Critical form and feedbacks in mountain-belt dynamics: the role of rheology as a tectonic governor.

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Abstract

The rheology that governs deformation within a convergent orogen also controls its topographic form, or in other words the co-varation of its height (or thickness) and its width. Under conditions pertaining to small orogens, and for rheologies ranging from linear viscous to Coulomb plastic, we show that this topographic form is extremely insensitive to the assumed pattern of crustal underplating or surface erosion, and is thus a robust property of the system. It can therefore be thought of as a 'critical' topographic form, directly analogous to the well-studied case of a Coulomb plastic rheology, which produces a wedge-shaped constant critical taper angle independent of its size. The robust tendency of the system to acquire this critical topographic form can be regarded as a 'tectonic governor' that strongly damps the response of the orogen to changes in climate or tectonic forcing. Scaling relationships can be derived for the variation of orogen width, height, and exhumation rate as a function of accretionary flux and precipitation rate. This study explores how sensitive these scaling relationships are to the assumed rheology. It is found that the scaling relationships are fairly constant for the wide range of rheologies considered: exponents vary by less than a factor of two for common choices of model parameters. These results suggest that the first order behavior of such orogens are likely to be well captured by the scaling relationships, and are relatively insensitive to alternative assumptions.

1 Introduction

For as long as the Earth Sciences have been studied, a major motivating goal has been to understand what controls the fundamental attributes of a mountain range such as its height, its width, and the pattern and magnitude of the deformation of rock within it. At the broadest level, the challenge reduces to characterizing the relative importance of tectonic, erosional, and climatic processes, the combination of which must ultimately be controlling the system. While these questions are obviously of enormous importance to many individual disciplines within the field, research has tended to approach the challenge from quite separate perspectives. For example, many studies exploring the role of climate in landscape evolution have assumed that rock uplift rates are an imposed and fixed property of the system (e.g., Howard et al., 1994; Whipple et al., 1999; Roe et al., 2003). In contrast, geodynamical studies exploring the processes setting the characteristic scales of the orogen have often neglected the role of erosion (e.g., England et al., 1985; Ellis et al., 1995). Only a few numerical models have been developed that have self-consistent representations of the climatic, tectonic, and erosional processes where the interactions are internally-determined (e.g., Beaumont et al., 1992, 2000). Moreover the interpretation of these numerical studies has been hampered by the absence of a theoretical framework capable of quantitative predictions.

It is only recently that the burgeoning interest in tectonic geomorphology has brought these separate approaches together, combining climate, erosion, and tectonics into a single picture. A series of recent studies has exploited the concept of a critical Coulomb wedge (Hilley et al., 2004; Whipple and Meade, 2004, 2006; Roe et al., 2006, 2008; Stolar et al., 2006, 2007; Tomkin and Roe, 2007). Under the assumption that the crust behaves as a Coulomb plastic material, an orogen maintains a wedge-shaped, self-similar critical form during orogenesis (e.g., Chapple, 1978; Davis et al., 1983; Dahlen, 1984; Dahlen, 1990). The power of this approach is that it provides an essentially geometric rule for the role of tectonics in orogen evolution. This geometric rule can be combined with models of fluvial or glacial erosion to yield scaling relationships for the relative importance of the accretionary flux or the climate in setting the scale of the orogen and the rock uplift rates within it. The lessons learnt from these conceptual models have proven extremely useful in understanding interactions among climate, erosion and tectonics in a number of setting, summarized recently in Whipple (2009).

Using this framework, Roe et al. (2008) show that the self-similar form acts as a powerful 'tectonic governor' acting to strongly damp the landscape response to changes climate and tectonic forcing. Mechanical governors came to the fore during the development of the first practical engines that turned power into work (Figure 1). It was recognized that a feedback (or governor) was necessary to regulate the process and that the degree of control could be described mathematically (Maxwell, 1868). In many dynamical systems it is not only the individual processes but the internal feedbacks that modulate how the system responds to changes. This recognition is the foundational concept of control systems theory, and is central to coupled system dynamics (e.g., Roe, 2009). In this paper we demonstrate how the overall form of a mountain belt, as dictated by its rheology, controls the response of the mountain range to changes in tectonic flux and climatically-driven erosion.

