Magnetosphere Configuration

- High latitude, open field lines, magnetospheric convection, magnetotail
- Middle altitudes: plasma sphere/radiation belts, ring current
- Ionosphere: aurora, field aligned currents, acceleration mechanisms, field aligned currents
- Global current systems linking them all
FIG. 10.4. Schematic diagram of plasma regions of the earth's magnetosphere as viewed in the noon–midnight meridian plane. The plasmasphere typically occupies much of the same region of space as the radiation belts. Frequently there is little or no gap between the inner edge of the plasma sheet and the outer boundary of the trapped radiation belts.
Next: Start learning about the difference between Cold and Hot plasma motions – they are VERY different in the magnetosphere
<table>
<thead>
<tr>
<th></th>
<th>Magnetosheath</th>
<th>Tail Lobe</th>
<th>Plasma-Sheet Boundary Layer</th>
<th>Central Plasma Sheet</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n \ (cm^{-3})$</td>
<td>8</td>
<td>0.01</td>
<td>0.1</td>
<td>0.3</td>
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<tr>
<td>$T_e \ (eV)$</td>
<td>150</td>
<td>300</td>
<td>1,000</td>
<td>4,200</td>
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<tr>
<td>$T_o \ (eV)$</td>
<td>25</td>
<td>50</td>
<td>150</td>
<td>600</td>
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<tr>
<td>$B \ (nT)$</td>
<td>15</td>
<td>20</td>
<td>20</td>
<td>10</td>
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<tr>
<td>$\beta$</td>
<td>2.5</td>
<td>$3 \times 10^{-3}$</td>
<td>$10^{-1}$</td>
<td>6</td>
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</tbody>
</table>
Geomagnetic substorm involves rapid changes in magnetotail energy storage, resulting in the dumping of energetic particles into auroral zones, and the sloughing off of the tail.
Next: Start learning about the difference between Cold and Hot plasma motions – they are VERY different in the magnetosphere
10.4 ELECTRIC FIELDS AND MAGNETOSPHERIC CONVECTION

FIG. 10.18. Typical patterns of (a) Birkeland current, (b) electric field, (c) horizontal ionospheric current, and (d) $E \times B$-drift velocity observed in the earth's ionosphere, as viewed from high above the North Pole. Local noon is toward the top of the page, local dusk to the left, and so forth. These patterns represent the primary ionospheric effects of magnetospheric convection. The

$$\phi(x) = \int E \cdot ds$$
Single Particle Motion in E and B fields

Everything follows from Lorentz Force:

1\textsuperscript{st} assume time independent fields

- Gyration
- ExB drift: charge and energy independent
- Grad-B and curvature drifts: depend on both charge and energy (explains How the Radiation Belts Work)
- Adiabatic invariants of the motion in a dipole field
Single particle motion in B field

Start with Lorentz force

\[ m \frac{d\mathbf{v}}{dt} = q \mathbf{v} \times \mathbf{B} \]

Simple harmonic oscillation

\[ \omega_c = \frac{eB}{m} \]

\[ \mathbf{r}_L = \frac{\mathbf{v}_L}{\omega_c} = \frac{m \mathbf{v}_L}{eB} \]

Larmor radius

Now add \( \mathbf{E} \) field and transform away \( \mathbf{E} \)

\[ \mathbf{F} = 0 = q \mathbf{E} + \mathbf{v} \times \mathbf{B} \]

\[ \Rightarrow \mathbf{E} = -\mathbf{v} \times \mathbf{B} \]

Gives

\[ \mathbf{v} = \frac{\mathbf{E} \times \mathbf{B}}{B^2} \]

Equation of \( \frac{eB}{m} \) drift at

\[ \mathbf{v}_L = \frac{\mathbf{E} \times \mathbf{B}}{B^2} \]

In general any force \( \mathbf{F} \) (substituting \( \mathbf{F} = q \mathbf{E} \))

\[ \mathbf{v}_d = \frac{\mathbf{F} \times \mathbf{B}}{qB^2} \]

for \( \mathbf{F} \) \perp \( \mathbf{B} \)
How the Radiation Belts Work

• Grad B drift = \(-\mu \boldsymbol{\nabla} \vec{B} \times \vec{B} / qB^2\)
• Force on Parallel Motion
  – Magnetic bottle
  – Pitch angle
  – Bounce Motion
• Longitudinal Drift
• Radiation Belt Organization
  – Loss cone
Non uniform $B$ [diagram]

$B_{out}$ $\rightarrow V_B$ $\rightarrow$ 

smaller gyro radius

larger gyro radius

Acts like a particle slowing down when it moves into $V_B$.

