The failure of earthquake failure models

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Abstract. In this study I show that simple heuristic models and numerical calculations suggest that an entire class of commonly invoked models of earthquake failure processes cannot explain triggering of seismicity by transient or "dynamic" stress changes, such as stress changes associated with passing seismic waves. The models of this class have the common feature that the physical property characterizing failure increases at an accelerating rate when a fault is loaded (stressed) at a constant rate. Examples include models that invoke rate state friction or subcritical crack growth, in which the properties characterizing failure are slip or crack length, respectively. Failure occurs when the rate at which these grow accelerates to values exceeding some critical threshold. These accelerating failure models do not predict the finite durations of dynamically triggered earthquake sequences (e.g., at aftershock or remote distances). Some of the failure models belonging to this class have been used to explain static stress triggering of aftershocks. This may imply that the physical processes underlying dynamic triggering differs or that currently applied models of static triggering require modification. If the former is the case, we might appeal to physical mechanisms relying on oscillatory deformations such as compaction of saturated fault gouge leading to pore pressure increase, or cyclic fatigue. However, if dynamic and static triggering mechanisms differ, one still needs to ask why static triggering models that neglect these dynamic mechanisms appear to explain many observations. If the static and dynamic triggering mechanisms are the same, perhaps assumptions about accelerating failure and/or that triggering advances the failure times of a population of inevitable earthquakes are incorrect.

1. Introduction

Earthquake-generated stress changes may increase the likelihood of other earthquakes (i.e., trigger), as evident in increases in seismicity rate following large, or "triggering," earthquakes [e.g., Reasenberg and Simpson, 1992; Toda et al., 1998; Anderson and Johnson, 1999; Stein, 1999]. The basic observation any triggering model must predict is that the change in seismicity rate caused by a triggering event lasts for a finite duration. This is so ubiquitous for aftershock sequences that it has been referred to as Omori's law, in which the number of earthquakes following a main shock decays approximately inversely proportional to time from the main shock [Utsu et al., 1995]. Remotely triggered sequences of finite duration also have been documented [Brodsky et al., 2000; Gomberg and Davis, 1996; Gomberg et al., 2001a, 2001b; Hill et al., 1993, 1995; Hough, 2001; Mohamad et al., 2000; Power et al., 2001; Protti et al., 1995; Singh et al., 1998]. In this context, remote refers to distances exceeding several times the length of the triggering fault.

I suggest that triggering by dynamic stresses, usually associated with transient oscillatory seismic waves, may occur at any distance and in a wide variety of settings. If true, an understanding of how dynamic triggering works should advance our understanding of earthquake initiation generally. Traditionally, aftershocks have been attributed to static stress changes associated with the final slip distribution, and remotely triggered sequences have been attributed to dynamic stresses. I propose that dynamic triggering must also be important at short distances. The first reason for this is that a fault affected by a stress perturbation does not "know" where the perturbation originated but only that the stress affecting it has been perturbed. The distance to the source of the stress perturbation therefore does not seem to be relevant. Second, the association of dynamic triggering with remote seismicity increases in part simply reflects the difficulty of observing it unambiguously at shorter distances, rather than any underlying physical process. At shorter distances, other changes, such as static stress or pore pressure changes, may be nearly as large. Additional observational bias leading to the perception that the correlation of rate changes with static stress changes at close distances is more significant than with dynamic stress changes may simply arise from the greater number of studies of static stress changes, owing to the relative simplicity of computing static deformations. Albeit few, dynamic triggering at aftershock distances has been inferred for a handful of earthquakes [Kilb et al., 2000; Gomberg et al., 2001a; Henry et al., 2000]. Finally, dynamic triggering does not seem to require a specialized environment. Although dynamic triggering appears to be more common in regions of geothermal activity, there are a growing number of observations in noneothermal areas, including the aforementioned ones at aftershock distances. Examples of dynamic triggering observations in noneothermal areas at remote distances include seismicity increases throughout Greece following the 1999 Izmit, Turkey, earthquake [Brodsky et al., 2000]; near Indio, California, after the 1999 Hector Mine, California, earthquake [Gomberg et al., 2001b]; at several sites in the western United States following the 1992 Landers, California, earthquake [Hill et al., 1993]; in Ohio after the 1811-1812 New Madrid earthquakes in the central United States;
When the triggering stress change is dynamic, many commonly invoked models of earthquake failure predict seismicity rate increases with a duration no longer than that of the transient (dynamic) stress change. In other words, garden-variety failure mechanisms with dynamic stress changes fail to explain a very fundamental observation. I refer to this class of failure mechanisms as "accelerating failure" mechanisms. These have the feature that under constant rate tectonic loading, the quantity characterizing failure accelerates non-linearly. Accelerating failure is a property of rate state frictional models in which the fault slip increases non-linearly until the slip velocity exceeds some threshold value (see Dieterich and Kilgore, 1996, for a summary). Sub-critical crack growth models also fail in this class because under constant rate loading the crack length accelerates until its growth rate reaches some critical value at which failure occurs [Atkinson, 1979; Das and Scholz, 1981]. In these types of models the quantity characterizing failure eventually will accelerate toward its failure threshold even if the loading is removed. Coulomb failure models have simpler failure criteria. The fault strength, or stress at which failure occurs, remains constant regardless of the applied loading (Figure 1). The notion of a constant failure stress threshold must be set aside to understand the behavior of accelerating failure models.

