Single Particle Motion in E and B fields

Everything follows from Lorentz Force:

1st assume time independent fields

- Gyration
- ExB drift: charge and energy independent
- Grad-B and curvature drifts: depend on both charge and energy (explains How the Radiation Belts Work)
- Adiabatic invariants of the motion in a dipole field

Single particle motion in B field

start with Lorentz Force

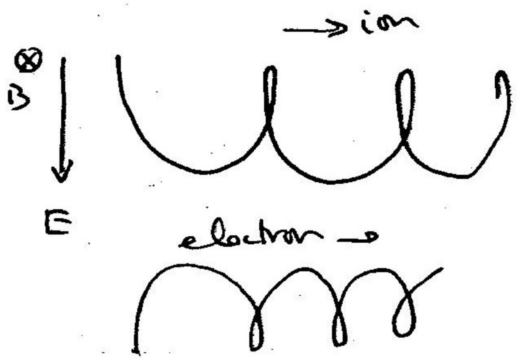
$$m \frac{d\bar{V}}{dt} = q \bar{V} \times \bar{B}$$

Simple harmonic Oscillator

$$\omega_c \equiv \frac{|e|B}{m} \qquad r_L \equiv \frac{V_\perp}{\omega_c} = \frac{m V_\perp}{eB}$$

Larmor radius

Now add $\vec{E}$ field and transform away E



ie $F = 0 = q\bar{E} + \bar{V} \times \bar{B}$

$\Rightarrow \bar{E} = -\bar{V} \times \bar{B}$

gives $\vec{V} = \frac{\bar{E} \times \bar{B}}{B^2}$

Gyration at $\frac{eB}{m}$ + drift at $\vec{V}_\perp = \frac{\bar{E} \times \bar{B}}{B^2}$

in general any force $\bar{F}$ (substitute $\bar{F} = q\bar{E}$)

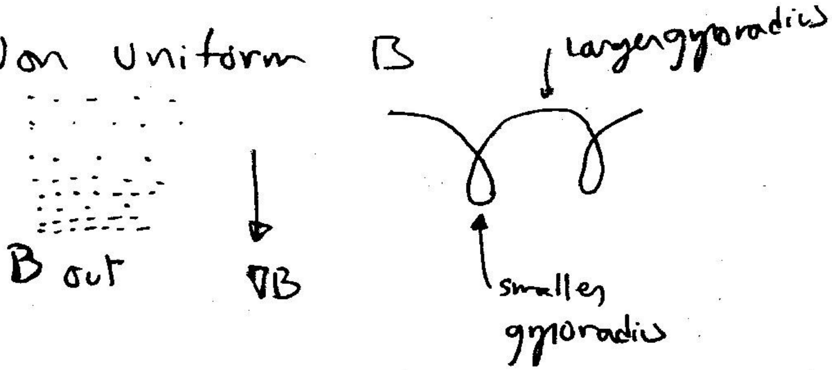
$$\vec{V}_d = \frac{\bar{F} \times \bar{B}}{q B^2}$$

for Force ⊥ B

How the Radiation Belts Work

- Grad B drift = $-\mu \nabla \vec{B} \times \vec{B} / qB^2$
- Force on Parallel Motion
  - Magnetic bottle
  - Pitch angle
  - Bounce Motion
- Longitudinal Drift
- Radiation Belt Organization
  - Loss cone

Non uniform B



Acts like a particle slowing down when it moves into ∇B.

$$\text{Force} = -\mu \vec{\nabla} B \quad \text{force on magnetic moment } \mu$$

$$\mu \equiv \frac{\epsilon_\perp}{B} = \frac{\frac{1}{2} m v_\perp^2}{B} \equiv \text{current} * \text{area}$$

assume $\nabla \propto \frac{1}{L}$ and $L >> r_L$

Guiding center equation

$$\vec{V}_G = v_\parallel \hat{b} + \underbrace{\frac{\vec{E} \times \vec{B}}{B^2}}_{E \times B} - \underbrace{\frac{\mu \vec{\nabla} B \times \vec{B}}{q B^2}}_{\text{Gradient drift}} + \underbrace{\frac{B}{q B^2} m v_\parallel^2 \frac{(\vec{B} \cdot \nabla)\vec{B}}{B^2}}_{\text{curvature drift}}$$

Parallel Motion of Guiding Center

- From Lorentz Force, looking at average motion parallel to B ($v_{\parallel} = \vec{v} \bullet \vec{B}/B$)

we get this equation of motion:

$$\overline{\frac{d}{dt} m v_{\parallel}} = e E_{\parallel} - \mu \frac{\partial B}{\partial \ell} \quad \Bigg| \quad \text{where } \frac{\partial}{\partial \ell} \equiv \frac{\vec{B}}{B} \cdot \vec{\nabla}$$

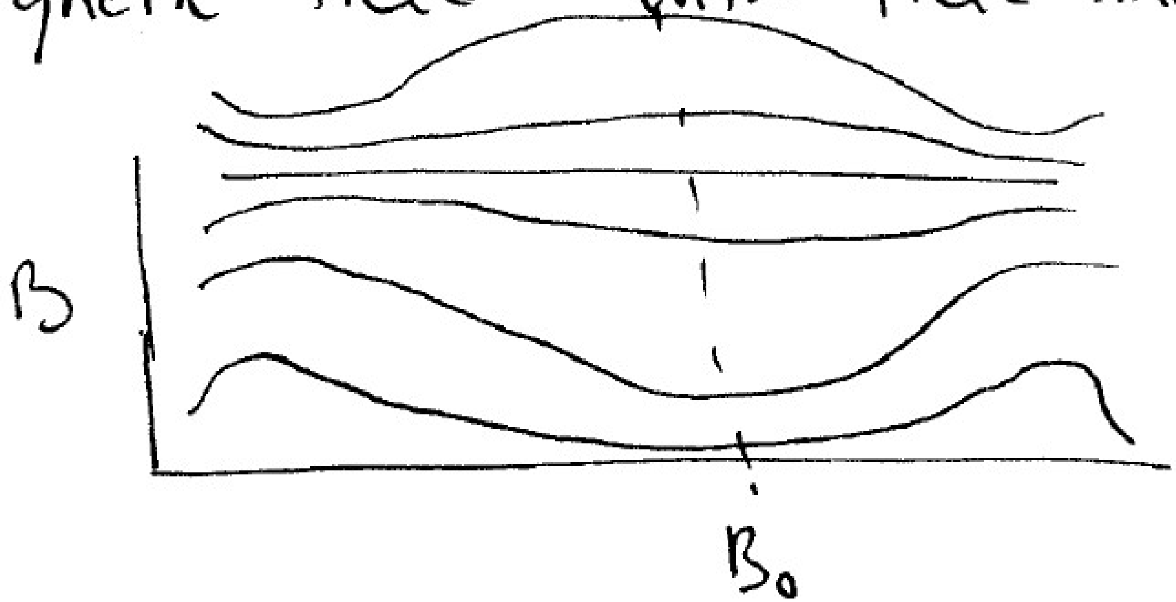
Where $\mu = \frac{1}{2} m v^2_{perp}/B$ = constant when $E_{\parallel} = 0$

(for derivation see http://earthweb.ess.washington.edu/bobholz/ess515/parallel_guiding_center_motion.pdf)

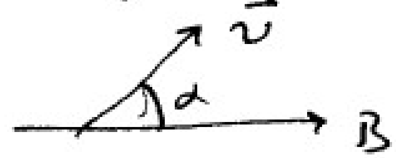
Magnetic Bottle

Magnetic Bottle

Consider a cylindrically symmetric magnetic field with field lines as



Suppose you start a particle on the axis at the place where field strength is B_0.
Suppose the velocity of the particle makes an angle α with respect to the magnetic field.
What Happens?

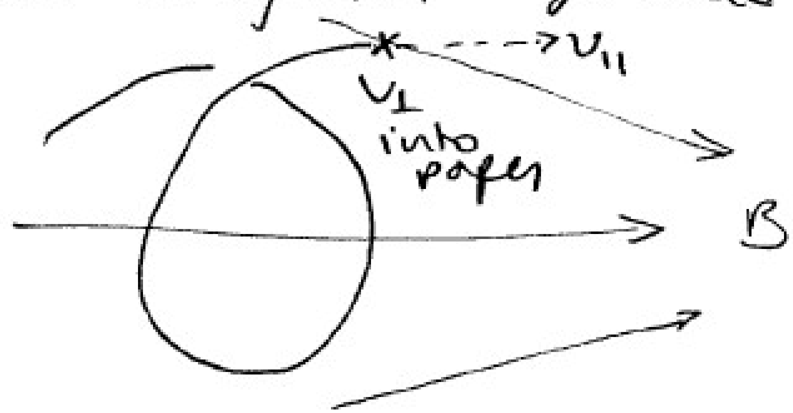


$\alpha \equiv$ pitch angle

$v_{\parallel} = v \cos\alpha$

$v_{\perp} = v \sin\alpha$

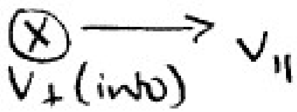
$v_{\parallel}$ carries the particle into a region of larger field strength. Some time later we have

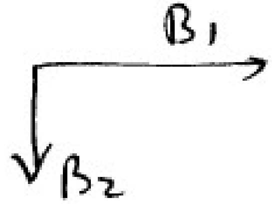


$v_{\parallel}$ to right
$v_{\perp}$ into paper

at the particle $\bar{B}$ has a component B_1 that is parallel to the guiding center field and a component B_2 that is perpendicular.

So we have





consider components of $\bar{v} \times \bar{B}$ (Lorentz Force)

$v_{\parallel} B_2$ causes force that increases $V_{\perp}$

$v_{\perp} B_2$ " " " decreases $V_{\parallel}$

Thus, $V_{\parallel}$ decreases while $V_{\perp}$ increases.
Since $V_{\parallel}^2 + V_{\perp}^2$ = constant (energy conserved), after a while $V_{\parallel}$ will be zero. But $v_{\perp} B_2$ continues to act, so $V_{\parallel}$ changes sign

ie: Particle enters magnetic field a certain distance, stops, turns around and comes out!

This is called <u>Mirroring</u>

We want to know the point where the particle turns around, or the Mirror Point

Equation for $v_\perp$ is $\mu =$ constant

or $$\frac{\frac{1}{2} m v_\perp^2}{B} = \text{const.}$$

if particle moves into region where B is larger, then its $v_\perp$ must get larger too.

Question? How far into the region of increasing magnetic field will a particle penetrate if it starts with pitch angle α_0 at a place where $B = B_0$?

$$\mu = \frac{\frac{1}{2} m v^2 \sin^2\alpha_0}{B_0} = \frac{\frac{1}{2} m v^2 \sin^2\alpha}{B}$$ ($v_\perp = v\sin\alpha$ initially)

as it penetrates, B increases, so does $\sin\alpha$ until, at the mirror point $\alpha = 90°$

and $v_{\parallel} = v\cos\alpha = 0$

$v_\perp = v\sin\alpha = v$ ← all energy in [illegible]

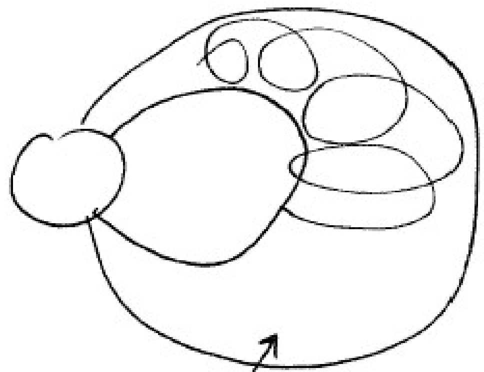
Define B at this point to be B_m

we have $$\frac{\sin^2\alpha_0}{B_0} = \frac{1}{B_m}$$

or $$\boxed{B_m = \frac{B_0}{\sin^2\alpha_0}}$$

All particles of Any energy, charge, mass will mirror at same point B_m if they start with same α_0 !