Figure 2 depicts the operation of the tectonic governor in the setting of a small convergent orogen. As an example, envision a climate change that increases precipitation, which increases erosion rates and so tends to reduce orogen size. The tectonic requirement of self-similar form means the reduction in relief is accompanied by a narrowing of the orogen. In turn, this narrowing focuses the same accretionary flux into a smaller area, which enhances the local rock uplift rates and so opposes the original forcing. The system response is therefore damped, and so the mechanism is a negative feedback (or governor) on topography. The key points are that the Coulomb-plastic rheology forces the orogen width and height to co-vary, and that this co-variation produces a negative feedback. The scaling relationships have been evaluated and confirmed in numerical models (Stolar et al., 2006, 2007) and the framework is particularly useful in that it allows for a formal characterization of the relative strengths of tectonic and climatic feedbacks (Roe, 2009; Roe et al., 2008).

There has thus been an almost total reversal of perspective: whereas the earlier geomorphological studies took the view that tectonics imposes the rock uplift rate, leaving erosion to set the form of the landscape, the more recent studies would argue that it is tectonics that sets the form of the landscape (via the geometry it imposes), and that it is erosion rates that set the rock uplift rates (because they must balance at steady state).

In this study we explore the extent to which rheology affects the results arising from this new perspective. Although there is evidence that many orogens do have a mean topographic form approximating that of a Coulomb critical wedge (e.g., Dahlen, 1984; Dahlen, 1990; Willett, 1999a; Whipple and Meade, 2004), the use a Coulomb-plastic rheology is a strong and possibly restrictive assumption about the behavior of the crust. Based largely on the work of Emerman and Turcotte (1983), we define a power-law viscous rheology which, with two free parameters, can be varied from linear viscous to Coulomb plastic. We solve for the topographic form resulting from such a rheology - the lower the exponent on the power-law rheology, the less sensitive the orogen height is to variations in orogen width. A striking result is that the topographic form is extremely insensitive to the pattern of crustal underplating or surface erosion. It can thus be thought of as a *critical topographic form*, and is a robust property of the system. The term critical is used here not in the sense of a critical yield stress in a plastic material, but in the sense that the tendency to maintain topographic form is a strong tendency, and is as a powerful control on the system dynamics.

Since this critical topographic form is the crux of the tectonic governor, we are able to explore how the tectonic control of the orogen varies as a function of the rheology controlling the deformation within it. Our analyses are predicated on an assumption that fluvial erosion controls the erosional yield from the system, and therefore apply to settings that have a well-developed channel network and moisture delivery.

2 General considerations

We begin by identifying the basic conditions of the dynamical system that are necessary for the existence of mutual interactions among climate, erosion, and tectonics. Tectonics is used here to refer broadly to the processes of crustal deformation that act to build an orogen. Of necessity the outline is qualitative, but it is to be hoped that it provides a useful framework for conceptualizing the problem. Figure 3a shows a typical schematic illustration of processes operating in orogenesis. Similar pictures are found in a variety of textbooks, and throughout the literature. Figure 3b shows a further distillation of the dynamics. As the recent series of studies cited above have demonstrated, these dynamics reduce to, in essence, a classic flux-balance problem moderated by the critical topographic form of the orogen.

