

Problem Set 5
ESS590 G&H
Winter 2017

Due by Friday 3/10 and accepted no later than Wednesday 3/15 for on-time grade

1. Nicholson p. 182 Exercise (mid page)
2. Nicholson p. 183 Exercise (top)
3. Nicholson p. 183 Exercise (2nd down from Top)
4. Nicholson p. 183 Exercise (3rd down from top)
5. Nicholson p. 183 Exercise (4th down from top - i.e. bottom Exercise)

6. Nicholson p. 210 problem 8.1 (about Alfvén waves).

7. This problem from J.D. Jackson Classical Electrodynamics (original edition 1962) prob 10.2 p. 343/344

A comparatively stable self-pinched column of plasma can be produced by trapping an axial magnetic induction inside the plasma before the pinch begins. Suppose that the plasma column initially fills a conducting tube of radius R_0 and that a uniform axial magnetic induction B_{z0} is present in the tube. Then a voltage is applied along the tube so that axial currents flow and an azimuthal magnetic induction is built up.

(a) **Show** that if quasi-equilibrium conditions apply, the pressure balance relation can be written:

$$\left[p(r) + \frac{B_z^2}{8\pi} + \frac{B_\phi^2}{8\pi} \right]_{r_1}^{r_2} + \frac{1}{4\pi} \int_{r_1}^{r_2} \frac{B_\phi^2}{r} dr = 0$$

(b) If the plasma has a sharp boundary and such a large conductivity that currents flow only in a thin layer on the surface, **show** that for a quasi-static situation the radius $R(t)$ of the plasma column is given by the equation:

$$\frac{R_0}{R} \ln \left(\frac{R_0}{R} \right) = \frac{1}{t_0} \int_0^t f(t) dt$$

where

$$t_0 = \frac{B_{z0} R_0}{c E_0}$$

and $E_0 f(t)$ is the applied electric field.

(c) If the initial axial field is 100 gauss, and the applied electric field has an initial value of 1 volt/cm and falls almost linearly to zero in 1 millisecond, **determine** the final radius if the initial radius is 50 cm. These conditions are of the same order of magnitude as those appropriate for the old British toroidal apparatus (Zeta), but external inductive effects limit the pinching effect to less than the value found here.