Altitude variation of glacier mass balance in Scandinavia

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[1] For each of ten glaciers in Norway and two in Sweden, vertical profiles of net balance $b_n(z)$, which are typically published as values at about a dozen altitudes, are strongly linear and nearly parallel from year to year. Separate linear functions fit the $b_n(z)$ from year to year with $r^2 \geq 0.89$ over the 12 glaciers. A family of parallel lines for each glacier that differ from year to year only by an amount Δb_n constant with altitude has $r^2 \geq 0.85$. There is an altitude z' on each glacier where the measured balance $b_n(z')$ correlates well with the glacier-total b_n with $r \geq 0.97$ over the 12 glaciers. A remarkable consequence of this and of the high correlation of b_n between many of the glaciers in the region is that measurements on one glacier (1775 meters on Hardangerjøkulen) provide a good estimate of b_n at several other glaciers. *INDEX TERMS:* 1827 Hydrology: Glaciology (1863); 1863 Hydrology: Snow and ice (1827); 9335 Information Related to Geographic Region: Europe. **Citation:** Rasmussen, L. A. (2004), Altitude variation of glacier mass balance in Scandinavia, *Geophys. Res. Lett.*, *31*, L13401, doi:10.1029/2004GL020273.

1. Introduction

[2] Altitude variation of net balance on a glacier has been the subject of much study in glaciology. *Meier and Tangborn* [1965] concluded from profiles $b_n(z)$ of measured net balance on South Cascade Glacier in 1958–64 that except for 1961 the profiles differed from each other by an amount Δb_n constant with altitude. This relation can be written as

$$b_n(z,t) = f(z) + \Delta b_n(t) \tag{1}$$

in which $f(z)$ is the characteristic curve for the glacier. *Lliboutry* [1974] developed a model in which from year to year b_n varied by a constant amount over the entire ablation zone of Glacier de Saint-Sorlin in France. *Kuhn* [1984] found that the variation of $b_n(z)$ from year to year on Hintereisferner in Austria is nearly constant with altitude. *Oerlemans and Hoogendoorn* [1989], however, developed an energy balance model that showed for it and for Kesselwandferner, also in Austria, that sensitivity to warming decreased markedly with altitude; another variation they considered can be written as

$$b_n(z,t) = f(z + \Delta z[t]) \tag{2}$$

Just as equation (1) is a shift of $f(z)$ in the b direction, equation (2) is a shift in the z direction. In the special case that $f(z)$ is linear, say $f(z) = \gamma z + \mu$, equations (1) and (2) give equivalent results when $\Delta z = \Delta b_n/\gamma$. The following analysis considers only measurements of mass balance and does not employ a model relating it to meteorological conditions.

2. Data Sources

[3] Net balance records from ten glaciers in Norway and two in Sweden (Figure 1 and Table 1) are analyzed. For the glaciers in Norway, glacier-total b_n are given through 2002 by *Kjøllmoen* [2003]. Altitude profiles $b_n(z)$ were supplied by Liss Andreassen in the Norwegian Water Resources and Energy Directorate (NVE) but are unavailable at Hansebreen for 1987; Hardangerjøkulen for 1977–1980 and 1984; and Storbreen for 1949, 1955, 1957–1960, and 1967. For Storglaciären and Rabots glaciär in Sweden, data through 2001 were supplied by Peter Jansson of Stockholm University except that $b_n(z)$ were unavailable for Rabots after 1995.

3. Vertical Profiles of Net Balance

[4] Glacier-total net balance b_n is usually calculated as a linear combination of measurements at discrete points over a range of altitudes

$$b_n = \sum_i \lambda_i b_n(z_i) \tag{3}$$

Here the λ_i, which sum to unity, are often taken as the fraction of the glacier's area represented by sites z_i. The sum of $\lambda_i f(z_i)$ is defined to be $\bar{f}$. Because $\Delta b_n(t)$ is constant spatially, the sum of $\lambda_i \Delta b_n(t)$ is $\Delta b_n(t)$. Thus, if equation (1) is substituted in equation (3), it gives

$$b_n(t) = \sum_i \lambda_i [f(z_i) + \Delta b_n(t)] = \bar{f} + \Delta b_n(t) \tag{4}$$

so that changes in b_n are equal to changes in Δb_n, regardless of the functional form of $f(z)$.