There is a flux of material, F, incorporated into the accretionary prism, a flux that is ultimately driven by plate convergence. Let V be the volume of the accretionary prism, and let its shape be loosely characterized in terms of its height, H, (which could equally well be its thickness) and width, L. Let Y be the total erosional yield from the orogen. Conservation of mass must apply:

$$\frac{dV}{dt} = F - Y. \quad (1)$$

In steady state the accretionary flux equals the erosional yield, otherwise any imbalance between the two leads to growth or decay of the orogen (e.g., Whipple and Meade, 2006; Stolar et al., 2006). Recycling of eroded material could be incorporated without any loss of generality (e.g., Whipple and Meade, 2003). The interactions between the three components of the system can be expressed in terms of three relationships (Figure 3b). The erosional yield is dependent on the climate and on the orogen shape. This can be written as

$$Y = Y(P, H, L). \quad (2)$$

Secondly, at the most general level the shape of the orogen, can be expressed qualitatively as

$$H = H(L), \quad (3)$$

and reflects the combination of erosion and deformation, isostasy and flexure. The role of tectonics is strongly reflected in the shape of the orogen. In the case of a wedge obeying a Coulomb plastic rheology, tectonics (meaning deformation) is the sole control of orogen shape. Given local isostatic compensation, the profile of a critical Coulomb wedge will always be at critical taper angle (e.g., Dahlen, 1989). The size of the wedge may change as the erosion changes, but the functional form of the wedge shape does not. We show in this study that this property of a robust topographic form also applies for more general rheologies as well. Note that if there is no deformational response to erosional unloading, the complete system of climate, erosion, and tectonics are at least partially decoupled.

Equation (3) also means that (2) encapsulates the relationship between local climate and tectonics, in so far the shape of the orogen controls orographic precipitation:

$$P = P(H, L), \tag{4}$$

where in general the connections between orogen shape and climate will involve more atmospheric variables than simply precipitation (e.g., Roe et al., 2008).

Equations (2), (3), and (4) are all necessary requirements for fully-coupled interactions between climate, erosion, and tectonics, and together they are sufficient requirements. The size, shape, and evolution of the orogen depends on the balance among the three, and on (1).

As noted above, in the broadest terms, orogen dynamics can be conceptualized as a classic flux-balance problem (Figure 3b). There is a source, F, a sink, Y, and a reservoir, V. However in contrast to, for example, a simple bucket acting as the reservoir, an active orogen is a rich dynamical system in which the size of the reservoir depends on the magnitudes of the fluxes, the interaction with the climate, and the rheology governing the shape of the orogen. It is this last factor that is the focus of this study.

3 Orogen-width scaling relationship

We outline the derivation of the scaling relationship between orogen width, accretionary flux, and precipitation rate. The steady-state size of the orogen is defined when the accretionary and erosional fluxes are equal.

We consider a system dominated by fluvial erosion, and follow essentially the same procedure as Whipple and Meade (2004), and Roe et al. (2006). However, because of some small differences in formulation, it is useful to briefly review the method below.

We use the notation and closely follow the outline of Roe et al. (2006). Although the relationship is derived considering only one side of the wedge, Whipple and Meade (2004) and Roe et al. (2009) show that a two-sided wedge obeys the same scaling relationship. The fluvial erosion rate follows a standard formulation (e.g., Whipple, 2004):

$$\dot{e} = KQ^m \left(\frac{dz}{dx} \right)^n , \tag{5}$$

where Q is the discharge and dz/dx is the along-channel slope of the river. Precipitation, P, is assumed uniform – Stolar et al. (2007) has shown that, for realistic precipitation rates, the orogen size and deformation pattern are relatively insensitive to the pattern of precipitation. Further using Hack's law for the relationship between channel length and drainage area (Hack, 1957) we can write $Q = Pk_a x^h$. K, k_a, h, m, and n are constants reflecting the rock erodibilty, the drainage network structure, and the dominant physical process governing fluvial erosion.