Another Example of Guiding Center Motion
Particles in a Magnetic Dipole Field
<u>The Radiation Belts</u>



as $v_{\parallel}$ carries particles away from the equator they enter region of increasing B field ∴ mirror and bounce back & forth

Additionally the guiding center drifts around the earth in longitude due to gradient & curvature drifts.

I Gyration

$$\tau_{gyration} \equiv \text{gyroperiod} = \frac{2\pi}{\Omega} = \frac{2\pi m}{Be} \sim$$

10^{-2} sec Protons at low alt

10^{-5} s e^- at low alt

10 sec p at $r = 10 R_e$

10^{-2} s e^- at $r = 10 R_e$

Bounce

bounce time $\equiv \tau_{bounce} = \oint \frac{ds}{v_{\parallel}}$

$$ds = \left(dr^2 + r^2 d\lambda^2\right)^{1/2} = r_e \cos\lambda \left(1 + 3\sin^2\lambda\right)^{1/2} d\lambda$$

$$v_{\parallel} = \left(v^2 - v_{\perp}^2\right)^{1/2}$$

but $\frac{v_{\perp}^2(r,\lambda)}{B(r,\lambda)} = \frac{v^2 \sin^2\alpha_e}{B_e} = \text{const}$

where α_e and B_e are equatorial pitch angle and field strength

so $v_{\parallel} = v\left(1 - \frac{\sin^2\alpha_e B(r,\lambda)}{B_e}\right)^{1/2}$

$$B(r,\lambda) = \frac{M}{r^3}\left(1 + 3\sin^2\lambda\right)^{1/2} \quad \text{(Dipole field)}$$

$$= \frac{B_e}{\cos^6\lambda}\left(1 + 3\sin^2\lambda\right)^{1/2} \quad \text{(because } r = r_e\cos^2\lambda \text{)}$$

field line equation

numerically integrating

$$\tau_{bounce} \sim 4\frac{r_e}{v} T(\alpha_e)$$

where $T(\alpha_e) = 1.3 - .56 \sin\alpha_e$
(not a strong dependence on α_e)

For $v = c$ and $T(\alpha) = 1$

r_e	1.5 Re	3 Re	6 Re
τ_{Bounce}	.13 s	.26 s	.52 s

III Longitudinal Drift

for $J = 0 = \nabla \times B$

$$V_G = \frac{\overline{B \times \nabla B}}{e B^3} \left(\mathcal{E}_\perp + 2 \mathcal{E}_\parallel \right)$$

integrate around drift path of 360° Longitude

gives $\tau_{drift} \cong \dfrac{44}{r_e * \mathcal{E}(\text{meV})}$ minutes

← in earth radii

That is, $r_e \equiv L R_e \equiv L$ earth radii

for $\mathcal{E} = 0.1$ MeV and $r_e = 10 R_e$

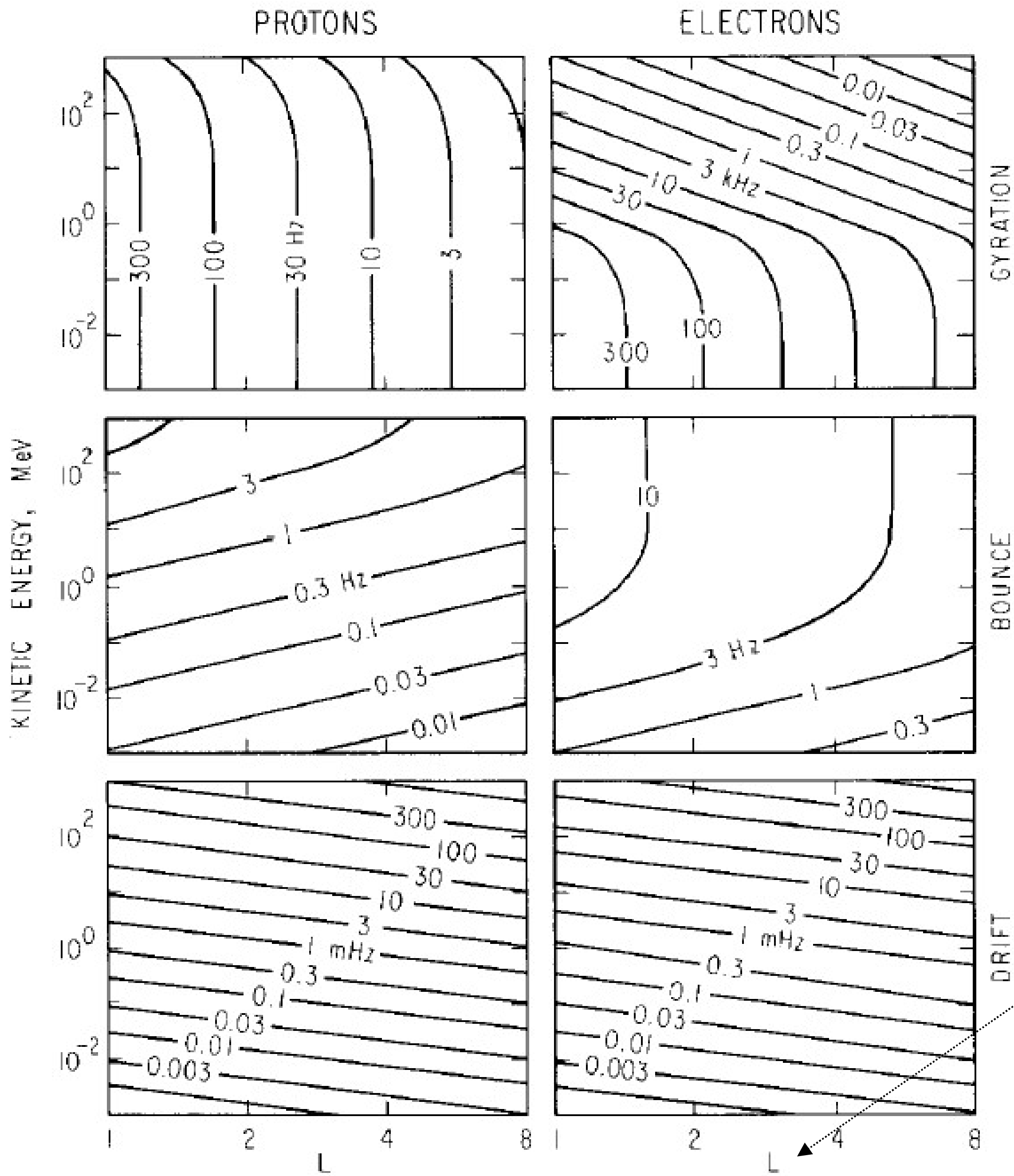
$\tau_{drift} \cong 44$ Minutes

So, in general

$$\tau_{drift} >> \tau_{bounce} >> \tau_{gyro}$$

Neglecting scattering and plasma instabilities particles can be trapped forever

in practice some species at some energies are trapped for 100 yrs!



L is the place the dipole field line crosses the equator, in units of Earth Radii

Fig. 6. Contours of constant adiabatic gyration, bounce, and drift frequency for equatorially mirroring particles in a dipole field. Adiabatic approximation

- Continue Bounce motion
- Longitudinal Drift
- Radiation Belt Organization:
  - Shielding layer
  - L-shell
- Field Line Equation: $r = LR_e \cos^2\lambda$
- Loss Cone
- Begin Large Scale Current

Single Particle Motion in dipole field

- $\mu = \frac{1}{2} m v_\perp^2 / B$ = Magnetic Moment =constant gave us gyration and drift.

Now:

- Pitch angle analysis gives us $B_m = B_o/\sin^2\alpha_o$
- Then add: Dipole field

  $B(r,\lambda) = (M/r^3)*(1 + 3\sin^2\lambda)^{1/2}$ (where λ is latitude)
- We will Find :

  Gyro period << Bounce period << Drift Period

Bounce

bounce time $\equiv \tau_{bounce} = \oint \frac{ds}{v_{\parallel}}$

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(because $r = r_e \cos^2\lambda$ field line equation)

numerically integrating

$$\tau_{bounce} \sim 4 \frac{r_e}{v} T(\alpha_e)$$

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Neglecting scattering and plasma instabilities particles can be trapped forever

in practice some species at some energies are trapped for 100 yrs!

to employ semi-empirical models to account for the associated adiabatic effects upon radiation-belt particles.

To sun ←

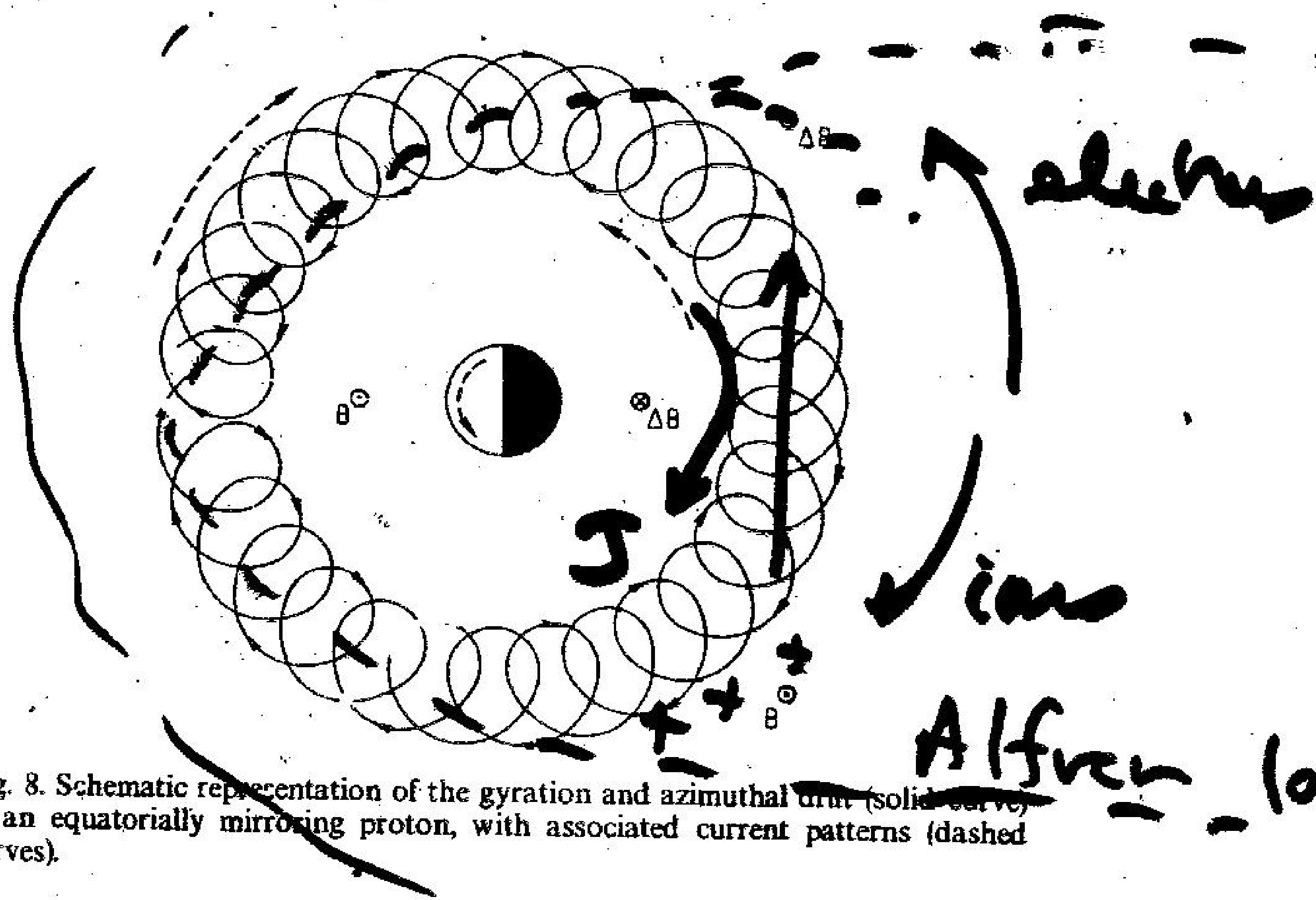

Fig. 8. Schematic representation of the gyration and azimuthal drift (solid curve) of an equatorially mirroring proton, with associated current patterns (dashed curves).