[5] Mean profiles $\bar{b}_n(z_i)$ for each of the 12 glaciers are shown in Figure 2, along with the best-fitting straight lines to them. Two with pronounced curvature overall are Rabots and Austdalsbreen, two with slight curvature overall are Langfjordjøkelen and Nigardsbreen, and two with curvature only at high altitude are Engabreen and Hardangerjøkulen.

[6] Four different functions $f(z, t)$ considered here to approximate the $b_n(z, t)$ are: separate quadratic functions each year, a characteristic quadratic with separate Δb_n displacements each year, separate linear functions each year, and a family of parallel lines formed from a characteristic linear function with separate Δb_n each year. Results over the period of record are given in Table 1 as, respectively,

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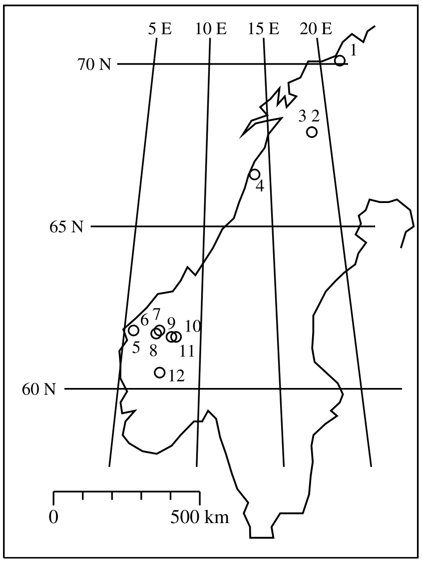

Figure 1. Glacier locations. Numbers indicate glaciers (see Table 1 for names).

r_2^2, $\bar{r}_2^2$, r_1^2, and $\bar{r}_1^2$. Goodness of fit [*Bevington*, 1969] is calculated by

$$r^2 = 1 - \frac{\sum\sum[f(z,t) - b_n(z,t)]^2}{\sum\sum[b_n(z,t) - \bar{b}_n]^2} \tag{5}$$

in which summation is over all measurements in all years and $\bar{b}_n$ is their mean. All four functions fit the measured profiles very well. Best are the separate quadratics (r_2^2), but only at Rabots and Austdalsbreen are they substantially better than separate linear functions (r_1^2). Separate linear functions are substantially better than a family of parallel lines ($\bar{r}_1^2$) only at Ålfotbreen and Gråsubreen.

[7] Nonlinear functions varying in the z direction (equation (2)) are not considered here. One reason is that, for Austdalsbreen for instance (Figure 2), nonlinear functions varying in the z direction cannot possibly accommodate anomalously large positive balance at the upper end of the altitude range. Another is that linear and quadratic functions varying in the b direction (equation (1)) fit the data so well.

[8] *Dyurgerov and Dwyer* [2000] concluded from records for 21 northern hemisphere glaciers, including seven in Scandinavia, that $b_n(z)$ is steeper in warm years; that is, $r < 0$ between db_n/dz and glacier-total b_n. *Oerlemans and Hoogendoorn* [1989] concluded the same in modeling mass balance at Hintereisferner and Kesselwandferner. *Funk et al.* [1997] found stronger gradients in warm years at Griesgletscher in Switzerland. At the 12 glaciers considered here, slopes of separate linear functions fit to each year's measured profiles $b_n(z)$ correlate poorly (Table 2) with b_n and only slightly better with mass turnover $b_t = b_w - b_s$. Here $b_w > 0$ is the winter balance and $b_s < 0$ is the summer balance, which are the seasonal components of net balance $b_n = b_w + b_s$.