In steady state the local rock uplift rate, U, balances this local erosion rate. Hence we can write

$$U = K{k_a}^m P^m x^{hm} \left(\frac{dz}{dx}\right)^n . \tag{6}$$

Assuming uniform rock uplift rate, (6) can be integrated from the toe of the orogen to a point near the divide, (x_c, z_c), where the discharge is so low that fluvial erosion no longer applies and hillslope processes (i.e., landslides, soil creep, etc.) take over as the dominant mechanism of erosion. This elevation gain is the fluvial relief:

$$z_c = \left(\frac{U}{K{k_a}^m P^m}\right)^{\frac{1}{n}} \left[L^{1-\frac{hm}{n}} - {x_c}^{1-\frac{hm}{n}}\right], \tag{7}$$

For the special condition of $hm/n = 1$ there is a different expression for the fluvial relief (e.g., Whipple et al., 1999; Roe et al., 2006). It can be shown that the scaling relationship we derive below also applies to this special case - physically it has to, since the scaling relationship must vary smoothly with small changes in model parameters, and the physics allows the limiting case of $hm/n \rightarrow 1$ to be approached from either side.

We refer readers to the earlier works (e.g., Hilley et al., 2004; Whipple and Meade, 2004; Roe et al., 2006) for details of the derivation, but a sketch of the solution is provided. Firstly, in fluvially-eroding orogens, the fluvial relief comprises much the greater part of the total relief, H, where we use total relief to mean the maximum elevation above the foreland. Hence $z_c \simeq H$. Secondly, the distance from channel head to the drainage divide is also typically small compared to the channel length, and so $x_c << L$. Finally if the rock upift rate is uniform, it is related to the accretionary flux, F, by: $U = F/L$. Substituting these three conditions into (7) and rearranging gives

$$H^n \propto \left(\frac{F}{LP^m}\right) L^{n-hm}, \tag{8}$$

where the model constants are subsumed into the proportionality sign. For an orogen where the dominant erosion process is fluvial, the above expression governs the relationship between H, L, P, and F in steady state.

Several results derived in Roe et al. (2006) show this relationship is quite general and not dependent on the specifics of the model. The exponents in the scaling relationship are not sensitive to the details of how the channel is connected to the drainage divide. Furthermore, it is not necessary that the rock uplift rate be uniform. Provided the rock uplift rate scales self-similarly with orogen size, the scaling relationship will remain unaffected. These analyses have also been evaluated and supported by the numerical modeling and theoretical results of Stolar et al. (2006, 2007).

The final step in deriving the orogen-width scaling relationship is to include how the deformation within the orogen controls the topographic form, or in other words, how it controls the relationship between H and L. In the next section it is shown that a power-law viscous rheology produces a relationship of the form

$$H \propto L^{\phi}. \tag{9}$$

Substituting this into (8), gives

$$F \propto P^m L^{1+hm} L^{n(\phi-1)}. \tag{10}$$

(10) is a general expression for the balance between F, L, and P, and holds provided that the height of the orogen can be expressed as a power-law function of L.

A Coulomb critical wedge has a constant taper angle, and so $H \propto L$ and $\phi = 1$. Therefore

$$L \propto F^{\frac{1}{1+hm}} P^{\frac{-m}{1+hm}}. \tag{11}$$

This is the relationship derived in Roe et al. (2006) and is essentially the same as in Hilley et al. (2004) and Whipple and Meade (2004). We set $h = 2$ (e.g., Hack, 1957; Montgomery and Dietrich, 1992), and consider three different combinations of (m, n): (1/3, 2/3), (1/2, 1), and (1, 2). The basis for these choices, which are suggested by a combination of theory and observations, is reviewed in Whipple and Tucker (1999). Taking these three combinations of erosion parameters, the exponents on F and P in (11) are (3/5, -1/5), (1/2, -1/4), and (1/3, -1/3), respectively (Roe et al., 2006). The low values of these exponents imply that the orogen width is quite insensitive to changes in precipitation rate and accretionary flux.

Equation (11) encapsulate the dynamics of the tectonic governor, as described above and in Roe et al., 2008. The next section develops the results necessary to assess the sensitivity of the system dynamics to the physics governing deformation within the orogen.