One of the few quantities relevant to the failure process that can be directly measured is the change in the rate of failure within a population of faults or, equivalently, the seismicity rate change. To understand the relevance, first the timing of failure for a single fault needs to be understood. In the absence of any stress changes associated with other earthquakes, stress is applied to a fault at constant rate due to plate tectonic motions. This tectonic loading ultimately leads to failure, after which the fault heals and the loading process repeats. I refer to the time between failures on a single fault as the cycle time $t_{cycle}$. The stress changes associated with another earthquake may perturb the tectonic loading and cause failure to occur earlier (or later) than it would have in the absence of the perturbation. The change in the cycle time is often referred to as clock advance $\Delta t$ (or clock delay).

In a general sense, an increase in failure rate due to a stress change can be thought of as resulting from clock-advanced failure on a collection of faults, or in other words, from a group of faults failing sooner than they would have in the absence of the stress perturbation. In simple Coulomb models, $\Delta t$ depends only on the stress change and the tectonic loading rate and is independent of the loading history prior to the time of the stress change, which I denote as time $t_0$ (Figure 1). In accelerating failure models, $\Delta t$ depends on $t_0$. To cause a change in the rate of failure with specific characteristics requires a specific dependence of $\Delta t$ on $t_0$ and some assumptions about the population of affected faults (see below). This implies that seismicity rate changes predicted for different failure models and stress changes will differ and thus provides an opportunity to test the models against observed rate changes.

Stress changes may trigger, or clock advance failure, in two ways: by permanently incrementing the tectonic load acting on a fault and by altering properties of the fault and/or its environment. Static stress changes can do both. Because dynamic stress changes are transient they can only do the latter (Figure 1). It has long been recognized that the stress changes caused by a triggering main shock must alter the triggered faults, or their surroundings, so that they do not all fail immediately but in a sequence with finite duration. Models that describe this process as a consequence of static stress changes have been developed [see Moreno et al., 2001]. Perhaps the most popular model invokes rate state frictional relations in which an evolving "state" variable represents alteration of contacts points across the fault surface [Dieterich, 1994]. This alteration of the fault surface clock advances the failure times of a population of faults in a specific way so that the failure rate is increased for a finite duration. Herein I show that not only Dieterich's model but also other accelerating failure models explain static stress triggering of aftershocks but cannot explain dynamic triggering. Thus either we must conclude that the physical processes underlying static and dynamic triggering differ, or we must rethink what we believe we understand about static triggering.

