# SM53A-1425

**Advances in Simulating and Modeling Magnetospheric Processes Using Data From Multiple Spacecraft III Posters**

## Abstract #1425

Comparisons of data with models of the magnetotail current sheet have mainly used the Harris model, which is not a suitable candidate. The absence of a magnetic field component normal to the sheet, a vanishing asymptotic density, and a uniform tangential drift across the magnetic field are among those assumptions of the Harris model which are violated in the real magnetotail.

We have constructed a "four-group" 1D fluid model where the ions approaching the center from opposite sides are separate groups; the electrons are a third group and trapped ions the fourth. The use of two groups for the center-crossing ions allows a straightforward treatment of the asymmetric case of different boundary conditions on opposite sides of the sheet. Trapped ions are necessary for a stable solution as has been shown by previous authors. We are testing use of the Chew-Goldberger-Low pressure relations for ions to obtain closure, whereas we assume electrons are massless and isothermal. We work in the deHoffman-Teller reference frame but the solution can be transformed into the earth's frame which then contains an electric field. This field leads to gain in particle energy in the new frame.

We give some examples of model solutions for the 1-D magnetotail and discuss what kinds of measurements are necessary to provide boundary conditions for using this model. We believe that this treatment offers a better model for comparison with data, leading to quantitative tests of both the model and the interpretation of data, and providing a better understanding of magnetotail processes.

# Multi-Fluid Model of 1D Magnetotail Current Sheet

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## Motivation for this Approach

Comparisons of data with the magnetotail current sheet structure have mainly used the Harris model of the current sheet.

The Harris model has important limitations:
- $B_z = 0$, so no particles cross the sheet from asymptotic sources
- It assumes a uniform mean y-component of drift velocity
- It assumes a simple Maxwellian distribution
- The particle density vanishes asymptotically

Kinetic and fluid approaches have been used for current sheet models:
- A fluid model can describe the fluid behavior reasonably well, but needs unknown equations of state for closure.
- A kinetic model is complicated to construct in that it would have to fit data with expressions involving constants of motion.

A multi-fluid model extends the versatility of the fluid model approach
- It can treat sources from opposite sides of sheet
- Boundary conditions can be symmetric or non-symmetric
- Fluid equations are easier to handle than kinetic equations

## Some Previous Magnetotail Current Sheet Work

Speiser, Lyons, Sestero, Schindler: Particle trajectories showed acceleration in "non-adiabatic" central region. Gain in speed is primarily in earthward direction. Self-consistent kinetic models used larger scales in x- and y-directions than in z-direction.

Kan: obtained exact 2-D kinetic solutions. His method was followed by others who obtained exact 2-D solutions for X-point structures (Reviewed by Lui).

Eastwood, Hill, Cowley et al, Francfort and Pellat, discussed fluid and kinetic aspects, showing that balance between **B** and $P_{||}$ was important.

Schindler, Sonnerup: The first adiabatic invariant, J, is conserved through the sheet in spite of failure of the magnetic moment, μ. J reverts to μ at exit from sheet.

Birn, Hesse, Lui, Sitnov, Zelenyi, et al: Recent work with fluid & kinetic treatments.

Tsyganenko et al; Models of tail **B** from spacecraft data. Central gradient in **B** due to local current but asymptotic behavior from dipole & remote currents.

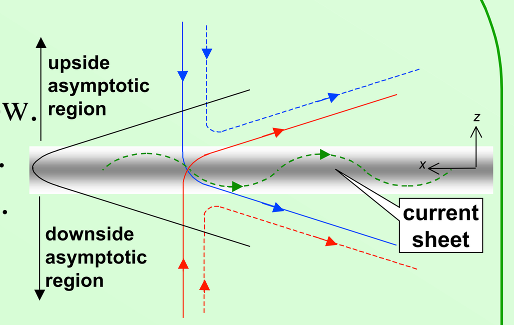
Baumjohann et al: data from traversals of the central sheet show a polytropic index close to 5/3 and $T_i/T_e$ staying nearly constant at about 7.

## Multi-fluid Model of 1D Current Sheet

**Ions represented as a multi-fluid**

Black line is a magnetic field line.
Solid blue is ions incident above, exiting below.
Solid red is ion incident below, exiting above.
Dashed blue is ion incident and exiting above.
Dashed red is ion incident and exiting below.
Dashed green is trapped ion.