Force $= -\mu \vec{V} \times \vec{B}$ force on a magnetic moment $\mu$

$\mu \equiv \frac{E}{L} = \frac{1}{2} m v^2$

$\equiv$ current x area

Assume $\nabla \propto \frac{1}{L}$ and $L \gg r_i$

Guiding center equation

$\vec{V}_G = U_{||} \hat{b} + \frac{E \times \vec{B}}{B^2} - \frac{\mu}{gB^2} \frac{V_B \times \vec{B}}{B} + \frac{B}{gB^2} \frac{m v^2}{L} \frac{(\vec{B} \cdot \hat{b})}{B^2}$

- $E \times \vec{B}$ Gradient drift
- $\frac{\mu}{gB^2} \frac{V_B \times \vec{B}}{B}$ Curvature drift
Parallel Motion of Guiding Center

- From Lorentz Force, looking at average motion parallel to B ($v_{\parallel} = \vec{v} \cdot \vec{B}/B$) we get this equation of motion:

$$\frac{1}{m} \frac{d}{dt} m v_{\parallel} = e E_{\parallel} - \mu \frac{\partial B}{\partial t} \left| \begin{array}{c} \text{where} \\ \frac{\partial B}{\partial t} = \frac{\vec{B} \cdot \vec{V}}{B} \end{array} \right.$$ 

Where $\mu = \frac{1}{2} m v_{\text{perp}}^2/B = \text{constant when } E_{\parallel} = 0$

(for derivation see http://earthweb.ess.washington.edu/bobholz/ess515/parallel_guiding_center_motion.pdf )
Magnetic Bottle

Consider a cylindrically symmetric magnetic field with field lines as...
Suppose you start a particle on the axis at the place where field strength is $B_0$. Suppose the velocity of the particle makes an angle $\alpha$ with respect to the magnetic field. What happens?

$\alpha$ = pitch angle

$v_{\parallel} = v \cos \alpha$

$v_{\perp} = v \sin \alpha$

$v_{\parallel}$ carries the particle into a region of larger field strength. Some time later we have ...

$v_{\parallel}$ to right

$v_{\perp}$ into paper

At the particle $\bar{B}$ has a component $B_1$ that is parallel to the guiding center field and a component $B_2$ that is perpendicular.
So we have

\[
\begin{align*}
\times \rightarrow v_{\perp} \\
v_{\perp} (\text{into})
\end{align*}
\]

consider components of \( \vec{v} \times \vec{B} \) (Lorentz Force)

\( v_{\parallel} B_2 \) causes force that increases \( v_{\perp} \)

\( v_{\perp} B_2 \) decreases \( v_{\parallel} \)

thus, \( v_{\parallel} \) decreases while \( v_{\perp} \) increases.

Since \( v_{\parallel}^2 + v_{\perp}^2 = \text{constant} \) (energy conserved),
after a while \( v_{\parallel} \) will be zero. But

\( v_{\perp} B_2 \) continues to act, so \( v_{\parallel} \) changes sign

ie: Particle enters magnetic field a certain distance, stops, turns around and comes out!

This is called \underline{Mirroring}. 
We want to know the point where the particle turns around, or the mirror point.

Equation for \( v_\perp \) is 

\[
\frac{1}{B} \frac{\Delta m v_\perp^2}{\beta} \text{ const.}
\]

If particle moves into region where \( B \) is larger, then \( v_\perp \) must get larger too.

