NOTES AND CORRESPONDENCE

The Effect of Concentrated PV Gradients on Stationary Waves: Correction

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ABSTRACT

This note corrects a numerical error in a prior work of Lindzen. The correction eliminates the strong sensitivity found in the earlier paper to the details of the concentration of potential vorticity gradients at the tropopause. Remaining sensitivities in the simple calculations presented are largely related to variations in the basic state's zonal wind.

1. Introduction

In attempting to make use of an earlier paper (Lindzen 1994), in which rather dramatic sensitivity was found for stationary wave response to the concentration of potential vorticity (PV) gradients at the tropopause, we discovered a numerical error in the implementation of the lower boundary condition. Regrettably, much of the interesting sensitivity discussed in Lindzen (1994) turns out to be an artifact of the error. There remains some significant sensitivity near the surface to the degree of mixing of PV within the troposphere; however, even this sensitivity is greatly diminished above about 1 km. At all levels, there is a 30% amplification in response for zonal wavenumbers greater than 3 when PV gradients are eliminated in the troposphere and concentrated at the tropopause. However, while significant, this is less dramatic than the earlier erroneous results and is dependent on how one assumes PV to be mixed. Some new features emerge involving sensitivity to stratospheric flow, but there are good reasons to expect that these features will not be important in practice. Our corrected results completely support the findings of Swanson and Pierrehumbert (1994), who found that the response is dominated by the stationary external mode and that the horizontal scale of this mode is not very dependent on the details of the PV distribution. However, the vertical structure of this mode and the specific response at specified wavenumbers are still dependent on details of the basic state.

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The equations, basic state specifications, and forcing are exactly the same as in Lindzen (1994) and will not be repeated in this note. In section 2, we will simply present the relevant corrected results. In section 3, we will discuss these results and briefly explore the relevance of properly accounting for PV mixing in calculating the response to stationary forcing.

2. Corrected results

Figure 1 shows the response to surface forcing as a function of zonal wavenumber, s, both at the surface and at 10 km for both completely adjusted and unadjusted basic states. Both states are described in detail in Lindzen (1994). The unadjusted state corresponds roughly to a standard atmosphere, while the adjusted state has been modified to have zero PV gradient below the tropopause with increased concentration of PV gradients at the tropopause. The adjustment was chosen to involve changes in both static stability and mean flow. Changes at 10 km are rather small, and this is essentially true at all altitudes above 1 km. However, for zonal wavenumbers greater than about 3.6, there is a fairly uniform increase in response amplitudes of about 30%. This continues to at least $s = 20$, though results are shown only for $s \leq 7$. Detailed calculations show that this change arises mostly from changes in basic state shear at the lower boundary that result in small shifts in the position of the external mode resonance. When PV gradients are eliminated purely by changing N^2, the increase is more nearly 15%, while eliminating PV gradients exclusively by changing the basic state zonal flow U results in greater increases. The increase, moreover, is different at different levels since there are changes in the vertical structure.