4 The topographic form and power-law rheologies

4.1 Defining a power-law rheology

On geologic time scales the crust deforms in response to the stresses that result from the overlying topography and from externally applied tectonic stresses. The rheology of the crust relates the stresses and strain rates characterizing this deformation. For a linear viscous rheology the strain rate is simply proportional to the stress. For a plastic rheology no deformation occurs until a threshold stress has been exceeded, beyond which deformation is undefined. In the case of a Coulomb-plastic rheology, this threshold stress increases linearly with depth reflecting the increasing strength of rocks with containing pressure. These different rheologies are illustrated in Figure 4.

It is possible to write a general expression for the rheology that encompass all of these possibilities in terms of two free parameters. First, let

$$\dot{\epsilon} = A(\tau/\tau_0)^{\alpha} \tag{12}$$

where $\dot{\epsilon}$ is the strain rate and τ is the deviatoric stress. A is a flow factor which we initially assume constant. In general A might also contain some temperature dependence reflecting that the hotter the crust is, the more easily it tends tends to deform (e.g., Stüwe, 2002). We explore the effect of such a dependence in Section 7. α determines the nonlinearity of the stress - strain rate relationship: if $\alpha = 1$, the fluid is linear; if $\alpha = \infty$ the fluid is plastic (see Figure 4).

Second, we define τ_0 as a normalizing stress, given by:

$$\tau_0 = \tau^* \left(\frac{z_s - z}{D_0} \right)^{\beta} \tag{13}$$

τ^* and D_0 are constants, $(z_s - z)$ is the depth below the surface, z_s, and β is the exponent governing the depth dependence of τ_0, such that $0 \leq \beta \leq 1$. The power-law depth variation of τ_0 is used here for mathematical convenience, and is not intended to represent any real process. Consider the case of $\alpha = \infty$: if $\beta = 0$, the threshold stress is constant and the material is perfect plastic; if $\beta = 1$, the material is Coulomb plastic.

By selecting different values of α and β, the rheology expressed in (12) and (13) runs the gamut from linear viscous to Coulomb plastic. In principle we could also specify a finite cohesion needed to be overcome before any deformation took place (e.g., Dahlen, 1990; Stüwe, 2002). The cohesion term for friction decreases in significance with increasing depth (e.g., Dahlen, 1990). Likewise, the topographically-induced shear stresses decay exponentially depth and inversely proportional to the wavelength of the topography (e.g., Liu and Zoback, 1992). Thus, cohesional strength accounts for steep slopes and large relief at short wavelengths, but has little influence on long-wavelength topographic form of an orogen (e.g., Dahlen, 1990).

4.2 The topographic form

The generalized formulation of the power-law rheology in (12) and (13) can be used to solve for the topographic form of an orogen governed by such a rheology. In Section 5 this topographic form will be combined with (10) to obtain a scaling relationship between accretionary flux, precipitation rate, and orogen width. We closely follow the scale analyses and approximations of Emerman and Turcotte (1983), who invoked the equations of lubrication theory to derive their results: we extend their analyses to include the potential for a depth-dependent rheology. The method of solution also ends up being similar to that for glaciers and ice sheets using the shallow-ice approximation (e.g., Paterson, 1994). Because of these similarities, the solution is only briefly outlined.

The main assumption that simplifies the solution is that the characteristic horizontal length scales are much larger than characteristic depth scales, or in other words, the aspect ratio of the orogen is large. It can be demonstrated (e.g., Emerman and Turcotte, 1983) that this implies that horizontal gradients in shear stress are negligible compares to those in the vertical. This is a different physical model from the thin sheet approximation (e.g., England and McKenzie, 1982), where the strain rates are vertically-averaged and the force balance is between the pressure gradient due to the topography, the longitudinal stresses and, possibly, some stipulated basal traction (e.g., Ellis et al., 1995).