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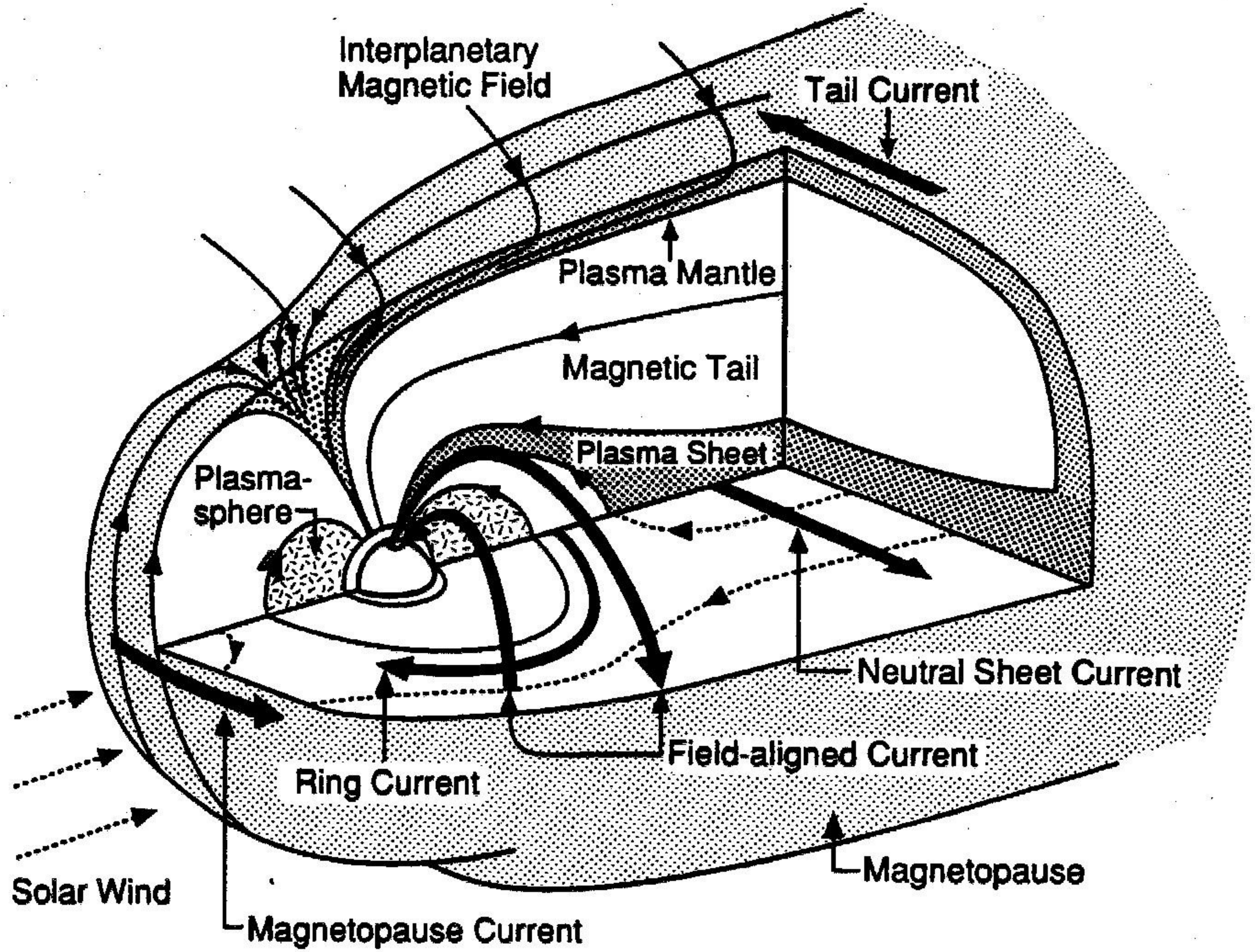
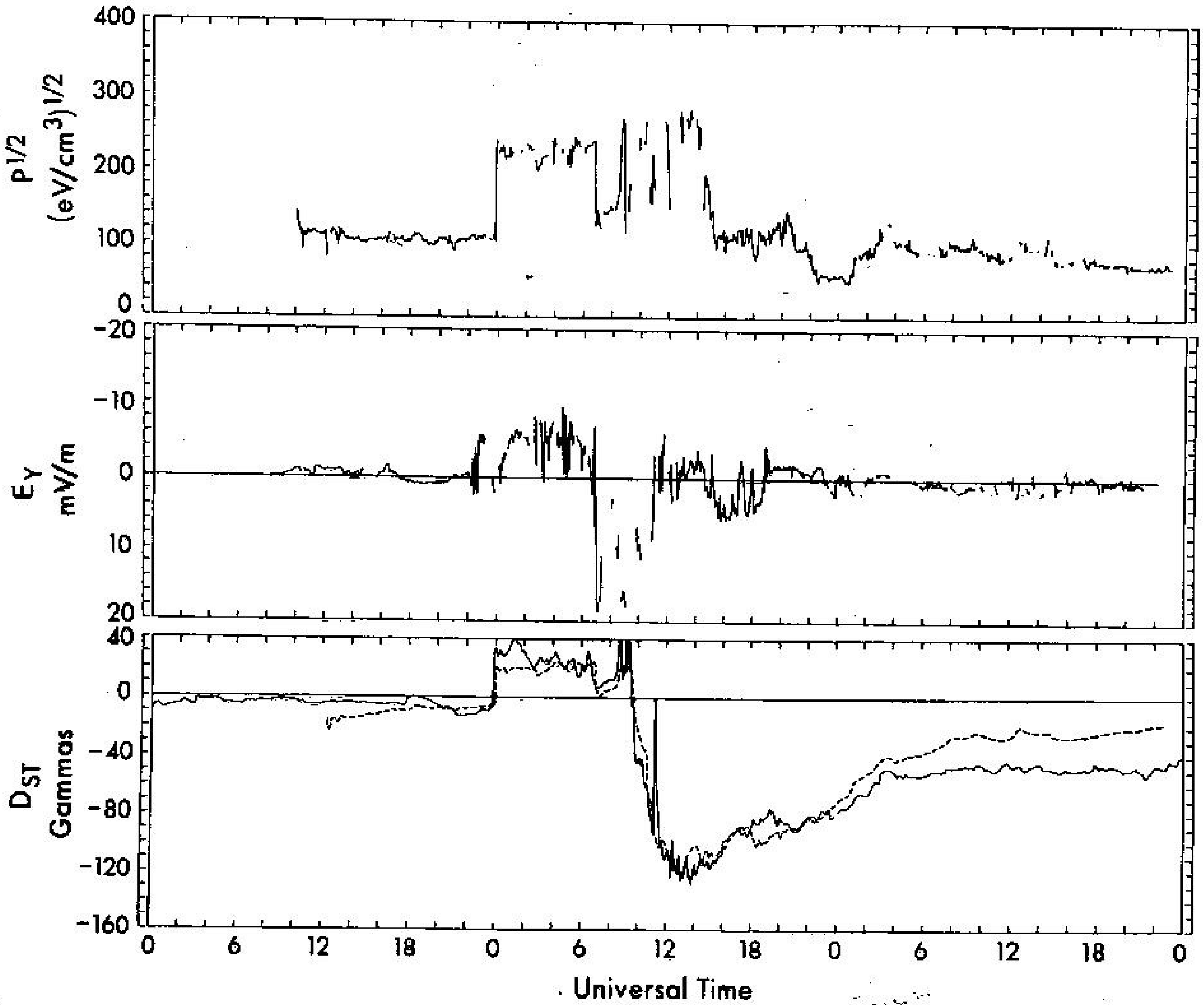
Simplif

V = ExB$_{drift}$ PLUS Grad-B$_{drift}$


Interplanetary
Magnetic Field
Tail Current
Plasma Mantle
Magnetic Tail
Plasma Sheet
Plasma-
sphere
Neutral Sheet Current
Field-aligned Current
Ring Current
Solar Wind
Magnetopause
Magnetopause Current



Equitorial Magnetic field perturbation

Radiation Belt organization

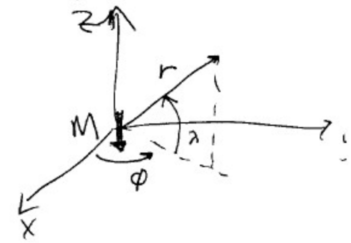
For a dipole B-field with
$\vec{M}$ = dipole moment of the earth

[$\vec{M}(\bar{r},t) = \oint \vec{\mu} f(\bar{r},\bar{v},t)\, d^3v$ = magnetic moment/volume

$\equiv$ Magnetization

For a dipole $\vec{B} = -\vec{\nabla}\psi$ remind: (Parks (3.10))

where $\psi = -\vec{M}\cdot\nabla\left(\frac{1}{r}\right)$ for dipole



in spherical coords:

$B_r = \frac{\partial}{\partial r}\psi = -2M\frac{\sin\lambda}{r^3}$

$B_\phi = \frac{1}{r}\frac{1}{\cos\lambda}\frac{\partial}{\partial\phi}\psi = 0$

$B_\lambda = -\frac{1}{r}\frac{\partial}{\partial\lambda}\psi = M\frac{\cos\lambda}{r^3}$

$|\vec{B}| = \sqrt{B_r^2 + B_\lambda^2 + B_\phi^2} = \frac{M}{r^3}\left(1 + 3\sin^2\lambda\right)^{1/2}$

from Parks 3.30 / 3.31 can write

$\frac{dr}{r} = 2\frac{d(\cos\lambda)}{\cos\lambda}$ integrate to get $\phi = \phi_0$ $r = r_0\cos^2\lambda$

Now Let $L \equiv \frac{r_0}{R_e}$ marks Equatorial distance of a particular Field Line

so $\boxed{r = L R_e \cos^2\lambda}$

over →

or, to put it another way,
the Latitude at the surface is given
by $r = R_e$ so $\cos^2\lambda = \frac{1}{L}$

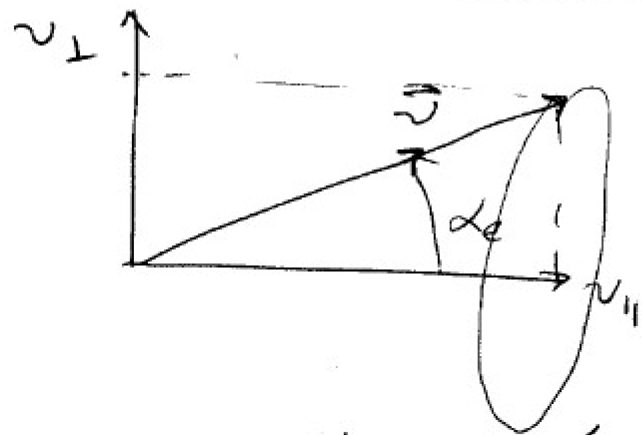
This is definition of <u>Invariant Latitude</u>

L can be defined more carefully for a distorted field.