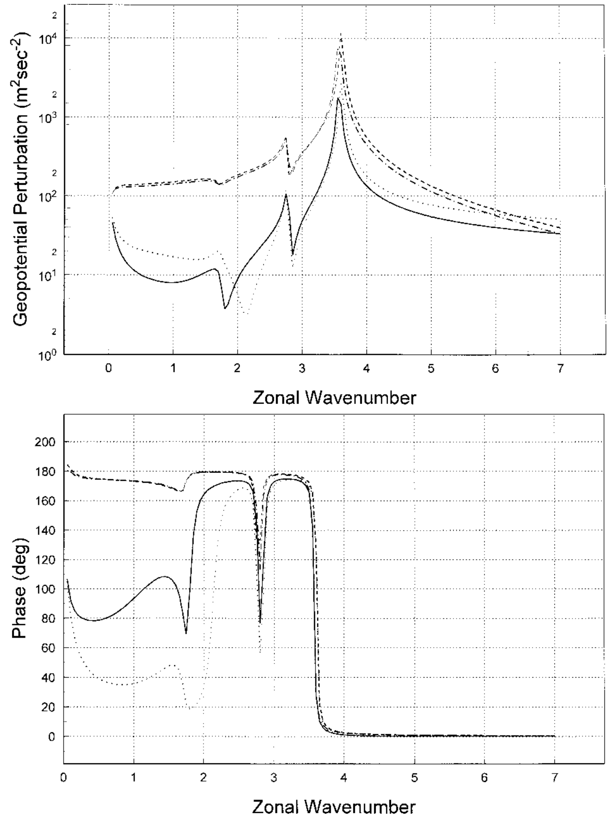


FIG. 1. Response to 100-m surface forcing as a function of zonal wavenumber *s.* (a) Amplitude of response; (b) phase of response. — corresponds to surface for unadjusted basic state, ··· corresponds to surface for adjusted basic state, -· corresponds to 10 km for unadjusted basic state, and – corresponds to 10 km for adjusted basic state. The illustrated results correspond to high resolution and low damping.

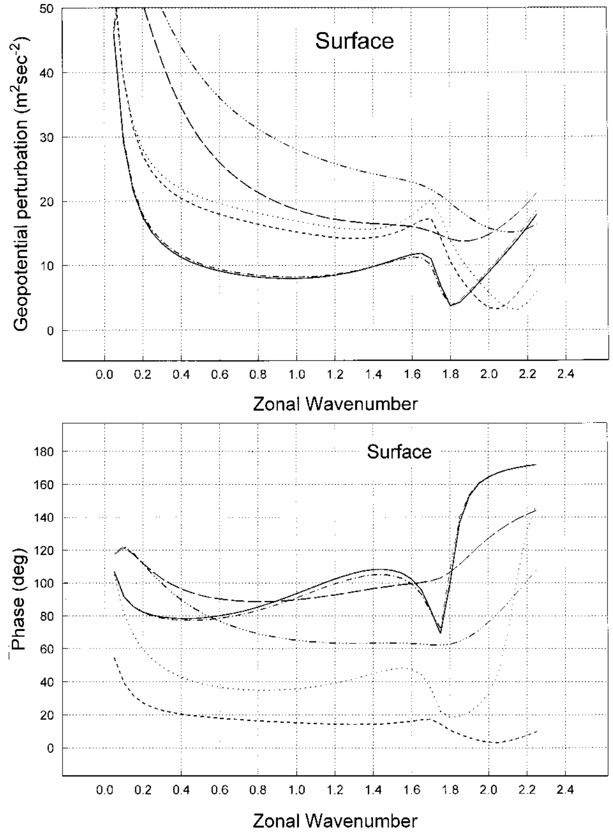
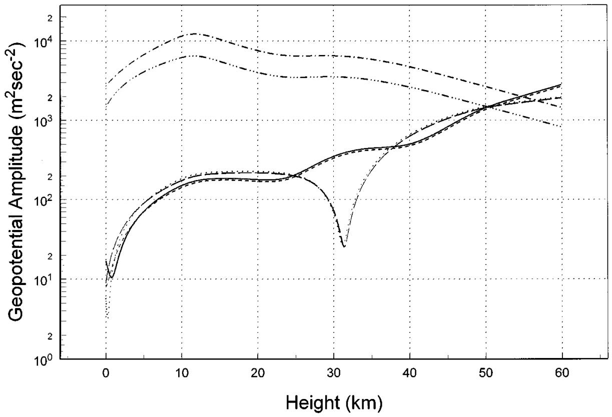


FIG. 2. Surface response to 100-m surface forcing as a function of zonal wavenumber *s.* (a) Amplitude; (b) phase. — corresponds to high resolution, low damping, and unadjusted basic state; ··· corresponds to high resolution, low damping, and adjusted basic state; -· corresponds to low resolution, low damping, and unadjusted basic state; - corresponds to low resolution, low damping, and adjusted basic state; — corresponds to high resolution, high damping, and unadjusted basic state; and -··· corresponds to high resolution, high damping, and adjusted basic state.