A schematic illustration of the forces acting on a element within the orogen is shown in Figure 5. The assumption of a large aspect ratio and the condition that the stress on the upper surface is zero means the fundamental force balance reduces to being between the shear stress on the lower face of the element, τ_{xz}, and the integral of the pressure gradient acting on the vertical sides of the element. Let x, z and u, w be the horizontal and vertical coordinates and rock velocities, respectively. The force balance can be expressed as

$$\tau_{xz} = \rho_c g(z_s - z)\frac{dz_s}{dx}, \tag{14}$$

where ρ_c is the density of the crust, g is the gravitational constant, and z_s is the surface elevation.

The large aspect ratio also means that the total strain rate is dominated by the vertical gradient in the horizontal velocity. Substituting into (12):

$$\frac{du}{dz} = A\left(\frac{\tau_{xz}}{\tau_0}\right)^{\alpha}. \tag{15}$$

The boundary conditions are that at the base of the orogen, $z = z_b$, the rock velocity is specified (u_b, w_b), and that the erosion rate at the surface of the orogen is $\dot{e}$. Although it is not necessary for the functional form of the topographic profile we derive, we can also consider a local isostatic balance, which relates z_s, z_b and the total thickness of the orogen, h:

$$\begin{aligned} z_s &= \left(1 - \tfrac{\rho_c}{\rho_m}\right) h \\ z_b &= -\left(\tfrac{\rho_c}{\rho_m}\right) h \end{aligned}, \tag{16}$$

where ρ_m is the density of the underlying mantle. Isostasy is important in deriving the time dependent behavior of the system, since it affects the amount of material needed to build an orogen of a given size (Whipple and Meade, 2006; Stolar et al., 2006). In isostatic equilibrium, u_b and w_b should be interpreted as the component of the mantle flow tangential and normal to the interface, respectively.

In this two dimensional framework the horizontal flux of rock, F, past any given point is given by the integral of the horizontal velocity:

$$F(x) = \int_{z_b}^{z_s} u(x,z)dz, \tag{17}$$

Using (13), (14), and (16), (15) can be integrated twice in the vertical, subject to the boundary conditions, to give :

$$F(x) = u_b h - \left\{ \frac{A(\rho_c g)^\alpha D_0^{\alpha\beta}(1-\rho_c/\rho_m)^\alpha}{\tau^*(\alpha(1-\beta)+2)} \right\} h^{\alpha(1-\beta)+2} \left(\frac{dz_s}{dx} \right)^\alpha . \tag{18}$$

The flux of material within the orogen can thus be be thought of as having two components: a translational component due to the basal traction – the first term on the right hand side of (18), and a deformation component due to the viscous rheology in the opposite direction – the second term on the right hand side of (18)).

Another constraint is conservation of mass, which requires at each point:

$$\frac{\partial h}{\partial t} = w_b - \dot{e} - \frac{\partial F}{\partial x}, \tag{19}$$

where $w_b - \dot{e}$ (i.e., underplating minus erosion) can be thought of as the net mass balance at a given point. In steady state $\partial h/\partial t = 0$, and so

$$F(x) = \int_0^x (w_b - \dot{e}) dx \tag{20}$$

In this one-sided wedge framework therefore, the net mass balance integrated from any given point to the orogen divide is balanced by the flow, or deformation, of rock within the orogen at that point. From (18) and (16), F can be expressed in terms of h and its first derivative, and so (20) is essentially a nonlinear diffusion equation that can be solved for the profile $h(x)$.

A second important assumption made by Emerman and Turcotte (1983) is that the thickness of the crustal layer being accreted into the wedge, δ, is much less than the total thickness of the orogen, h (i.e., including the crustal root). Crucially, Emerman and Turcotte demonstrate this means that in solving for the profile shape of the wedge, the integral on the right hand side of (20) can be neglected: the integral scales as $w_b L$, which is the rock uplift integrated over the width of the orogen. Conservation of mass means that this must also equal the accretionary flux, or $u_b \delta$. Since $\delta << h$, the integral is much less than than the first term on the right hand side of (18). Therefore the steady-state solution for the profile of the orogen (19) boils down to solving $F = 0$. Using (18) and (16), this gives:

$$u_b h - A' h^{\alpha(1-\beta)+2} \left(\frac{dh}{dx} \right)^{\alpha} = 0, \tag{21}$$

where all of the constants have been subsumed into A'.