So, for a given L shell most particles will be undergoing 3 distinct motions — (gyration, bounce + ∇B drift) <u>but</u> if <u>mirror point</u> is $< 1\,R_e$ they will be lost due to scattering from atmosphere.

Actually, the loss altitude $\sim R_e + 100\,\text{km}$ where probability of scattering becomes high

Which Particles are Lost?



$d\Omega$ solid angle

Loss cone

if mirror point is < Re + 100 km, then particle is Lost. Find this α_e

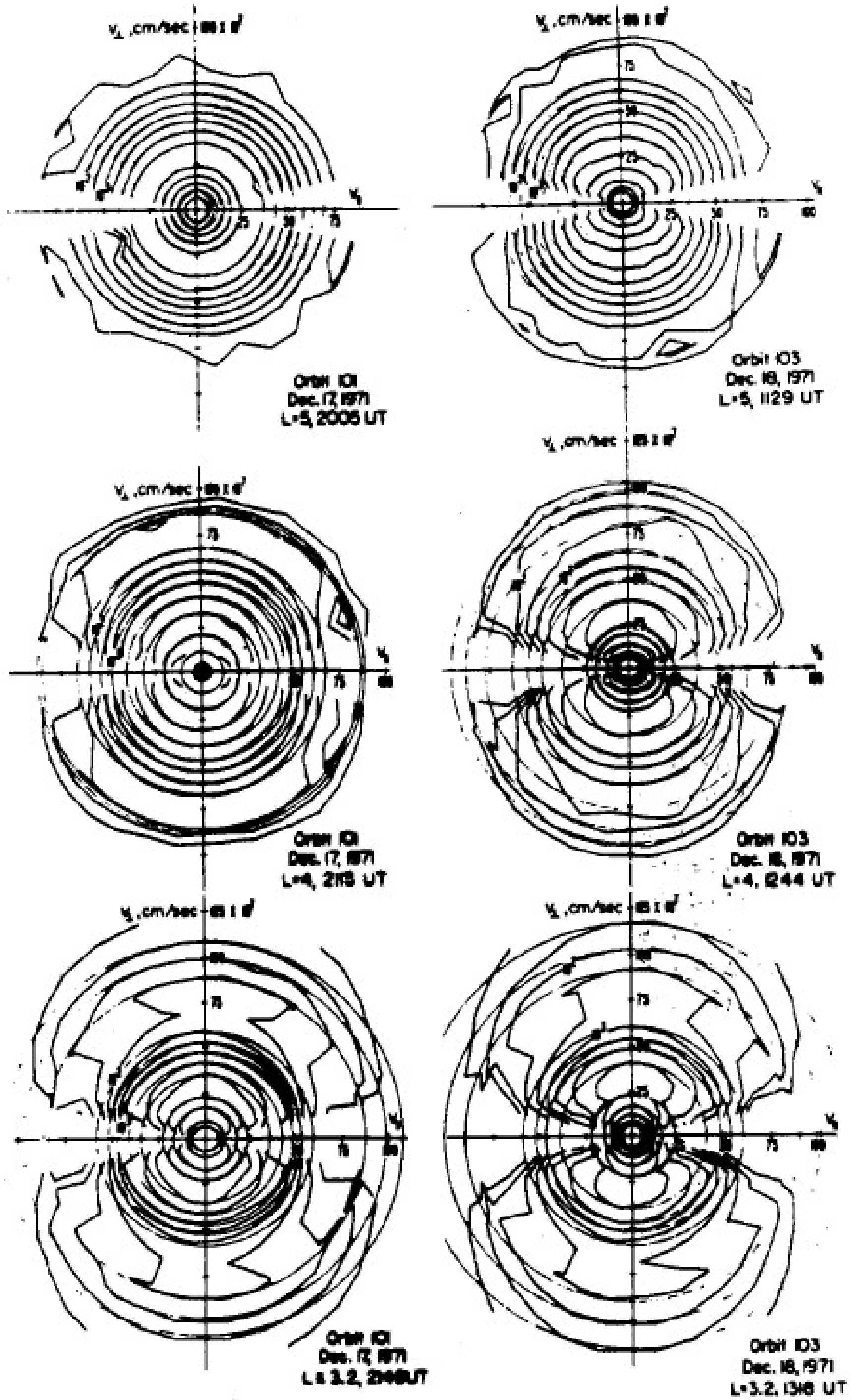
using μ = constant = $\frac{v^2 \sin^2\alpha_e}{B_e}$

find α_e such that $\alpha = 90°$ at B for r = Re + 100 km

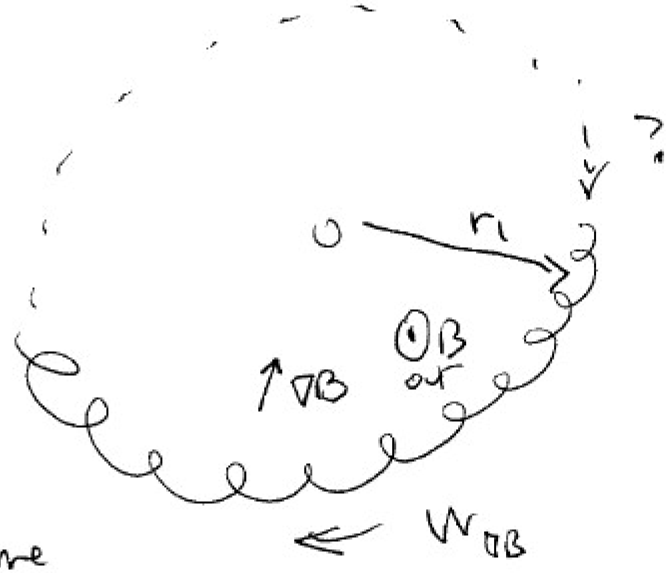
we find that $\alpha_e \sim 3°$ at L = 6

so MOST particles are trapped.

show Pitch angle Distributions along field line


v⊥, cm/sec
v∥
Orbit 101
Dec. 17, 1971
L=5, 2005 UT
Orbit 103
Dec. 18, 1971
L=5, 1129 UT
Orbit 101
Dec. 17, 1971
L=4, 2113 UT
Orbit 103
Dec. 18, 1971
L=4, 1244 UT
Orbit 101
Dec. 17, 1971
L≅3.2, 2148UT
Orbit 103
Dec. 18, 1971
L=3.2, 1318 UT

L-shell drift: How can you tell the drift motion always returns guiding center to starting point?



Answer: if energy is conserved then B_1 with μ = constant, if start from r_1 where $B = B_1$

$$\mu = \frac{\mathcal{E}}{B_1} \text{ at } r_1$$

if returned at $r \neq r_1$, $B \neq B_1$

so $\mu \neq$ constant

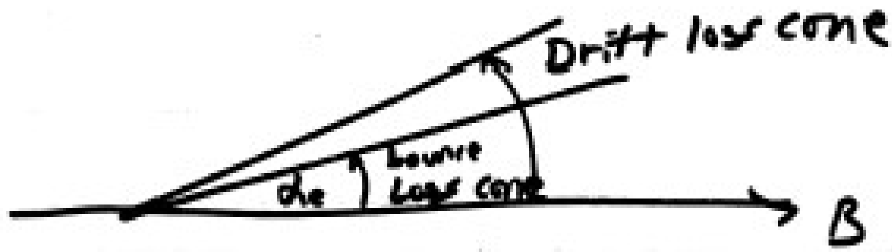
assume equatorially mirroring

then $\mathcal{E} = \mathcal{E}_\perp$ always

L defines a closed shell for perfect dipole

Ah! But the field is not a perfect dipole:

Drift loss cone



Drift loss cone = largest α_e at equator which equals the bounce loss cone at some longitude.

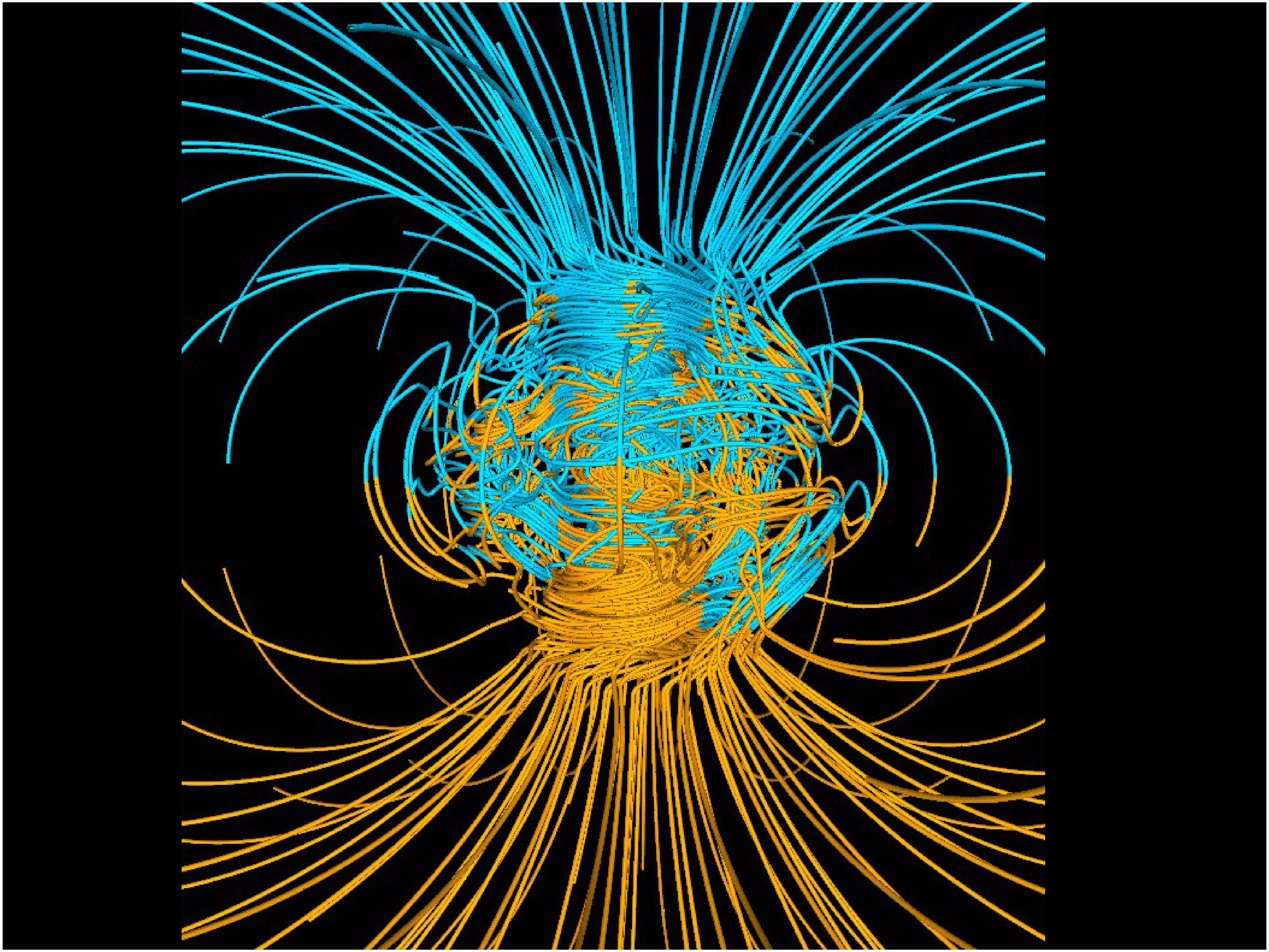
That is: all particles in the bounce loss cone are lost in 1 bounce period

while

Particles in the drift loss cone are lost sometime during their drift period.
Namely, somewhere in drift period they see the Smallest Earth field so their mirror point is lowest.

waves are constantly
affecting particles' pitch angles.