**Question**: How far into the region of increasing magnetic field will a particle penetrate if it starts with pitch angle \( \alpha_0 \) at a place where \( B = B_0 \)?

\[
M = \frac{\frac{1}{2} m v_\perp^2 \sin^2 \alpha_0}{B_0} = \frac{\frac{1}{2} m u^2 \sin^2 \alpha}{B}
\]

\( v_\perp = \text{initial} \) \( u \)

As it penetrates, \( B \) increases, so does \( \sin \alpha \).

Until \( \beta \)

At the mirror point \( \alpha = 90^\circ \)

and \( u_\parallel = u \cos \alpha = 0 \)

\( u_\perp = u \sin \alpha \)

Define \( B \) at this point to be \( B_m \)

we have \( \sin^2 \alpha_0 = \frac{1}{B_0} \)

so \( B_m = \frac{B_0}{\sin^2 \alpha_0} \)

All particles of any energy, charge, mass will mirror at same point \( B_m \) if they start with same \( \alpha_0 \).
Another Example of Guiding Center Motion
Particles in a Magnetic Dipole Field

The Radiation Belt

A few cosmic particles away from the equator they enter region of increasing B field:
Mimic and bounce back and forth.

Additionally, the guiding center drifts around the earth in longitude due to gradient and curvature drifts.

1. Gyration

\[ T_{\text{gyration}} = \frac{2\pi \tau}{e} = \frac{2\pi \tau m}{Be} \sim \]

- \(10^{-2}\) sec proton at level
- \(10^{-5}\) sec \(e^-\) at level
- \(10^{-3}\) sec \(p\) at \(r=10Re\)
- \(10^{-2}\) sec \(e^-\) at \(r=10Re\)
bounce time \( t_{\text{bounce}} = \oint \frac{ds}{\nu_{\|}} \)

\[ ds = (dr^2 + r^2 d\lambda^2)^{1/2} = r_e \cos \lambda (1 + 3 \sin^2 \lambda)^{1/2} d\lambda \]

\[ \nu_{\|} = \left( \frac{\nu^2 - \nu_+^2}{\nu_+^2} \right)^{1/2} \]

but \[ \frac{\nu_+ (r, \lambda)}{B (r, \lambda)} \approx \frac{\nu \sin \theta_e}{B_e} = \text{const} \]

where \( \theta_e \) and \( B_e \) are equatorial pitch angle and field strength.

so \[ \nu_{\|} = \nu \left( 1 - \frac{\sin^2 \theta_e B (r, \lambda)}{B_e} \right)^{1/2} \]

\[ B(r, \lambda) = \frac{M}{r^2} \left( 1 + 3 \sin^2 \lambda \right)^{1/2} \text{ (Dipole Field)} \]

\[ = \frac{B_e}{\cos \lambda} \left( 1 + 3 \sin^2 \lambda \right)^{1/2} \text{ (because } r = r_e \cos \theta \text{ field line equation)} \]

numerically integrating

\[ t_{\text{bounce}} = 4 \frac{r_e}{\nu} T(\alpha_e) \]

where \( T(\alpha_e) = 1.3 - 1.5 \theta_e \sin \theta_e \)

(nota strong dependence on \( \alpha_e \))

for \( \nu = c \) and \( T(\alpha) = 1 \)

\[ r_e \quad 1.5 r_e \quad 3 r_e \quad 6 r_e \]

\[ t_{\text{bounce}} \quad 1.3 s \quad 1.26 s \quad 0.52 s \]
III Longitudinal Drift

for \( J = 0 = \nabla \times B \)

\[ V_c = \frac{\beta x \Phi B}{eB^3} (E_\perp + 2E_{\parallel}) \]

integrate around drift path of 360° longitude

gives \( T_{\text{drift}} = \frac{44}{R_e \times E(\text{mev})} \) minutes

That is, \( R_e = L \times R_e \equiv L \) earth radii

For \( E = 0.1 \text{ Mev} \) and \( R_e = 10 \text{ Re} \)

\[ T_{\text{drift}} \approx 44 \text{ minutes} \]

So, in general

\[ T_{\text{drift}} >> T_{\text{bounce}} >> T_{\text{gyro}} \]

Neglecting scattering and plasma instabilities, particles can be trapped forever.