2. Heuristic Model

The affect of a stress change on seismicity rate may be understood by first considering the response of a single fault under tectonic loading and the effect of a perturbing stress on
Crack Length, $x$, vs. Time in Cycle

Crack Growth Rate, $dx/dt$, vs. Time in Cycle

**Figure 2.** Time histories of the length and its rate of change, or growth rate, of a crack evolving according to the theory of subcritical crack growth (equation (1)). The response to three loading histories are shown; tectonic loading alone (shaded curves) for which $t_{cycle} = 108.7$ years, a dynamic perturbation added earlier ($t_0 = 90$ years is chosen arbitrarily) to the tectonic load (thin solid curves), and the same perturbation at later time ($t_0 = 100$ years, thick solid curves). Comparison of $\Delta t$ for the two perturbations shows that $\Delta t$ increases as $t_0$ increases. See text for an explanation of why this happens. Model parameters are defined in the text, with specific values selected to be appropriate to earthquakes; these include $k_0 = 91$ bar cm$^{1/2}$, $V_0 = 0.415 \times 10^{-3}$ cm/yr, $C = 1.286$, and $n = 20$ [Das and Scholz, 1981]; $d\tau/dt = 0.25$ bar/yr is from Gross and Burgmann [1998]. I chose an initial crack length and stress of $X_0 = 10$ cm and $\tau'_{init} = 0$, respectively, stress change amplitudes $\Delta\tau = 5$ bars, and $t_w = 50$ days simply to produce a response large enough to be easily visible.

In accelerating failure models the fault is moving toward failure at a faster rate later in its cycle (greater values of $t_0$), so that an equal transient perturbation applied later advances it farther toward failure, i.e., causes a bigger $\Delta t$, than an earlier one. I illustrate this with an example calculated for a relatively simple specific accelerating failure model, subcritical crack growth. In many materials found in Earth, cracks will propagate quasi-statically at a rate described by a single equation, written

$$dx/dt = (C/k_0)\tau' x^n,$$

where $x$ is the crack length, $C$ is a geometric factor, and $k_0$ and $V_0$ are material properties determining the minimum stress intensity factor and rate of crack growth below which no growth occurs, respectively [Atkinson, 1979; Das and Scholz, 1981, and references therein]. The stress corrosion index $n$
Figure 3. Equivalent time histories of the length and growth rate of a crack as in Figure 2, except that the stress perturbations are static. Comparison of $\Delta t$ for the two perturbations shows that $\Delta t$ decreases as $t_0$ increases (see text for explanation).

typically exceeds 10. Clearly, from (1) the longer the crack becomes, or the greater the applied stress $\tau$, the faster the crack grows.

Figure 2 shows that when the stress perturbation is dynamic, clock advances increase as $t_0$ increases. Time histories of a crack growing under tectonic loading alone and two additional cases in which equal dynamic perturbations are added at different times in its cycle are plotted. For simplicity, I use a dynamic stress change described by a boxcar time function with amplitude $\Delta \tau$ and width $t_w$. The perturbation causes the crack growth rate $dx/dt$ (equation (1)) and thus the length $x$ both to increment rapidly at $t_0$ and grow until the perturbation ceases at $t_0 + t_w$. Because the crack length has grown during the perturbation and $dx/dt$ is proportional to $x^{n/2}$, $dx/dt$ is greater after the perturbation than it would have been in its absence, and failure is clock-advanced (Figure 2). However, the increment in $x$ is smaller for an earlier perturbation because $dx/dt$ is smaller earlier in the cycle, and the change in $dx/dt$ is smaller. Thus the clock advance is smaller too.

Figure 3 shows why the clock advances decrease as $t_0$ increases for a static stress change. Figure 3 is equivalent to Figure 2 except that the stress perturbations are static, represented by step time functions with amplitude $\Delta \tau$. As in the dynamic case, the increment in $\tau$ causes a rapid increment in the crack length and in $dx/dt$. However, because the load change is permanent, $dx/dt$ remains higher and the crack continues growing at a faster rate than had no perturbation occurred. For the earlier perturbation the incremented $dx/dt$ is smaller than for the later perturbation (because $x$ is smaller), and it takes more time to reach failure. However, relative to the rate with no perturbation, the crack perturbed earlier spends a greater fraction of its total time to failure at an elevated rate and thus experiences a greater change in its failure time or equivalently, a greater clock advance.

