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A comparison of competing explanations for the 100,000-yr ice age cycle

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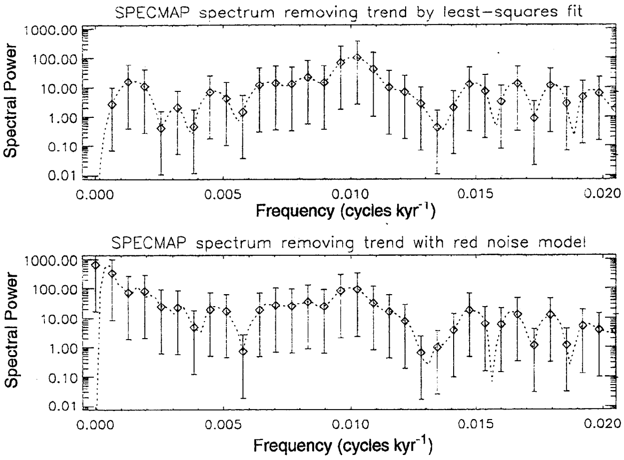
Abstract. There currently exists no consensus as to the cause of the ice ages of the late Pleistocene. Many of the competing hypotheses have been formulated into mathematical models which enables a rigorous comparison to be made. Spectral analysis fails to distinguish rival models. Using regression analysis, we examine the relative performance of several models, each representative of a different type of modeling approach, as explanations of both the global ice volume and also the time rate of change of the global ice volume. We find there is no objective evidence in the record in favor of any particular model. The respective merits of the different theories must therefore be judged on physical grounds.

An explanation for the 100,000-year (100-ky) cycle in global ice volume during the late Pleistocene (the last 700-ky or so) is one of the major problems confronting paleoclimatology, and has recently been a source of renewed controversy. The traditional focus of research is the Milankovitch hypothesis — that the ice ages are a consequence of the temporal variations of the latitudinal distribution of insolation (solar heating), caused by the periodic changes in the earth's orbital parameters [*Milankovitch,* 1941]. The principal parameters and their periods are: eccentricity (400-ky, 125-ky, and 95-ky), obliquity (41-ky), and climatic precession (19-ky and 23-ky). There is general acceptance that the Milankovitch hypothesis is at least partially correct; global ice volume during the late Pleistocene has responded approximately linearly to the obliquity and climatic precession parameters [e.g. *Imbrie et al.,* 1992]. However, the global ice volume data show a dominant 100-ky 'saw-toothed' cycle, whereas insolation time series would predict most of the spectral power to be concentrated at the obliquity and climatic precession time scales. *Muller and MacDonald* [1997] (hereafter MM97) have recently suggested an alternative mechanism for the ice ages, noting that variation in the inclination of the earth's orbital plane relative to the invariable plane of the solar system also has a period of 100-ky. They postulate that the changes in accretion of extraterrestrial dust in the atmosphere could thus be a driver of the climate system on the 100-ky time scale.

The key observational evidence advanced by MM97 in favor of their hypothesis is the (dis)similarity between the power spectra of various proxies for global ice volume over the past 900-ky and the power spectrum of orbital inclination (eccentricity). However, results from spectral analysis may well be misleading when applied to signals which are known to be non-sinusoidal in a record which is short relative to the time scales of interest. Figure 1 shows the standard periodogram of the SPECMAP index (a proxy for global ice volume for the last 780-ky [*Imbrie et al.,* 1984]) as computed by MM97 using a box-car window: diamonds show discrete spectral estimates at the individual Fourier frequencies, vertical bars show the 95% range of uncertainty in these estimates given the length of the data series [e.g. *Wei,*


SPECMAP spectrum removing trend by least-squares fit

SPECMAP spectrum removing trend with red noise model

Figure 1. Power spectra for 780-ky SPECMAP proxy for global ice volume, using a standard periodogram and a box-car window on the data. The error bars represent the 95% range of uncertainty, and the dashed line is the power spectra obtained by zero padding 1000-ky either end of the data set. In the upper panel the trend was removed using a least squares fit. In the lower panel the trend was removed using an AR(1) noise model.

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1990] and the dotted line shows a continuous 'interpolated' spectrum obtained by zero-padding the data for 1000-ky at either end of the data set before Fourier transforming. We stress that any apparent additional information in the continuous spectrum (e.g. the regular bumps at higher frequencies) is illusory — it is a simple artifact of the zero-padding procedure. Given the uncertainties in the individual spectral estimates, such power spectra clearly cannot be said to have settled the issue in favor of inclination. A key problem with spectral analysis of series of this type is that results are extremely sensitive to the methods used to pre-process the data. The upper panel, which corresponds closely to column 3 in Figure 3 of MM97, shows the spectrum after the trend has been removed by a simple least-squares fit. The lower panel shows the corresponding result after removing a maximum-likelihood estimate of the trend based on an AR(1) noise model for the stationary variability [e.g. *Wei,* 1990]. Clear discrepancies emerge at the lowest frequencies, indicating the dangers of drawing conclusions about the presence or absence of power at 400-ky periods in a 780-ky data set.