4. Estimating Glacier Total Balance From a Few Point Measurements

[9] *Fountain and Vecchia* [1999] analyzed mass balance records from South Cascade Glacier in Washington and Maclure Glacier in California, concluding that small glaciers require high spatial density of measurements and that large glaciers require low density, with 5 to 10 being needed for alpine glaciers <10 km^2. That paper contains a good discussion of the relation between uncertainty in the glacier-total balance and that in measurements at individual sites. *Trabant and March* [1999] analyzed records from Wolverine Glacier and Gulkana Glacier in Alaska, concluding that three measurement sites — one low in the ablation area, one near the equilibrium line altitude (ELA), one in the accumulation area — is the minimum number, even for relatively small glaciers. *Funk et al.* [1997] in analyzing 34 years of measurements on Griesgletscher concluded that one site per km^2 is needed. Glaciers in Scandinavia, by contrast, apparently do not require such a large number of sites.

[10] If a family of curves of the form of equation (1) fit the measured profiles perfectly, regardless of the functional form of $f(z)$, every measured $b_n(z)$ would correlate perfectly

Table 1. Results (Percent r^2) of Fitting $b_n(z)$ Profiles for Individual Years, Beginning in Year t_0[a]

i	Glacier	N Lat.	E Lon.	t_0	r_2^2	$\bar{r}_2^2$	r_1^2	$\bar{r}_1^2$
1	Langfjordjøkelen	70.1	21.8	1989	99	97	96	95
2	Storglaciären	67.9	18.6	1964	94	92	89	87
3	Rabots glaciär	67.9	18.6	1982	97	93	91	89
4	Engabreen	66.6	13.8	1970	99	96	97	94
5	Ålfotbreen	61.8	5.6	1963	99	92	97	91
6	Hansebreen	61.8	5.7	1986	97	94	96	94
7	Austdalsbreen	61.8	7.3	1988	99	95	90	88
8	Nigardsbreen	61.7	7.1	1962	99	98	98	97
9	Storbreen	61.6	8.1	1949	100	97	98	96
10	Hellstugubreen	61.6	8.4	1963	98	96	96	95
11	Gråsubreen	61.6	8.5	1963	97	85	93	85
12	Hardangerjøkulen	60.5	7.5	1963	99	97	98	96

[a]The fit of individual quadratic polynomials is r_2^2, and of characteristic quadratic with separate $\Delta b_2(t)$ constant with altitude each year is $\bar{r}_2^2$. The fit of individual linear functions is r_1^2, and of characteristic linear functions adjusted by an amount $\Delta b_1(t)$ constant with altitude each year is for $\bar{r}_1^2$. The subscript indicates the degree of the polynomial, 1 for linear and 2 for quadratic. All r^2 significant at 99 percent level.

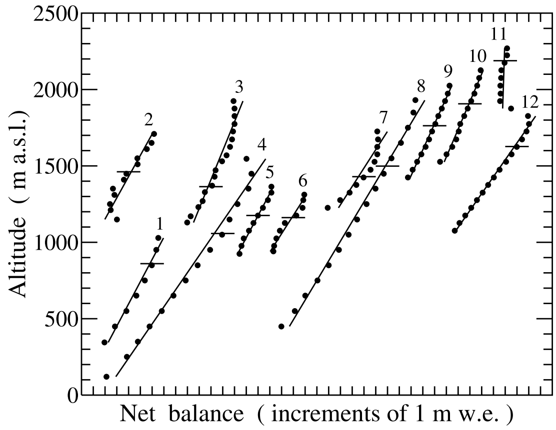

Figure 2. Mean altitude profiles $b_n(z)$, mean ELA (horizontal line), and best linear fit. Numbers indicate glaciers (see Table 1 for names, period of record).

Table 2. Altitude z_i' Where Measured Balance $b_n(z_i')$ has Best Correlation r_i With Glacier-Total Balance b_n, and rms of Error $b_n^* - b_n$ is rms_i[a]