In practice some species at some energies are trapped for 100 yrs.
Fig. 6. Contours of constant adiabatic gyration, bounce, and drift frequency for equatorially mirroring particles in a dipole field. Adiabatic approximation.
Radiation Belt organization

For a dipole $B$-field with $M = \text{dipole moment of the earth}$

$$\vec{M} = \iiint \vec{B} \cdot \hat{r} \, dr \, d\theta \, dz$$

Magnetic field $\vec{B} = -\vec{\nabla} \psi$

where $\psi = -\vec{M} \cdot \hat{r} \cdot r$ for dipole

in spherical coords:

$B_r = \frac{3}{r^3} \psi = -2 M \sin^2 \lambda$

$B_\phi = \frac{1}{r \sin \lambda} \frac{2}{2 \phi} \psi = 0$

$B_\lambda = -\frac{1}{r^3} \frac{2}{2 \phi} \psi = M \frac{\cos^2 \lambda}{r^3}$

$|\vec{B}| = \sqrt{B_r^2 + B_\phi^2 + B_\lambda^2} = \frac{M}{r^3} \left(1 + 3 \sin^2 \lambda\right)^{1/2}$

from Eqs 3.30 and 3.31 can write

$$\frac{dr}{r} = 2 \frac{d(\cos \lambda)}{\cos \lambda}$$

Integrate to get

$$\phi = \phi_0 \quad r = r_0 \cos \lambda$$

Now let $L = \frac{r_0}{R_e}$ marks equatorial distance of a particular field line

so $r = L R_e \cos^2 \lambda$
or, to put it another way, the latitude of the surface is given by \( r = R_e \) so \( \cos^2 \theta = \frac{1}{2} \)

This is definition of Invariant Latitude

\( L \) can be defined more carefully for a distorted field.

So, for a given L shell, most particles will be undergoing 3 distinct motions—gyration, bounce + VB drift—but its mirroring point is \(< 1 \) \( R_e \); they will be lost due to scattering from atmosphere.

Actually, the loss altitude \( \sim R_e + 100 \text{km} \) where probability of scattering becomes high.
Which particles are lost?

If minimum point is < \( R + 100 \text{ km} \) then particle is lost.

Find this \( \Delta \epsilon \)

Using \( M = \text{constant} = \frac{v^2 \sin^2 \Delta \epsilon}{B \epsilon} \)

Find \( \Delta \epsilon \) such that \( \epsilon = 90^\circ \) at \( B \) for \( r = R + 100 \text{ km} \)

We find that \( \Delta \epsilon \approx 3^\circ \) at \( L = 6 \)

So most particles are trapped.

Show pitch angle distributions along field lines.
L-shell drift: How can you tell the drift motion always returns guiding center to starting point?

Answer: If energy is conserved then with \( M = \text{constant} \), if start from \( r_i \) where \( B = B_1 \),

\[
M = \frac{E}{B_1} \quad \text{at} \ r_i
\]

If returned at \( r \neq r_i \), \( B \neq B_1 \) so \( M \neq \text{constant} \).

L defines a closed shell for perfect dipole.
Adiabatic Invariant

\[ I(3, s) = \oint d\theta \tilde{p}(3, \theta, s) \cdot \frac{\partial \tilde{q}(3, \theta, s)}{2\pi} \]

usually abbreviated

\[ I = \oint \tilde{p} \cdot d\tilde{q} = \text{by Stokes} \int d^3 p d^3 q \]

\[ \tilde{q} = \text{constant} \]

but \( \int d^3 p d^3 q \) has same value for any canonical \( \tilde{p}, \tilde{q} \)

\( \Rightarrow \) since transformation from unperturbed to perturbed \( \tilde{p}, \tilde{q} \) is canonical

\[ \oint d^3 p d^3 q = \int d^3 p d^3 q = \oint \tilde{p} \cdot d\tilde{q} \]

\[ \tilde{q} = \text{constant} \]