So, there is a random diffusion
of particles from larger to smaller
pitch angles, where particles get lost
inside the loss cone, or drift
loss cone.



Contours of constant Magnetic field Strength

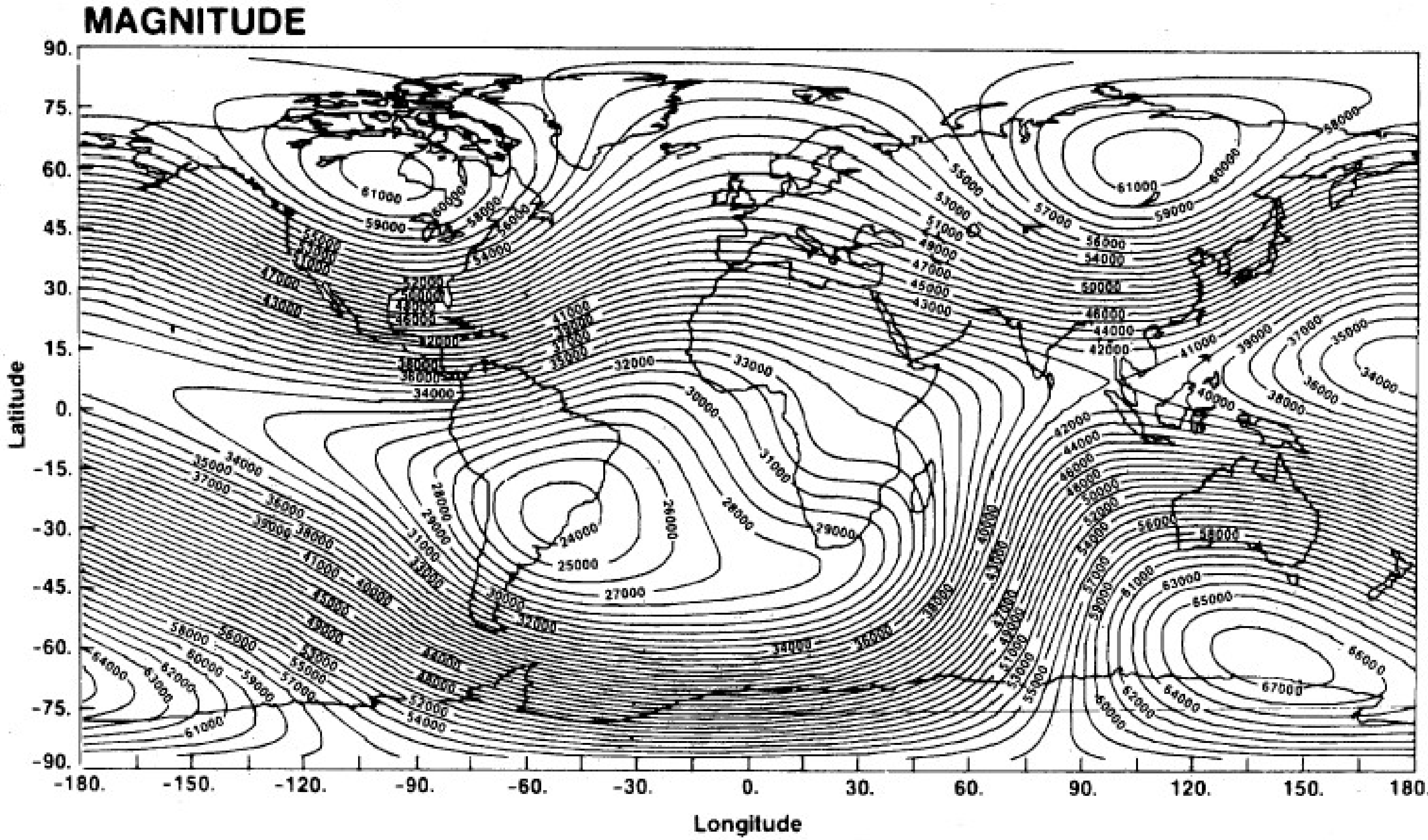


Figure 4-11. Contours of constant total field B at the surface of the earth from the model IGRF 1980.0.

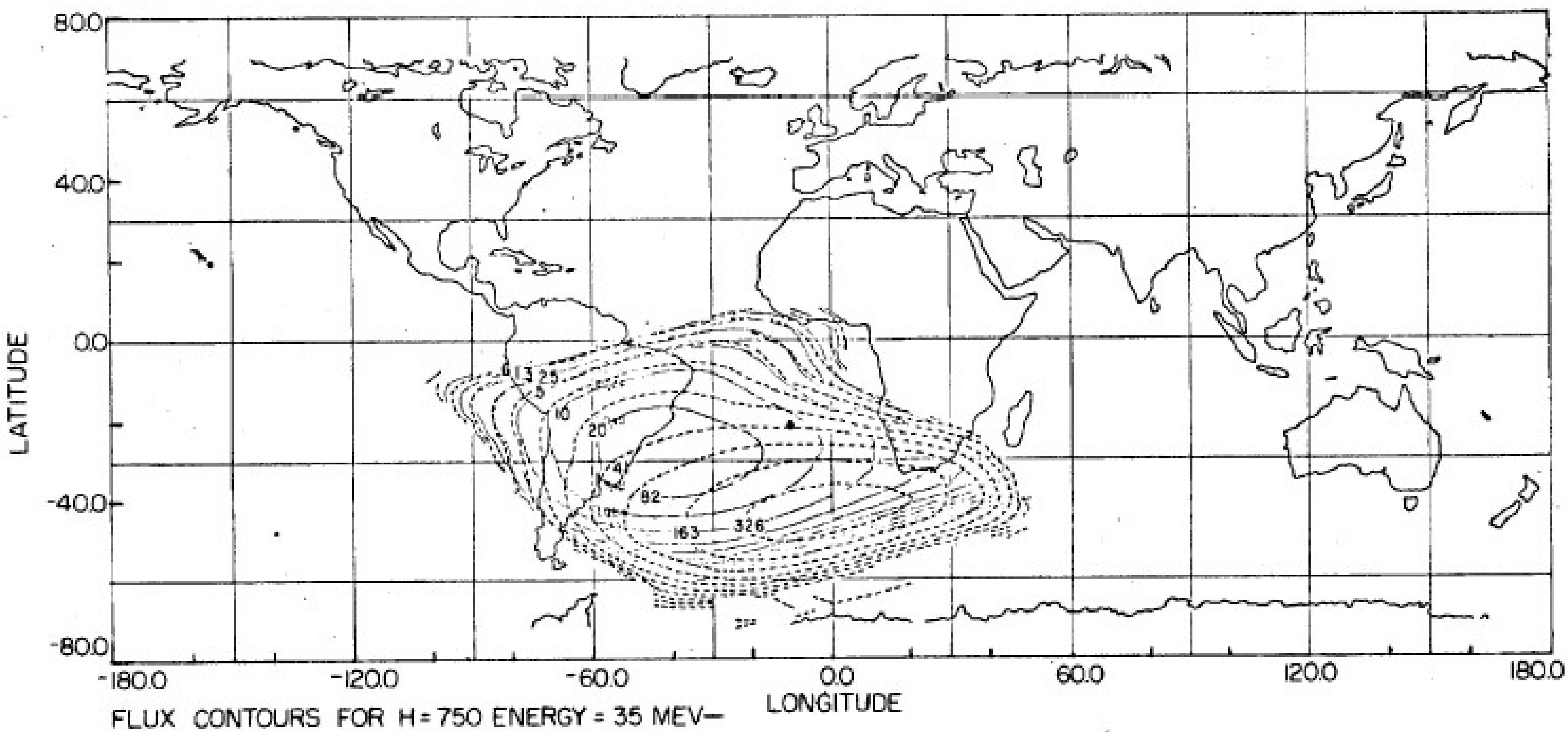


Figure 5-33. Proton isointensity flux contours as measured in the South Atlantic anomaly at an altitude of 750 km. The solid lines depict 28-45 MeV proton (ion) fluxes and the dashed lines 5-7 MeV proton fluxes. The flux units are particles/(cm^2-s-MeV).

Magnetic Field Minimum

Precipitating >80MeV protons

Single Event Upsets in the memory for Topex Satellite

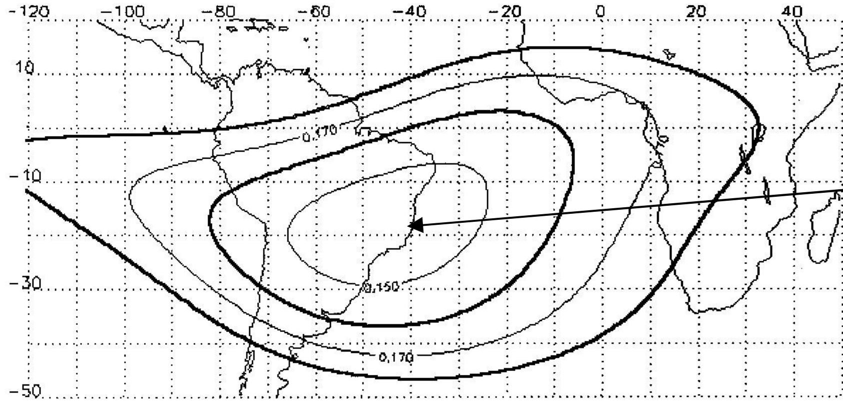


Figure 1. DGRF 65, Epoch 1970, 1336 km magnetic field contours.

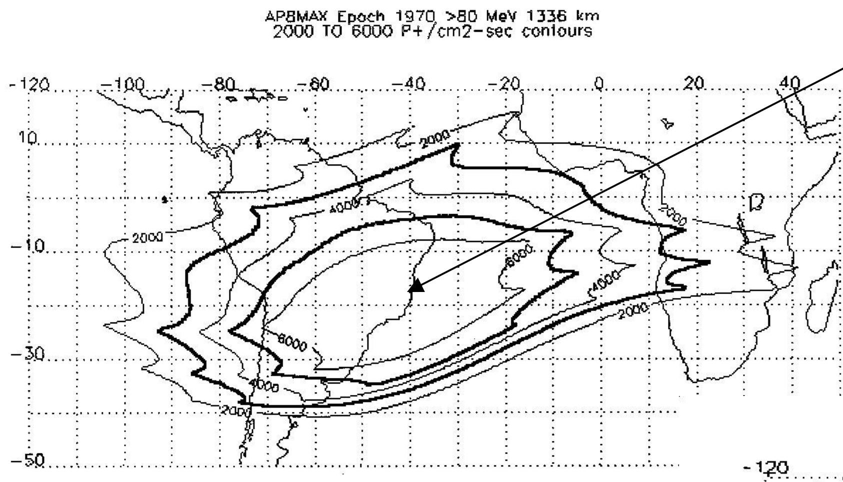


Figure 2. AP-8 MAX Epoch 1970 > 80 MeV protons at 1336 km.