Ion Fluid Streams



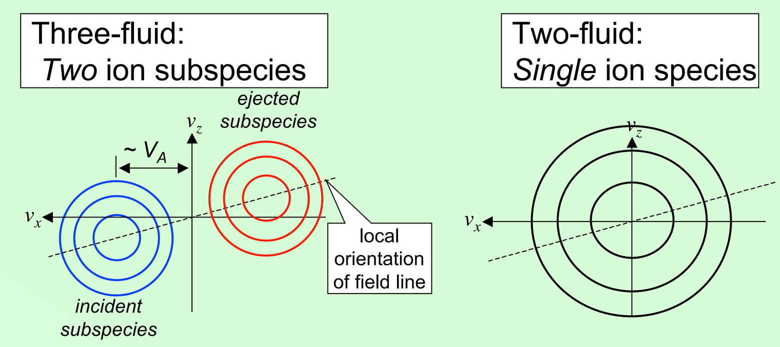

**Ions in stationary-x-point frame.**

Though not shown in the adjacent figure, ions have tangential ($u_y$) as well as normal velocity.

It is assumed that the various ion streams do not interact directly with each other. Rather, the interactions are through the magnetic and electric fields.

The behavior is analyzed in the deHoffman-Teller frame where there is no out-of-plane electric field, $E_y = 0$. Ions flow parallel to **B** far from the sheet.

## The Three-Fluid Model Allows Highly Anisotropic Ion Distributions




Distribution function contours in deHoffman-Teller frame

The two-fluid model lumps all the ions together into a single grouping: thus it cannot represent cases with significant anisotropy

## Physics Included in Model

1. mass conservation for each fluid
2. momentum conservation for each fluid, including ion inertia, pressure, and electromagnetic forces), but no fluid-fluid interactions, except through macroscopic electromagnetic fields
3. equations of state for each fluid (pressure related to density)
4. Ampere's law in terms of charge fluxes, Faraday's law, no magnetic monopoles

This gives a system of 19 equations in 19 unknowns ($\mathbf{B}$, $\mathbf{u}_\alpha$, $n_\alpha$, $p_\alpha$, $\phi$)

## Model Assumptions

1. one-dimensional ($\partial/\partial x = \partial/\partial y = 0$)
2. steady state ($\partial/\partial t = 0$)
3. quasi-neutrality ($\Sigma\, q_\alpha n_\alpha = 0$)
4. massless electrons
5. $E_x = E_y = B_y = 0$ far from current sheet
6. isothermal electron fluid ($p_e = n_e k_B T_e$, with $T_e =$ *constant*)
7. ion fluids described by a polytropic equation of state ($p_i n_i^{-\gamma} =$ *constant*)
8. no trapped ions; however, there are two symmetric, identical, singly charged, ion fluids: ($n_1 = n_2$; $u_{1y} = u_{2y}$ ; $u_{1x} + n_{2x} = 0$; $u_{1z} + n_{2z} = 0$ )

Assumptions 7 and 8 can be modified to model the observations more accurately

## General Solution for Model

One numerical integration to determine spatial structure. Other relations between variables (n, $B_x$, $u_x$, $u_y$) are algebraic.

$$n \equiv n_e = 2n_1 = 2n_2$$

$$\mathbf{u}_e = 0$$

$$K \equiv n_1 u_{1z}$$

$$u_x = \frac{B_z}{8\pi m K} B_x$$

$$u_y^2 = \hat{u}_y^2 + \frac{4K^2}{\hat{n}^2}\left(1 - \frac{\hat{n}^2}{n^2}\right) + \frac{B_z^2}{2\pi m \hat{n}}\left(\frac{\hat{n}}{n} - 1\right) + \frac{4\hat{p}}{\hat{n} m}\left(\frac{\gamma}{\gamma - 1}\right)\left(1 - \frac{n^{\gamma-1}}{\hat{n}^{\gamma-1}}\right) + \frac{B_z^2 \hat{p}}{4\pi m^2 K^2}\left(\frac{n^\gamma}{\hat{n}^\gamma} - 1\right) + \frac{2k_B T_e}{m}\log\left(\frac{\hat{n}}{n}\right) + \frac{\hat{n} k_B T_e B_z^2}{8\pi m^2 K^2}\left(\frac{n}{\hat{n}} - 1\right)$$

$$B_x^2 = \hat{B}_x^2 + 8\pi\left(\frac{4mK^2}{\hat{n}}\left(1 - \frac{\hat{n}}{n}\right) + 2\hat{p}\left(1 - \frac{n^\gamma}{\hat{n}^\gamma}\right) + \hat{n} k_B T_e\left(1 - \frac{n}{\hat{n}}\right)\right)$$

$$B_y = E_x = E_y = 0,\ B_z = \text{constant}$$

$$E_z = -\frac{k_B T_e}{en}\frac{dn}{dz}$$

$$\int dz = \frac{c}{4\pi e}\int\left(\frac{dB_x}{dn}\right)\frac{dn}{n u_y(n)}$$

Hatted quantities are evaluated at a fixed location (initial conditions)

Formulation uses cgs units

## Model Extensions

Assume a third trapped ion population is also present, and assume ions are described by CGL pressure equations.