Many of the various other hypotheses for the cause of the ice ages have been formulated into mathematical models, which may be characterized broadly into groups [e.g. *Imbrie et al.,* 1993] Typically, the performance of these different models is evaluated by comparing their spectral characteristics with the ice volume record, or even by just overlaying plots of the model predictions and the data. There is thus a clear need to quantifiably compare the ability of models from these different groups to account for the late Pleistocene ice volume. Since the signals concerned may be non-sinusoidal, the most natural approach is to use a standard regression model [e.g. *Wei,* 1990] (spectral analysis may be thought of a special case of regression where signals are sinusoidal). A regression analysis based, as here, on an autoregressive (AR) process for the residuals also allows for a self-consistent account of the climate system's response to additional (unknown) noise forcing. This is essential if confidence intervals are to have any meaning.

We have taken six models, chosen to be each representative of a different modeling approach, and each characterized by just a few tunable parameters. As a proxy for global ice volume, we choose the composite stack of deep-sea cores created using graphic correlation methods by *Prell et al.* [1986]. We choose this stack, in preference to the more often used SPECMAP stack, because it has had no smoothing applied to it, which would introduce extraneous auto-correlations. The data were linearly interpolated onto a 5-ky grid. We perform the regression analysis for the last 600-ky which is the interval of time over which the 100-ky cycles are most clearly established. The first model (referred to as ECC) is a linear combination of the eccentricity, obliquity, and climatic precession orbital parameters [*Berger,* 1978] - what might be termed the basic Milankovitch hypothesis. Allowing for arbitrary amplitude and lag of the orbital parameters in the ECC model gives 6 free parameters. The second model (M&M) is motivated by Muller and MacDonald's inclination hypothesis and is a linear combination of inclination, obliquity, and climatic precession. We again allow for arbitrary amplitude and lag of the orbital parameters in the M&M model, which therefore also has 6 free parameters.

Two other approaches, pursued by Saltzman among others [e.g. *Saltzman and Sutera,* 1984], treat the 100-ky periodicity as an internal climate oscillation; either phase locked to orbital forcing, or a free oscillation of independent phase. To represent the phase locked oscillation we take the forced oscillator model of *Maasch and Saltzman* [1990], which is the result of a set of postulated feedbacks between global ice volume, atmospheric CO_2, and ocean circulation/heat content. The summer insolation at $65N$ is assumed to represent the external forcing on the ice volume. The model is denoted as STZ and has 5 free parameters. For the phase independent 100-ky climate oscillation (OSC model) we simply take the unforced Maasch and Saltzman model and include a linear combination of orbital precession and obliquity (giving 9 free parameters). In yet another approach, *Imbrie and Imbrie* [1980] exploit the asymmetric growth and decay of ice sheets to extract 100-ky power from the envelope of the climatic precession signal (the I2 model, 4 free parameters). Lastly, *Paillard* [1998] has recently modeled the ice ages as being the consequence of three possible climate states, transitions between which occur if the high latitude insolation crosses certain threshold values (the PLD model, 7 free parameters).

In each of the above models, we treat the residuals as either a first- or second-order autoregressive process (AR(1) or AR(2)). The AR(1) process, or 'red noise', is the simplest self-consistent model of climate variability [*Hasselmann,* 1976], and we also consider the AR(2) model to assess AR(1) model adequacy. Higher order AR noise models did not produce significantly better explanations of the data. The models' free parameters were adjusted to optimize the fit to the data, taking into account the properties of the noise estimated from the residuals, by minimizing the corrected mean square prediction error [e.g. *Wei,* 1990]. The assumption of a linear, additive model of residuals is clearly restrictive, but given the short data record, we wish to confine ourselves to the simplest self-consistent model available to avoid the problem of over-parameterization.