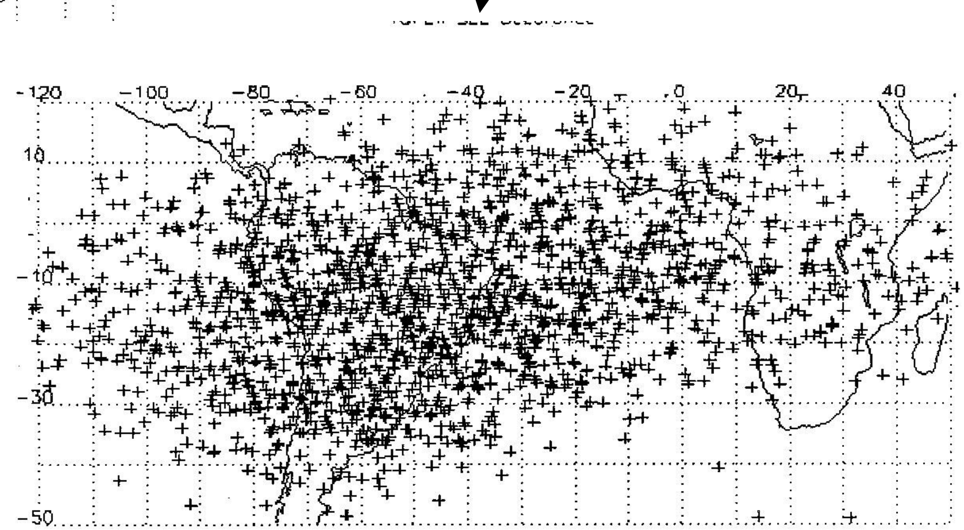


Figure 3. TOPEX SEE geographical distribution.

Advanced subject

For details see adiabatic_invariants.pdf

Adiabatic Invariant

$$I(\bar{z},\delta) = \oint d\theta\, \vec{p}(z,\theta,\delta) \cdot \frac{\partial \vec{q}(z,\theta,\delta)}{\partial\theta}$$

usually abreviated

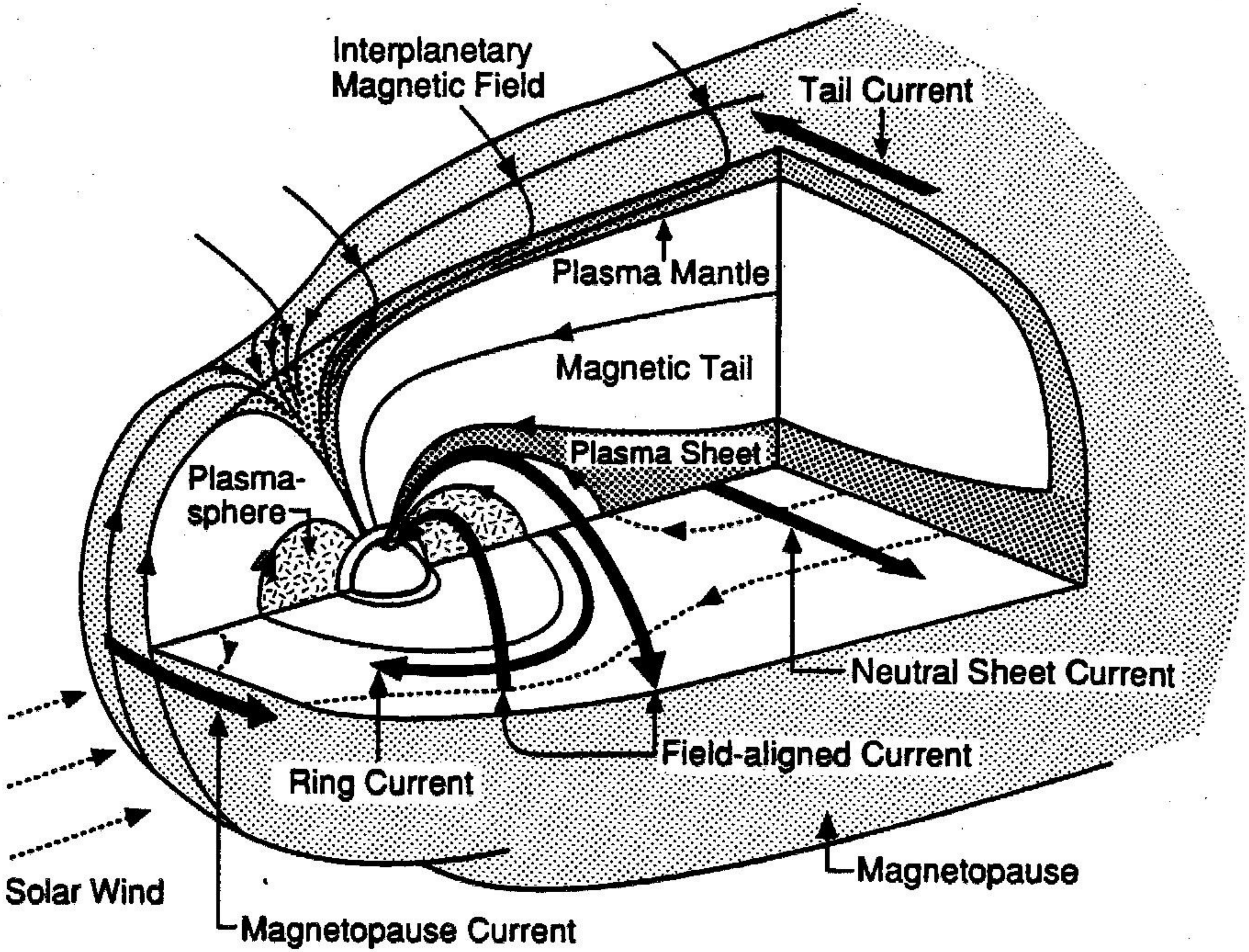
$$I = \oint_{\bar{z}=\text{constant}} \vec{p}\cdot d\vec{q} \overset{\text{by Stokes theorem}}{=} \int d^3p\, d^3q$$

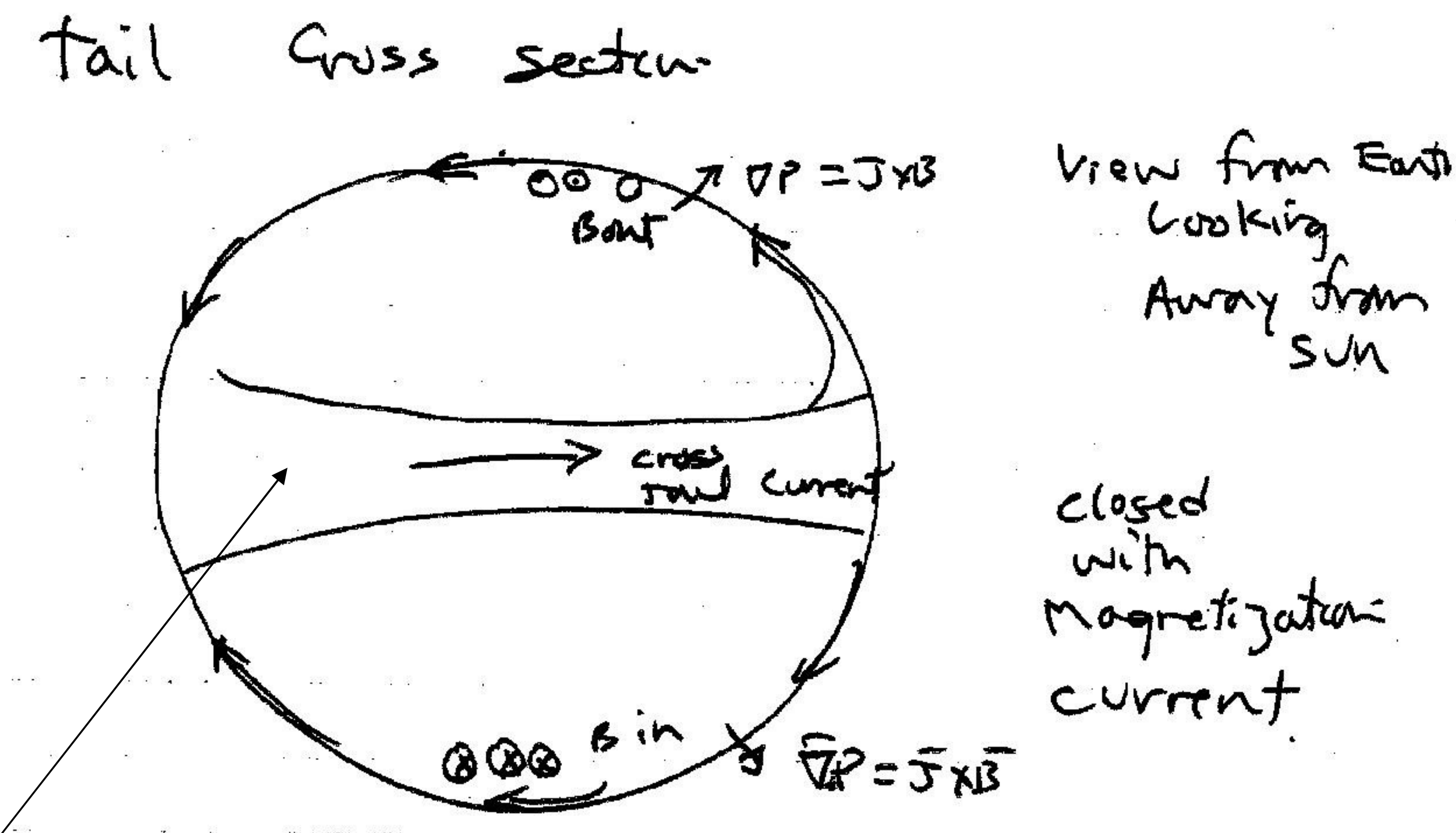
but $\int d^3p\, d^3q$ has same value for any canonical $\vec{p},\vec{q}$

∴ since transformation from unperturbed to perturbed $\vec{p},\vec{q}$ is canonical

$$\underset{\text{actual motion}}{\int d^3p\, d^3q} = \underset{\text{unperturbed motion}}{\int d^3p\, d^3q} = \underset{\substack{z=\text{constant} \\ \text{unperturbed motion}}}{\oint \vec{p}\cdot d\vec{q}}$$