- Requires large normal magnetic field, $B_z = 0.3$
- Needs large anisotropy with high $\beta_{p\perp}$ and low $\beta_{p||}$
- High asymptotic field-aligned flow, Mach number = 2.8
- Current and density profiles differ are hollow at the center while $B_x$ and $J_y$ are very unlike the Harris solution.

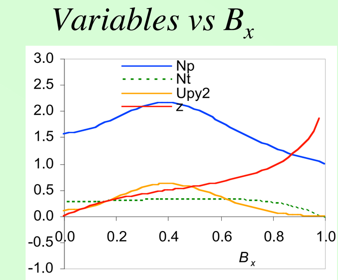
*Input parameters*

| $B_z$ | $\beta_e$ | $\beta_{P,\ perp}$ | $\beta_{T,\ perp}$ |
|---|---|---|---|
| 0.3 | 0.2 | 14 | 0.8 |
| | | $\beta_{P,\ par}$ | $\beta_{T,\ par}$ |
| | | 0.3 | 0.3 |

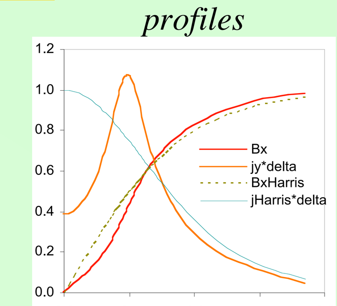
*Output point values*

| $M_A$ (inf) | δ | $N_p$ (0) |
|---|---|---|
| 2.818 | 0.909 | 1.570 |

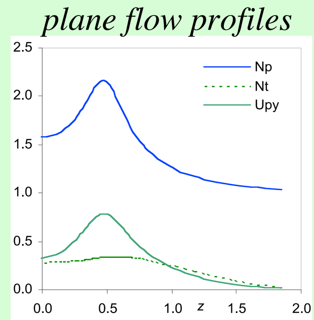
*Variables vs $B_x$*




*Field and current profiles*




*Density and out-of-plane flow profiles*




Now, allow trapped ions to have isotropic, instead of CGL, pressure.

Results are similar to above, except trapped ion density vanishes.

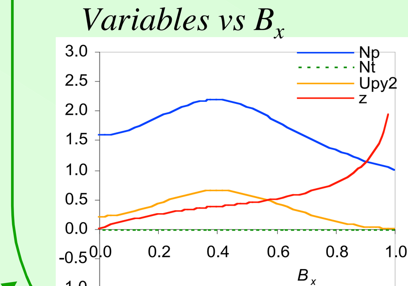
*Input parameters*

| $B_z$ | $\beta_e$ | $\beta_{P,\ perp}$ | $\beta_{T,\ perp}$ |
|---|---|---|---|
| 0.3 | 0.2 | 14 | 0.8 |
| | | $\beta_{P,\ par}$ | $\beta_{T,\ par}$ |
| | | 0.3 | 0.3 |

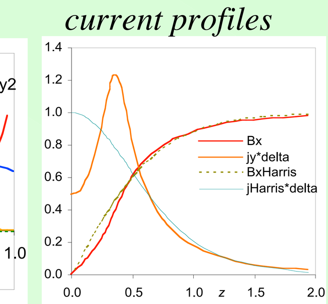
*Output point values*

| $M_A$ (inf) | δ | $N_p$ (0) |
|---|---|---|
| 2.818 | 0.708 | 1.576 |

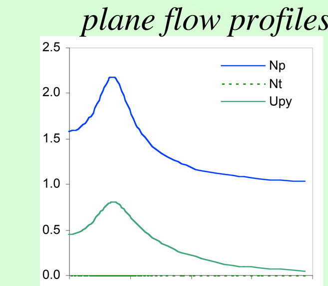
*Variables vs $B_x$*




*Field and current profiles*




*Density and out-of-plane flow profiles*




## Example Solution for Model

Asymptotic $B_z/B_x = 0.1$ and Alfven Mach number = 1.16

Near sheet, current and number density profiles are similar to Harris model

Unlike Harris model, the out of plane flow, $u_y$, falls to zero, while density does not

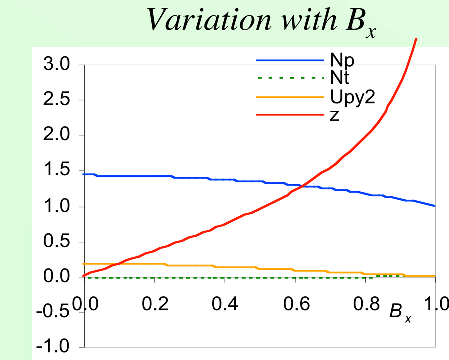
*Input parameters*

| $B_z$ | $\beta_e$ | $\beta_{P,\ perp}$ | $\beta_{T,\ perp}$ |
|---|---|---|---|
| 0.3 | 0.2 | 1 | 1 |