Surface changes are most notable at low zonal wavenumbers. This is illustrated in Fig. 2, where we also show the effects of reduced vertical resolution in our numerical solution algorithm (from δz = 100 m to δz = 2.5 km, where δz is the grid interval) and increased applied linear damping (from .01 days^{-1} to .05 days^{-1}). Although both changes alter the results, the differences between adjusted and unadjusted states remain. Moreover, it is clear that the results are no longer strongly dependent on the ability to resolve the concentrated PV gradients at the tropopause. The surface effects appear again to be more related to the change in surface shear associated with adjustment than to the elimination of q_y (the basic state PV gradient) in the interior of the troposphere. For $s \leq 3$, the vertical structure of the response to surface forcing is largely unaffected by reduction of tropospheric q_y and the concentration of gradients at the tropopause, though the use of a logarithmic plot in Fig. 3a de-emphasizes changes. Significant changes in structure were found for $s > 3$. In point of fact, both adjusted and unadjusted states have large concentrations of q_y at the tropopause.

This is seen in Fig. 3a, which shows amplitude as a function of height for zonal wavenumber, s, equal to 1, 2, and 3.6, the last corresponding to a wavenumber close to the resonant external mode. The change in overall amplitude for s = 3.6 is of no practical consequence and is associated with slight changes in the precise value of the resonant wavenumber. Figure 3b shows the phase as a function of height for the unadjusted basic state; the results are not different above 1 km for the adjusted state. We see that the character of the response is different for each wavenumber. The external mode is essentially trapped everywhere above the tropopause. Wavenumber 1 propagates vertically everywhere, and wavenumber 2 propagates in the neighborhood of the minimum in zonal basic flow at 30 km but is trapped above about 50 km. This leads to a reflection that in turn produces a node in amplitude at about 30 km and a phase structure characteristic of a standing wave. The situation is more clearly illustrated in Fig. 4, which shows an approximate measure of the index of refrac-



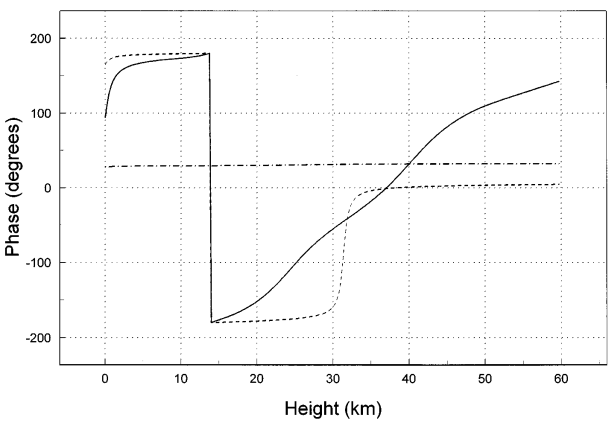


FIG. 3. Vertical structure of response for $s = 1$, $s = 2$, and $s = 3.6$. (a) Amplitude; (b) phase. — corresponds to $s = 1$ for adjusted basic state; - corresponds to $s = 2$ for adjusted basic state; -· corresponds to $s = 3.6$ for adjusted basic state; - corresponds to $s = 1$ for unadjusted basic state; — corresponds to $s = 2$ for unadjusted basic state; and -··· corresponds to $s = 3.6$ for unadjusted basic state. The illustrated results correspond to high resolution and low damping.