Although the examples shown in Figures 2 and 3 are for specific cases, the opposite dependencies of $\Delta t$ on $t_0$ for static and dynamic stress perturbations applies to all accelerating failure models. This may be easily understood recognizing that a static stress change can be considered as a sum of transient boxcar perturbations. Because a fault is closer to and is moving toward failure a faster rate later in its cycle, an equal transient perturbation applied later is more effective at advancing the time of failure. In other words, $\Delta t$ increases with increasing $t_0$. Figure 4a illustrates schematically this dependence of $\Delta t$ on $t_0$ for two fault stress histories, which include tectonic loading with boxcar dynamic stress perturbations. Figure 4b shows the
analogous stress histories for static stress perturbations. An earlier applied static perturbation differs from a later one only by having an additional boxcar transient added at the beginning. Clearly, this extra perturbation should lead to additional clock advance, or $\Delta t$ increasing with decreasing $t_p$.

How then does clock advance affect seismicity rate? Consider first some population of identical faults that fail under tectonic loading alone at rate $r_0$. Note that for this to occur the faults must be at different points in their loading cycles at any given time (Figures 5a and 5b). When perturbed by a static stress change, faults that are early in their cycles are clock-advanced more than those that are late in their cycle, resulting in an increased failure rate (i.e., a sort of shoveling of failure times toward the failure end; Figure 5a). As Figure 4 illustrates, the opposite occurs as a result of a dynamic perturbation, leading to a decrease in failure rate (Figure 5b). The exception to this is when failure occurs immediately during the transient, and because the system does not yet “know” that the perturbation is transient, it responds as predicted for a static stress change. Thus the rate increases only for the duration of the transient and it then decreases. Because the rate change is determined only by the type of stress perturbation, these results do not depend on assumptions about the fault population; that is, although identical faults are considered in the example of Figure 5, the same rate change behavior results for a more complex population.

This heuristic model and interpretation can be quantified by again considering first a population of faults with identical properties that fail at rate $r_0$ under tectonic loading alone. Gomberg et al. [2000] show that the change in seismicity rate due to a stress perturbation may be estimated from

$$r(t)/r_0 = 1 - dt_0/dt, \quad (2)$$

in which $t_0$ is the perturbed time of failure marked from the start of each triggered fault’s loading cycle and $t$ is the time that has passed since the triggering perturbation onset at $t_0$ (e.g., a main shock origin time) or, equivalently, $t$ is the delay from immediate failure (Figure 6a). As Figures 5 and 6b illustrate, the change of $t_p$ with $t$ (i.e., the slope, or $dt_0/dt$ in equation (2)) is a measure of the change in failure or seismicity rate. To see how this relates to the relationship between clock advance and the loading history prior to the perturbing stress, equation (2) may be rewritten in terms of $t_0$ instead of the delay time $t$. Noting that $t = t_p - t_0$, equation (2) becomes

$$r(t)/r_0 = [1 - dt_0/dt_0]^{-1}, \quad (3a)$$

or in terms of clock advance, $\Delta t = t_{cycle} - t_p$.

$$r(t)/r_0 = [1 + d\Delta t/dt_0]^{-1}. \quad (3b)$$

Equations (3a) and (3b) show explicitly that the change in seismicity rate depends only on how the perturbed failure time, or clock advance, varies with how far along in its cycle a fault is at the time of the perturbation. The dependence is determined primarily by the constitutive properties of the faults when it is very close to failure [Gomberg et al., 2000] and thus the rate change will be essentially the same for a population composed of faults with varying cycle times or initial conditions. Figures 6b and 6c illustrate the relationship between (2) and (3) schematically for the case in which initially the faults of the population are uniformly distributed in their loading cycles (i.e., at any point in time each fault is at a different point in its cycle) such that the initial failure rate is constant. This is the case employed in the widely used model of Dieterich [1994]. It can be shown that the approximate analytic equations that he derives may obtained from the very general relations described by (2) or (3) [see Gomberg et al., 2000].