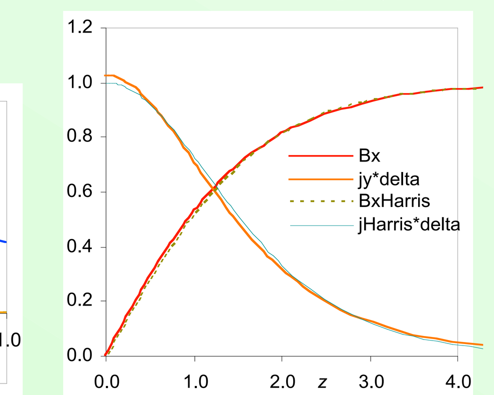
*Output point values*

| $M_A$ (inf) | δ | $N_p$ (0) |
|---|---|---|
| 1.122 | 1.601 | 1.475 |

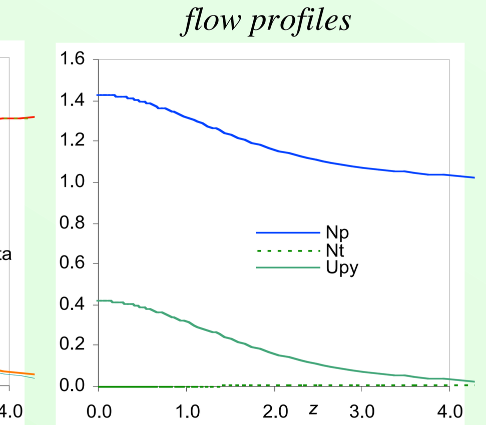
*Variation with $B_x$*




*Field and current profiles*

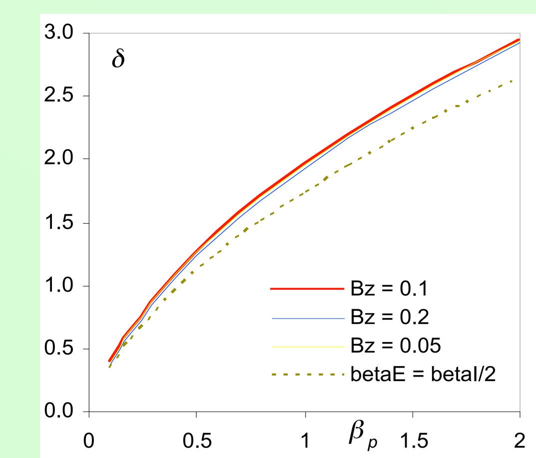


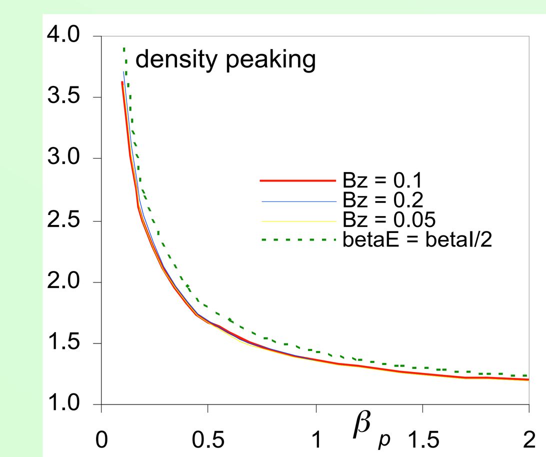

*Density and out-of-plane flow profiles*




## Parameter study for solutions

1. Current sheet thickness, δ, depends roughly on the square root of ion pressure.
2. Weak dependence of solutions on $B_z/B_x < 0.1$
3. Ion pressure broadens the sheet. Cooler electrons reduce thickness slightly.
4. The density at the center of the current sheet decreases with large ion pressure.




## Conclusions: Usefulness of Fluid Current Sheet Models

- The isotropic ion model is an improvement on the Harris model: $B_z$ is taken into account; it allows passing particles with asymptotic non-zero density and asymptotically vanishing out-of-plane drift speed, $u_y$.
- They give rise to predictive relationships between the ion pressure, the current sheet thickness, and the density peaking factor.
- Comparison of data with models can be used to help determine the equations of state that may apply under different conditions.
- They are simpler to construct than kinetic models. Also, moments of measured distribution functions (**u**, n, p) will provide averages over peculiarities and/or fluctuations in measurements, avoiding some of the difficulties of working with the full velocity distribution function.

## Acknowledgments

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