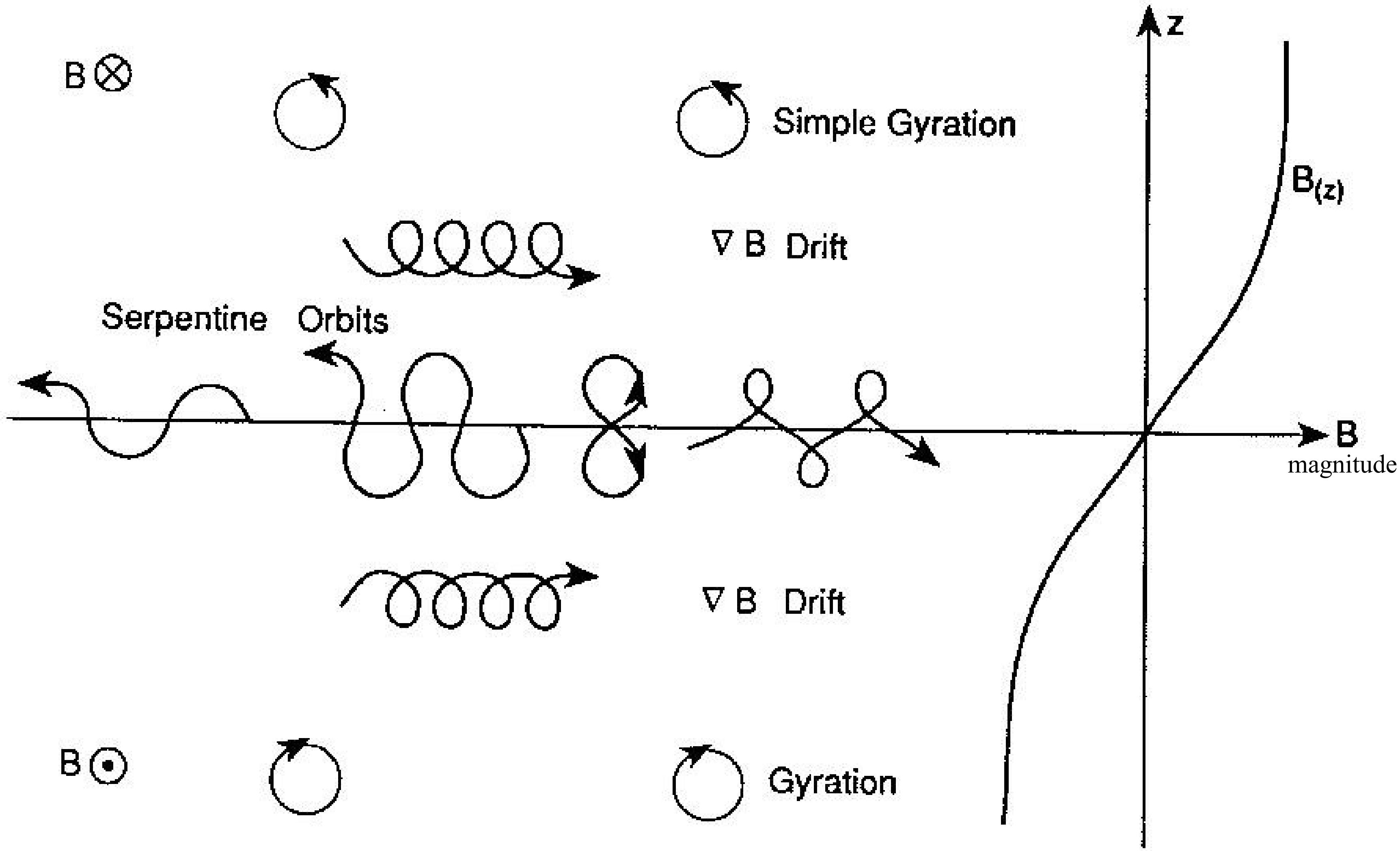
- Now, lets look at magnetotail Tail curents
- Then combine cold and hot plasma drifts
- Cold:
  - Sunward convection on closed field lines
  - Plasmasphere co-rotatation
- Hot
  - Ring current
  - Partial ring current/Alfven layer
- Then: Aurora and ionosphere


Interplanetary
Magnetic Field
Tail Current
Plasma Mantle
Magnetic Tail
Plasma Sheet
Plasma-
sphere
Neutral Sheet Current
Field-aligned Current
Ring Current
Magnetopause
Solar Wind
Magnetopause Current



How can there be a current

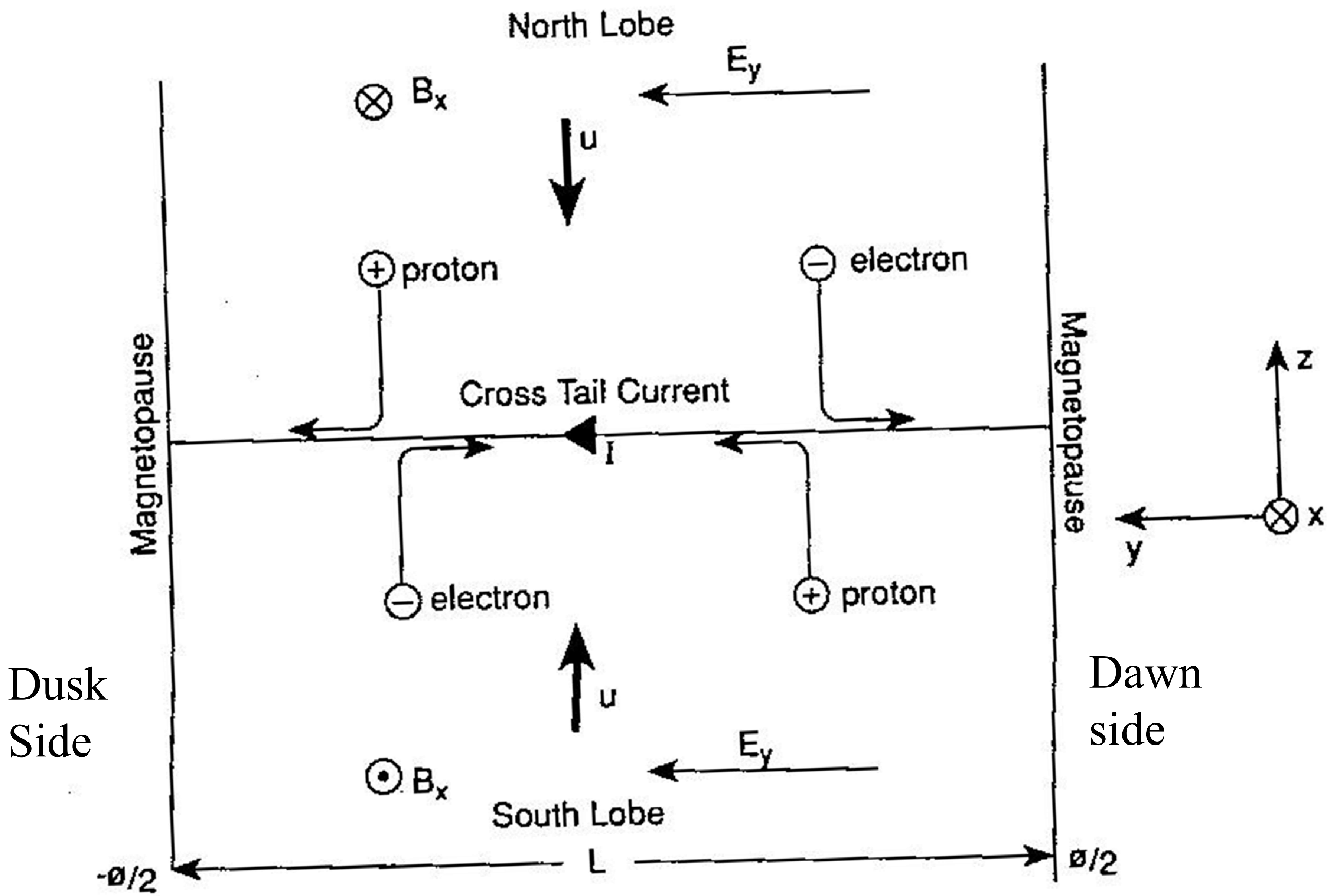
Like this: charge moving ACROSS the B –field?