3. Examples: Two Specific Models

The generality of (2) and (3) allows the seismicity rate change to be computed numerically for a variety of physical models of failure. I demonstrate this and the validity of the above heuristic model using two examples, a rate state frictional model and a subcritical crack growth model. In both cases I assume for simplicity the same fault population described in the previous paragraph; for this special population, calculations for a single fault can be used directly to compute seismicity rate changes. It also permits direct comparison with the Dieterich [1994] model results. For a single fault the perturbed failure time $t_p$, or equivalently $\Delta t$, is calculated as a function of $t_0$. Schematically, this corresponds to the stressing history of a suite of identical tectonically loaded faults with perturbations added at different values of $t_0$. Figure 6c shows this for a static stress change.). Figure 7a plots $t_p(t_0)$ graphically for rate state frictional model calculations.
I will now review how to transform a calculation of $t_\rho(t_0)$ for a single fault to one appropriate for calculation of the failure rate of a population of faults [Gomberg et al., 2000]. First, note that it is the delay $t$ from immediate failure at origin time of the triggering event, $t_0$, that gives rise to a triggered sequence of finite duration. Thus a 45° line defines immediate failure in Figure 7a (the case when $t_\rho = t_0$), and the delay time $t = t_\rho - t_0$ is simply the difference between this line and the computed failure curve $t_\rho$. Just as $t = t_\rho - t_0$ is rescaled to $t_0$ to obtain (3) from (2), the failure curve $t_\rho(t_0)$ can be replotted as a function of $t$ instead of $t_0$ (Figures 7a and 7b). In terms of the heuristic model, this corresponds to shifting the stress histories of the faults in Figure 6c so that failure occurs at some finite constant rate, $r_{\alpha}$ and all faults are perturbed at the same $t_0$ (as in Figure 6b). Figure 7b now corresponds to a system appropriate for a triggering event clock-advancing failure on a suite of faults (e.g., a main shock/aftershock sequence). The corresponding rate change can be computed by taking the derivative of $t_\rho(t)$ or of $t_\rho(t_0)$ as proscribed by (2) or (3), respectively. The resulting rate change is plotted as a function of $t$. If the stress perturbation is static the clock advances cause a rate increase that decays as observed in aftershock sequences (Figure 7c). Notably, however, if the perturbation is dynamic (transient), the rate increases only for a time equal to the duration of the transient (Figure 7c). One could have anticipated this result from Figures 5b and 7b and equation (2), noting that for the transient the slope $dt_\rho/dt$ changes sign once $t$ exceeds the duration of the transient.
Figure 6. (a) (left) Schematic time histories of stresses applied to a single fault for a single cycle. Under tectonic loading alone, failure occurs after $t_{\text{cycle}}$ from the previous earthquake (dark line). In relation to the start of the cycle when a static stress step is added at $t_0$ (e.g., a main shock origin time), failure occurs at time $t_p$. The parameter $t$ is the time that has passed since $t_0$ or, equivalently, is the delay from immediate failure or the time to failure from a main shock at $t_0$. (b) Figure 5a repeated, as a reminder that the slope of the line marking failure (left-leaning dashed line) is a measure of the failure rate. This line represents $t_p$ and illustrates equation (2) schematically; that is, the change in seismicity rate is proportional to the slope of the failure line or $dt_p/dt$. (c) The same population of faults is shown as in Figure 6b except that the timescale has been shifted so that the cycles of all the faults start at the same time. This shows how calculations for a single fault can be used to calculate the rate change for a population of faults. Instead of calculating the change in failure times on a population of faults distributed throughout their cycles due to a triggering event at $t_0$, failure times are calculated for a single fault for values of $t_0$ distributed from zero to $t_{\text{cycle}}$. The equivalence of populations in Figures 6b and 6c, differing only by a known change of timescale, $t = t_p - t_0$, shows the equivalence of equations (2) and (3).

