the time scale for further cooling of the lava flow if conduction is the only process operating.
   (b) Estimate the time scale if percolation of rainwater is the only process operating.
   (c) Discuss the implications of these estimates for the partial differential equation that you would have to solve if you wanted to know how the actual cooling takes place.

   You may assume that the thermal diffusivity of lava is 2 10^{-7} m^2/s, that the cracks are less than 10 cm apart, and that the rainfall in southeastern Hawaii is 1.5 meters/year.

3. Approximate a spreading ocean ridge as two fluid quarter spaces diverging horizontally from each other. (Mass is conserved by a uniform source at the vertical boundary between the quarter spaces.) Assume that the temperature on the top surface of the quarter spaces is 0° C and the temperature of the source and at great depth is 1400° C. Estimate the thickness of the cold ``lithosphere" as a function of the age of the surface material. Assume a “half” spreading rate of 5 cm/year and a thermal diffusivity of 10^{-2} cm^2/sec.

Problem for 514 students only

4. Consider a uniform fluid with constant velocity tangential to a porous wall. Normally, the "Blasius" boundary layer will grow downstream. Show that adding a constant velocity flow INTO the porous wall can make the boundary layer thickness constant downstream. It has the additional advantage of making the non-linear boundary layer differential equation linear. This is an example of where adding a complication leads to a simplification. Derive the variation of tangential velocity as a function of distance from the wall.