The results for the both the AR(1) and AR(2) residual models are shown in Figure 2. The first column of Table 1 gives the confidence level, based on the standard F-statistic [e.g. *Press et al.,* 1990] at which we can rule out the various models as being significantly worse than the best available explanation of the ice volume using an AR(1) process for the residuals, taking into account the number of free parameters in each model. While the OSC model is the nominally the 'best' explanation of the ice volume, only the I2 model can be

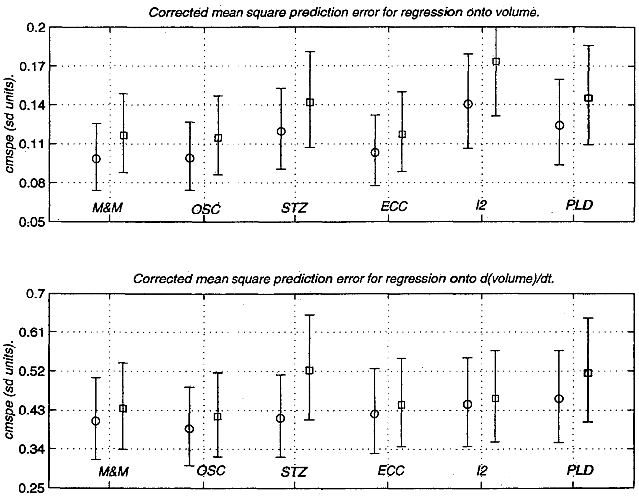

Figure 2. Corrected Mean Square Prediction Error (cmspe) for the regression analysis of the different models described in the text onto global ice volume (upper panel), and the time rate of change of the global ice volume record (lower panel). The error bars are the 95% range of uncertainty. The squares are for regression analysis done using an AR(1) residuals model and the circles are for regression analysis using an AR(2) residuals model. The smaller the cmspe, the better the explanation of the data. However, if the error bars overlap substantially, the significance of the difference between the model fits is low.

ruled out as doing worse at the 95% confidence level. The ECC model does surprisingly well, but this result is misleading: it is likely we can reject the ECC model on physical grounds because the optimum lag in the eccentricity (49-ky) is unphysical, and in addition there is no known mechanism by which the eccentricity alone can cause glaciation: by itself eccentricity only affects globally averaged insolation, and then only by a very small amount [e.g. *Crowley and North*, 1991].

For all the models, the explanation of the data is improved by using an AR(2) process for the residuals rather than an AR(1) process. The F-test significance of the improvement varies between 75% in the ECC model and 87% in the I2 model. The AR(2) processes represent weak internal oscillations which were all found to have a period around 100-ky, suggesting that none of the models (including inclination) are a complete explanation of the 100-ky power in the data.

All the models presented (and any other insolation based model) are based on mechanisms which act to warm or cool the climate system. They should therefore be more properly considered as models of the rate of change of global ice volume rather than for the the global ice volume itself. Taking rate of change of global ice volume deemphasizes the 100-ky periodicity in the data, which is just a statement that the changes in the heating of the climate have been concentrated at the obliquity and climatic precession time scales. We repeat the above regression analysis but instead treating the models as explanations of the rate of change of global ice volume. We do not expect the results to necessarily be the same: for example, a sinusoidal forcing is a better explanation of a sawtooth cycle than the rate of change of a sawtooth cycle.

The data was linearly interpolated onto a 4-ky grid, and centered differencing was used to obtain the time rate of change of the data. The time step was lessened because of the increased variability at shorter time scales, although doing this did not significantly affect the results. The OSC, STZ, I2, and PLD models are formulated in terms of the rate of change of ice volume. In this test, it is the OSC model which again proves to be the 'best' explanation of the data, as shown in Figure 2. However, none of the models can be ruled out at the 95% confidence level, as shown in the second column of Table 1. We would however again reject the ECC model on physical grounds: the optimum eccentricity lag, at 24-ky, is still unrealistically large.

We have found, comparing six very different theories of ice-age cycles, that the global ice volume record does not provide objective evidence in favor any particular theory over the others – indeed, the equivalence of all the theories considered in terms of their agreement with data is quite remarkable. Given the small number of parameters in each of the models presented here, we suggest it is likely that any model that uses only a few parameters to generate a 100-ky cycle, together with the obliquity and climatic precession indices, will do comparably well. Our results therefore demonstrate that spectral analysis or regression modeling against the available ice volume data cannot resolve the 100-ky question in favor of any one hypothesis. Credibility given to a particular theory must therefore be based on additional criteria. Further study and modeling efforts should examine in detail the physical plausibility of the different mechanisms invoked in the rival models. The

Table 1. F-test significances of model performance

Model	Free parameters	F-test sig. (regressing on ice volume volume)	F-test sig. (regressing on rate of change of ice volume)
M&M	8	52.9	60.0
OSC	11	—	—
STZ	7	86.7	91.0
ECC	8	54.9	64.5
I2	6	98.5	71.2
PLD	9	89.2	89.8

F-test significances of each of the model's performance relative to the model which was the best explanation of the data (the OSC model in both cases). A value of 50% would mean the two models are equally good explanations of the data. The results presented are all for an AR(1) residuals model, and the free parameters include those for the residuals model.

prediction of other paleo-evidence, such as the concentrations of atmospheric CO_2, and the onset of the oscillations themselves around 750-ky ago, must also be given due weight.

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