To illustrate the generality of this result, equivalent calculations are performed for a subcritical crack growth model (Figure 8). Again, calculations are performed for a single crack, and the time axis is rescaled to obtain results appropriate for a population. For this population an analytic expression for the rate changes may be easily derived for static and dynamic stress perturbations described as step and boxcar functions, respectively (Appendix A). Notably, the crack growth model predicts rate changes indistinguishable from those of a rate state frictional model. Perhaps this should not be surprising as a close connection between crack growth and frictional sliding processes has been suggested previously on the basis of laboratory studies [see Lockner, 1998, and references therein].

4. Conclusions

In summary, simple heuristic models suggest that dynamic triggering cannot be explained by an entire class of commonly
Figure 7. Changes in failure times and rates calculated numerically for (left) static and (right) dynamic stress perturbations added to tectonic loading for a fault modeled as a spring-slider system obeying rate state frictional laws. (a) Graph of the variation is shown of perturbed failure times ($t_f$, solid circles) due to stress perturbations applied at different times in the fault’s cycle, or $t_p(t_0)$. The prediction of the approximate analytic formula of Dieterich [1994] agrees with the complete solution for static perturbations applied late in the cycle (where the approximation is valid). Failure times along the 45° dashed line would imply instantaneous failure ($t_f = t_0$), and delays from this equal $t = t_f - t_0$. This delay is equivalent to the time to failure from a main shock at $t_0$ and gives rise to aftershock sequences of finite duration. For the dynamic case (right), these delays are too small to be visible at this scale, and fluctuations about $t_f = t_{cycle}$ at early values of $t_0$ are due to numerical error. (b) The same failure curves are shown as in Figure 7a but are plotted as a function of $t = t_f - t_0$ or $t_f(t)$. (c) Seismicity rate change calculated numerically according to equation (2) is shown from the results plotted in Figures 7a and 7b. Notably, the transient only perturbs the rate for a duration as long as the transient itself (right). In all calculations for the static case, model parameters are identical to those of Gomberg et al. [2000, Table 1] for the slowness state evolution law and $a = 0.01$. The same parameters are used for the dynamic calculations and a transient duration of 10,000 s. This is much longer than expected for seismic wave transients and is used simply so that the duration of the rate increase is clearly visible on this plot; the effect of shortening the transient duration would simply be to shorten the duration of the rate increase.
Figure 8. The analog to Figure 7, except for a crack undergoing subcritical crack growth. Failure times and rates can be computed analytically according to formulae in Appendix A. Model parameters are identical to those in Figure 2. The predicted rate change in Figure 8c also can be fit with an Omori law decay function.
invoked earthquake failure models. These “accelerating failure” models share the feature that under tectonic loading, the quantity characterizing failure (e.g., fault slip or crack length) accelerates nonlinearly with time at the rate at which it grows exceeds some critical value. These heuristic models have been illustrated for a few specific cases using analytic and numerical calculations. Some of the failure models belonging to this class have been used to explain static stress triggering of aftershocks [Dieterich, 1994; Gross and Kisslinger, 1997; Gross and Burgmann, 1998; Toda et al., 1998]. If dynamic triggering does not require highly specialized conditions, then we must either conclude that the physical processes underlying dynamic triggering differs or rethink what we believe we understand about static triggering. If the former is the case, we might appeal to physical mechanisms relying on oscillatory deformations. Candidate mechanisms might include cyclic fatigue, increased pore pressures in saturated fault gouge as in liquefaction [Beeler et al., 2001], or rectified diffusion [Sturtevant et al., 1996]. Even if the mechanisms differ, then we must wonder why models of static triggering, which neglect additional dynamic triggering mechanisms, appear to explain the observations. If the physical processes underlying static and dynamic triggering are the same, we need to reexamine the models of static triggering that rely on accelerating failure processes. Instead or additionally, perhaps aftershocks and remotely triggered events are not inevitable clock-advanced events but rather failures that would not have happened otherwise.