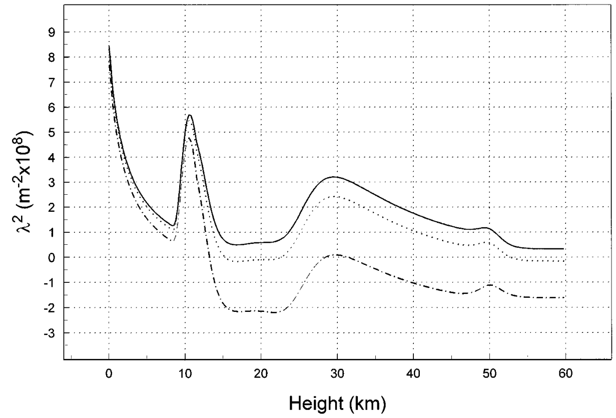


FIG. 4. Approximate index of refraction as a function of height. — corresponds to $s = 1$; ·· corresponds to $s = 2$; and -· corresponds to $s = 3.6$. The illustrated results are for unadjusted basic state.

tion, λ^2, as a function of height for the three zonal wavenumbers; that is,

$$\lambda^2 = \frac{N^2(z)}{f^2}\left(\frac{q_y^2(z)}{U(z)} - k^2 - l^2\right) - \frac{1}{4H^2}, \qquad (1)$$

where k is the zonal wavenumber [where $k = s/a\cos(\phi)$, a being the radius of the earth, and ϕ latitude], l is the meridional wavenumber (which we take to be π/a), N is the Brunt–Väisälä frequency, and f is the Coriolis parameters at 45°N. Here, H is a reference scale height which is taken to be constant. Propagation is associated with positive values for λ^2.

3. Discussion

The corrected results almost completely eliminate the sensitivity to concentrated PV gradients at tropopause level that was the focus of Lindzen (1994). The corrected results are much less demanding on model resolution. The analytic results, used by Lindzen (1994) to explain the incorrect results, were simply meant to illustrate the role of a region of concentrated positive index of refraction embedded in either a trapping or propagating environment. For the former, the existence of the delta function region of positive index permitted the existence of a resonance that would not otherwise have existed, given the simple lower boundary condition. The correct lower boundary condition does allow an external mode; the error in Lindzen (1994) left one with a situation more nearly like that in the simple model. For the propagating environment, the simple model results arose from the fact that the delta function region produced a partial reflection of upward propagating waves where the phase of the reflected wave depended on wavenumber and indices of refraction both outside and inside the region of concentration. The actual problem corresponds to only a small part of the parameter space explored in the simple analytic model ($1 \leq \bar{x} \leq 1.5$), and within this restricted region variations are small.

The remaining sensitivities in the simple stationary wave problem considered are largely determined by the distribution of U rather than q_y. The fact that the horizontal wavenumber of the external mode is relatively independent of various assumptions by no means implies that the structure of these modes is independent of the basic state chosen. For example, the external mode amplitudes are strongly concentrated at the tropopause level (viz. Fig. 3a), rather than smoothly decaying away from the surface (apart from the usual exp $[z/(2H)]$ growth) as would be the case for a constant U and N^2 (Lindzen 1968). The smaller features near $s = 2$ and $s = 3$ in Fig. 1 are associated with ducted waves trapped in the middle atmosphere. It is unlikely that these features will prove important in practice since they depend on reflecting surfaces that are horizontal, whereas in reality such surfaces will be deformed (Lindzen and Hong 1974). There is also increased radiative damping in the stratosphere. It is probably inappropriate to conclude from the present note that the ability of a model to simulate PV mixing is unimportant to its ability to simulate stationary waves. The 30% increase in response associated with $s > 3.6$ is large enough to be of practical importance. Moreover, as shown in Nigam and Lindzen (1989) and in Da Silva and Lindzen (1993), there remains a strong sensitivity in the response to the Himalayas to the position of the subtropical jet. Preliminary calculations suggest that PV mixing does tend to concentrate the jet in the Tropics, and thus an indirect influence may remain to the extent that the position of the jet is dependent on the intensity with which PV is mixed. More broadly and obviously, stationary waves do propagate away from their sources, and the nature of this propagation depends on the index of refraction, which is dependent on q_y. Of course, the present one-dimensional study cannot fully display this dependence.

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