i	Glacier	z_i'	$S(z_i')$	$\bar{b}_n(z_i')$	r_i	rms_i	α	β	r_{12}	rms_{12}	$\overline{db_n/dz}$	r_n	r_t
1	Langfjordjøkelen	750	0.61	−0.62	**98**	0.15	0.825	−0.21	45	0.70	6.9	26	−37
2	Storglaciären	1430	0.52	−0.10	**97**	0.15	0.897	0.02	**64**	0.49	7.2	28	19
3	Rabots glaciär	1410	0.40	0.29	**97**	0.12	0.817	−0.27	**68**	0.34	5.3	−32	**82**
4	Engabreen	1250	0.43	1.27	**98**	0.22	0.973	−0.51	**63**	0.91	8.7	18	**60**
5	Ålfotbreen	1175	0.69	−0.01	**99**	0.18	0.985	0.30	**87**	0.69	6.3	10	20
6	Hansebreen	1175	0.42	0.08	**98**	0.24	0.924	−0.42	**79**	0.77	8.2	15	−5
7	Austdalsbreen	1425	0.70	−0.05	**99**	0.16	1.007	0.01	**86**	0.54	8.4	**−65**	25
8	Nigardsbreen	1650	0.42	1.15	**99**	0.14	0.966	−0.66	**88**	0.47	8.0	17	15
9	Storbreen	1675	0.72	−0.47	**98**	0.12	0.885	0.19	**87**	0.33	6.0	−21	7
10	Hellstugubreen	1875	0.57	−0.18	**98**	0.13	0.994	−0.14	**80**	0.34	5.6	−36	**50**
11	Gråsubreen	2025	0.62	−0.41	**99**	0.08	0.950	0.09	**62**	0.46	1.9	**−45**	36
12	Hardangerjøkulen	1775	0.35	0.99	**99**	0.13	0.968	−0.71	**99**	0.13	8.6	−14	24

[a]Best fitting linear relation is $b_n^* = \alpha b_n(z_i') + \beta$. Approximate fraction of glacier area above z_i' is $S(z_i')$, and mean value of balance there over period of record is $\bar{b}_n(z_i')$. Correlation r_{12} is between b_n and Hardangerjkulen $b_n(z_{12}')$, with rms_{12}. Mean slope $\overline{db_n/dz}$ is in meters per year per kilometer. Correlation of each year's best fitting slope with b_n is r_n, with mass turnover $b_t = b_w - b_s$ is r_t. Both $\bar{b}_n(z_i')$ and rms in water equivalent meters. Correlations r are in percent, significant at 99 percent in boldface.

with the glacier-total b_n, as equation (4) shows. Reported values for the 12 glaciers do not have this property but do come very near to it. On each glacier there is a routinely reported measurement $b_n(z_i)$ that correlates with glacier-total b_n with $r \geq 0.97$ over the 12 glaciers (Table 2). That $\alpha \approx 1$ and $\beta \approx 0$ indicates measurements approach the relation of equation (4). Measurements at the mean ELA have $r \geq 0.93$.

5. Glacier to Glacier Correlation

[11] Correlations of glacier-total b_n over the 66 pairs formed from the set of 12 glaciers vary from +0.09 to +0.96 with median $r = 0.71$. Slopes db_n/dz of separate linear functions fit to each year's $b_n(z)$ correlate poorly, with $-0.60 \leq r \leq +0.70$ and 22 pairs with $r < 0$. Extremely strong correlation between b_n and the measured value at one site on the glacier (Table 2) combined with positive glacier to glacier correlation of b_n permits estimating b_n at one glacier from the measured value at one site on another glacier. Correlation of b_n at 1175 meters on Hardangerjøkulen with the glacier-total b_n at the other 11 glaciers is also shown in Table 2.

6. Conclusions

[12] Mass balance records in Scandinavia have several remarkable properties. Vertical profiles of net balance from year to year are fit very well by a family of parallel straight lines, whereas elsewhere profiles often have pronounced $d^2 b_n/dz^2 < 0$ curvature. Slopes of individual straight lines fit to profiles in separate years correlate poorly with the glacier-total b_n in those years, whereas analyses of other glaciers find steeper profiles in warm years, when the balance is more negative. Just one measurement on a glacier, near the middle of its altitude range, correlates extraordinarily well with the glacier-total b_n, whereas analyses of other glaciers suggest that three to a dozen sites on a glacier are needed.

[13] Two factors may contribute to this regular behavior compared with other regions of the world, one climatic and one topographic. Summers are neither hot nor long, so there is not strong ablation low on the glacier, which produces curvature in $b_n(z)$ elsewhere. Winters are wet and cold, so there is an abundance of snowfall at all altitudes. Mountains are not very high, so precipitation does not diminish much high on the glacier.

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