Appendix A: Seismicity Rate Change for Subcritical Crack Growth

In many materials found in Earth, cracks will propagate quasi-statically in a manner described by a single equation:

\[ dx/dt = (C/k_0)x^{n/2}, \]  

(A1)

in which \( x \) is the crack length, \( C \) is a geometric factor, and \( k_0 \) and \( V_0 \) are material properties determining the minimum stress intensity factor and rate of crack growth below which no growth occurs, respectively [Atkinson, 1979; Das and Scholz, 1981, and references therein]. The stress-corrosion index, \( n \), typically exceeds 10 or more. To model tectonic loading at constant rate, \( d\tau/dt \), with some perturbation applied at time \( t_o \), I assume that for a static stress perturbation the shear stress \( \tau \) evolves

\[ \tau(t) = (d\tau/dt)t + \Delta\tau H(t - t_o), \]  

(A2)

in which \( H(t) \) represents a step function with amplitude \( \Delta\tau \). For a transient, for simplicity, I assume a boxcar time function of width \( t_w \), so that the total shear stress equals

\[ \tau(t) = (d\tau/dt)t + \Delta\tau t \quad t < t_0, \]  

\[ \tau(t) = (d\tau/dt)t + \Delta\tau t_0 \leq t \leq t_0 + t_w. \]  

(A3)

Expressions for the perturbed failure time \( t_p \), and its rate of change with respect to \( t_o \), are derived by substituting (A2) or (A3) into (A1) and defining \( t_p \) as the time at which the velocity \( dx/dt \) becomes infinite. This can then be used to compute the seismicity rate change according to (3a). While some lower velocity would be more accurate, the acceleration near the end of the cycle becomes so rapid that the difference is insignificant. The expressions for \( t_p \) and \( dt_p/dt_o \) are most simply written by first defining the failure time under tectonic loading alone as \( t_o \), a “Coulomb” clock advance as \( \Delta t_o = \Delta\tau (d\tau/dt)^{-1} \), a similarly normalized initial stress as \( \tau_{init} = \tau_{init}(d\tau/dt)^{-1} \), and \( N = n + 1 \). In the absence of any perturbing stress change the failure time equals

\[ t_o = (2(n + 1)/(n - 2))(k_0/(Cdr/dt))^N \cdot V_0 \cdot (n - 2) + \tau_{init} \]  

(A4)

When at static stress step is added at \( t_o \), the perturbed failure time is

\[ t_p = [(t_o + \tau_{init}) - (t_0 + \tau_{init}) + (t_0 + \Delta t_o + \tau_{init})^N - (\tau_{init} + \Delta t_o)]. \]  

(A5a)

When at transient boxcar stress change of duration \( t_w \) is added at \( t_o \), the perturbed failure time is

\[ t_p = [(t_o + \tau_{init}) - (t_0 + \tau_{init}) + (t_0 + \Delta t_o + \tau_{init})^N - (\tau_{init} + \Delta t_o) + (t_0 + \Delta t_o + \tau_{init})^{N - N} - \tau_{init}]. \]  

(A5b)

The derivative \( dt_p/dt_o \) in the expression for the seismicity rate change (equation (3a)) is simply derived from (A5), yielding

\[ dt_p/dt_o = \left[-(t_0 + \tau_{init})^N + (t_0 + \Delta t_o + \tau_{init})^N \right] \left[(t_0 + \tau_{init}) - (t_0 + \Delta t_o + \tau_{init})^{N - N} - (\tau_{init} + \Delta t_o) \right] \]  

(A6a)

for the static stress change and

\[ dt_p/dt_o = \left[-(t_0 + \tau_{init})^N + (t_0 + \Delta t_o + \tau_{init})^N - (t_0 + \Delta t_o + \tau_{init})^N \right] \left[(t_0 + \tau_{init})^N - (t_0 + \Delta t_o + \tau_{init})^N \right] \left[(t_0 + \tau_{init})^N - (t_0 + \Delta t_o + \tau_{init})^N \right] \]  

(A6b)

for the transient.

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References


Dieterich, J. H., and B. Kilgore, Implications of fault constitutive


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