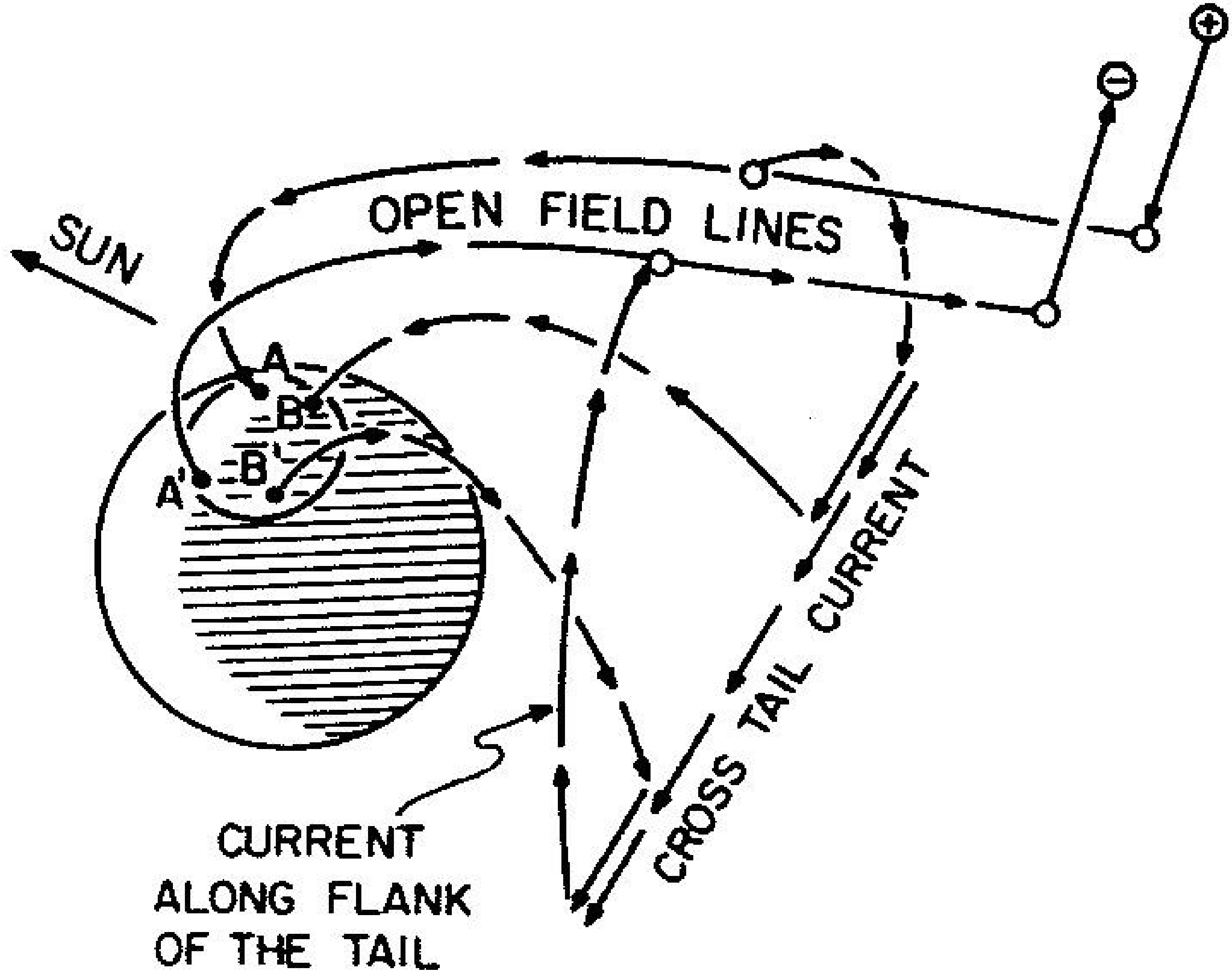
s

View from the Tail looking back at Earth

Dusk ← dawn

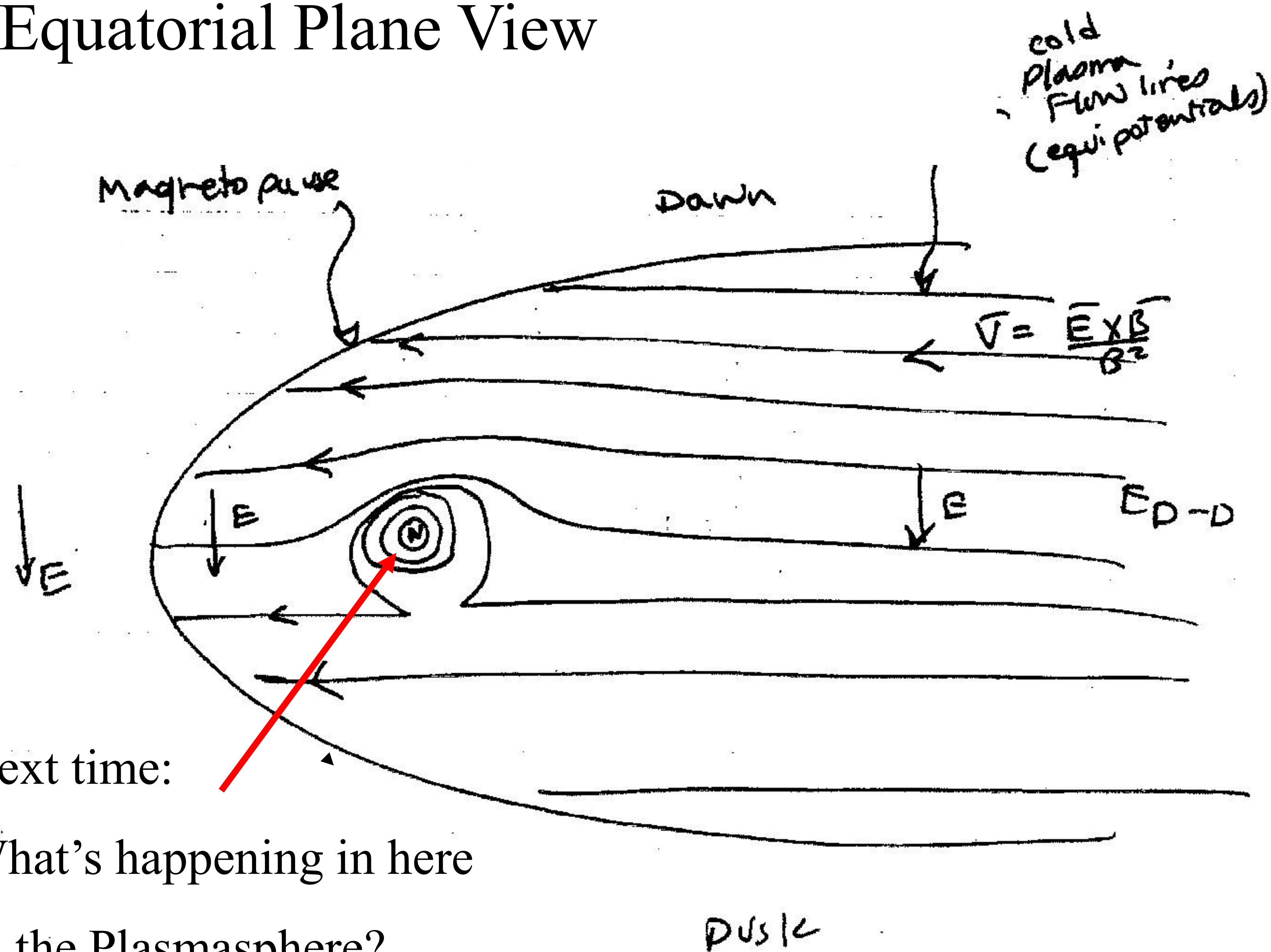


View from Tail towards Earth



SUN
OPEN FIELD LINES
A
B
B'
A'
CROSS TAIL CURRENT
CURRENT
ALONG FLANK
OF THE TAIL

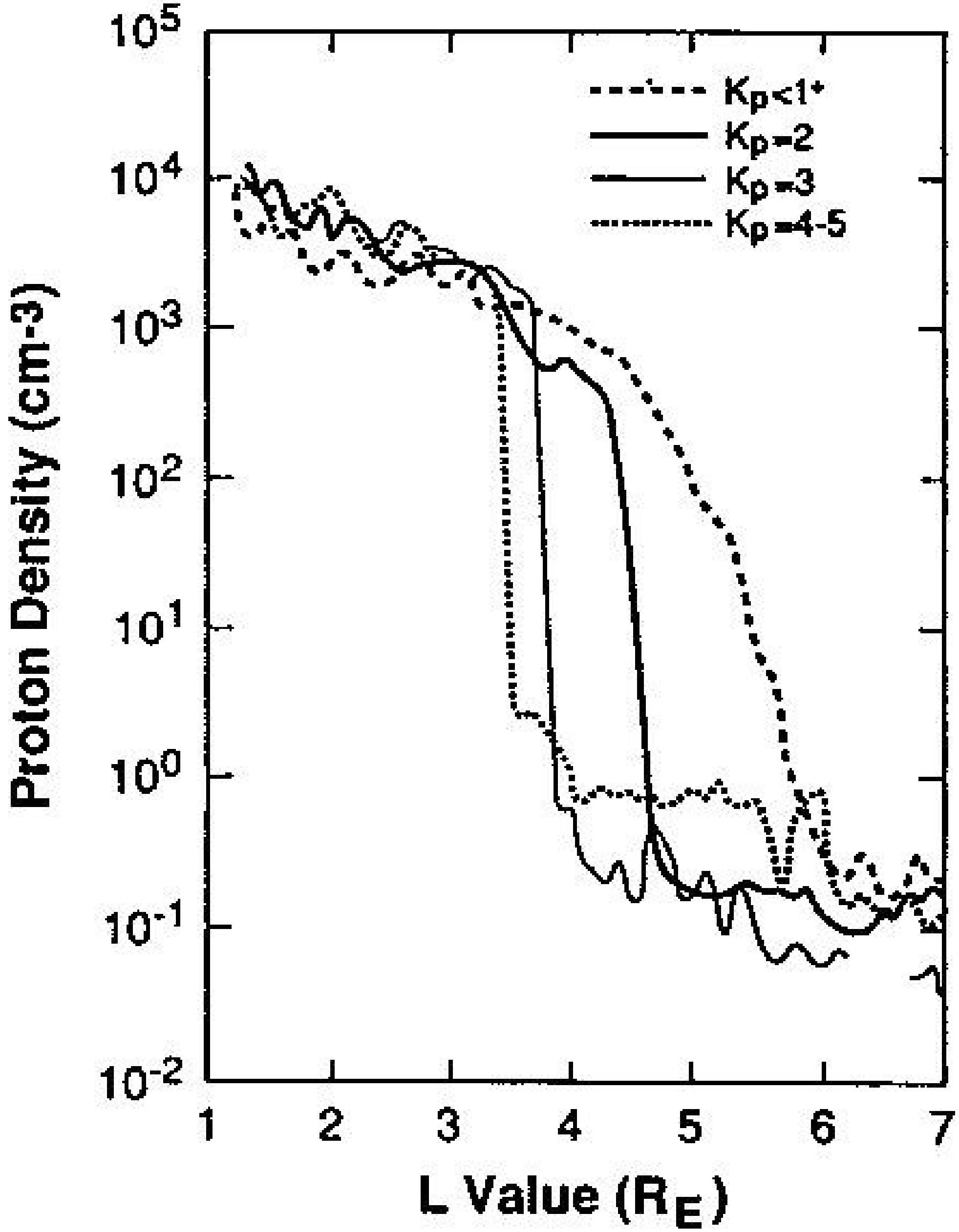
Equatorial Plane View



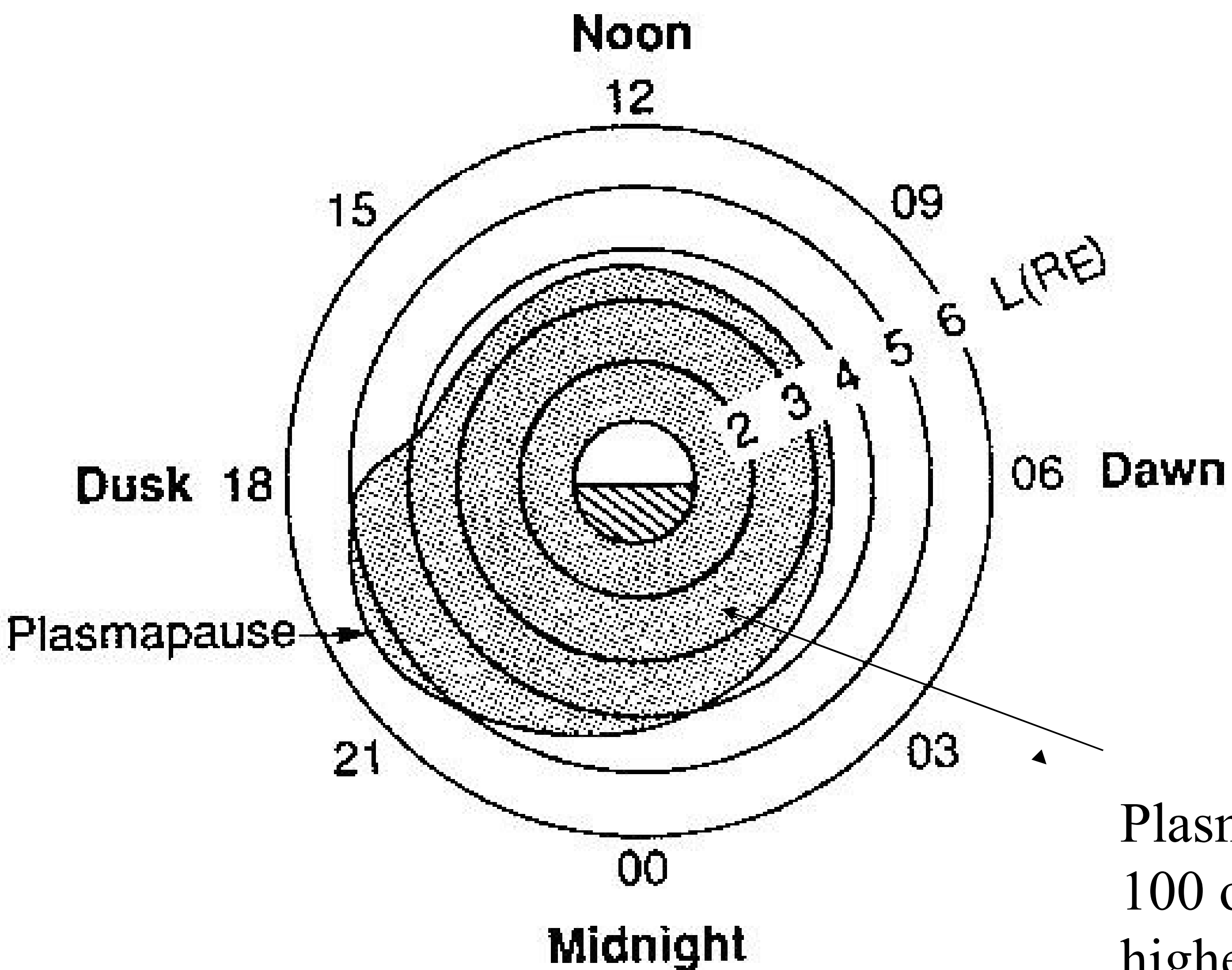
Next time:

What's happening in here

In the Plasmasphere?



Magnetic activity index



Plasma density 100 or 1000 times higher than outer magnetosphere