It seems counter-intuitive to solve for $F = 0$ when the orogen depends on a mass flux for its existence, and we emphasize that this approximation is only valid in calculating the shape of the wedge profile, and cannot be made in calculating the size of the orogen (Emerman and Turcotte, 1983). Relatedly it is shown in Section 6 that the shape of the orogen profile is extremely insensitive to the pattern of erosion and underplating. The solution to (21) can be regarded as a ‘critical’ topographic form - the profile that the orogen will attain for the particular rheology governing its deformation.

Using (16), (21) can be rearranged and expressed as a first order differential equation for h. Integrating this from $x = 0$ to $x = L$, assuming $h = 0$ at $x = L$, gives

$$h = A''(L - x)^{\frac{\alpha}{\alpha(2-\beta)+1}}, \tag{22}$$

where A'' is a constant[1].

Hence, letting H be the maximum height of the orogen (above $z = 0$), the height and width of the orogen scale according to:

$$H \propto L^{\frac{\alpha}{\alpha(2-\beta)+1}}. \tag{23}$$

For a linear-viscous rheology $\alpha = 1$ and $\beta = 0$, giving $H \propto L^{\frac{1}{3}}$, as found by Emerman and

[1] Emerman and Turcotte (1983) solve for the return flow at the surface of the wedge, assuming no frontal accretion or erosion. For the rheology used here, this velocity is given by $u = u_b/(\alpha(1-\beta)+1)$, predicting some surface extension, even as the limit of a Coulomb plastic rheology is approached (e.g., Buck and Sokoutis, 1994; Willett 1999b).

Turcotte (1983). In the case of a Coulomb-plastic rheology $\alpha = \infty$ and $\beta = 1$, and so $H \propto L$. Thus the analysis reproduces the critical taper angle of a Coulomb critical wedge. These height-width relationships are consistent with the physics: a linear-viscous wedge is less able to resist deformation than a Coulomb-plastic wedge, and so changes in height are less than changes in width for the viscous case.

5 Orogen scaling-relationships for a general rheology

Having obtained the solution for the topographic form for a general rheology, it can now be used to calculate how the orogen-width scaling relationship changes as a function of the assumed rheology. From (23), the expression for ϕ in (10) is given by:

$$\phi = \frac{\alpha}{\alpha(2-\beta)+1} \tag{24}$$

Rearranging (10) as an explicit equation for L, the exponents on F and P are a function of h, m, n, and ϕ. Table 1 presents these exponents for Coulomb-plastic and linear-viscous rheologies, and for the three different sets of commonly-assumed fluvial erosional parameters. As might be expected, the exponents are larger for the linear-viscous rheology than for the Coulomb-plastic rheology: a linear-viscous orogen deforms more easily and hence the orogen width is more sensitive to changes in forcing.

A striking result is that, even though the two end-member rheologies are very different, there is relatively little impact on the exponents in the orogen-width scaling relationship. In other words, the impact of precipitation and tectonic forcing on the width of an orogen is quite insensitive to the specific rheology governing the deformation. The reason for this result can be seen by comparing the two exponents on L on the right hand side of (10). The first exponent, $1 + hm$, reflects two factors: the hm comes from the dependence of fluvial erosion on discharge and also the fact that upstream drainage area accumulates with downstream distance; and the 1 reflects that the local erosion rates must be integrated over the whole orogen width to balance the total incoming accretionary flux. The effect of rheology on the topographic form is folded into the second exponent, $n(\phi - 1)$. For the wide range of rheologies considered in Section 4.1, $1/3 \leq \phi \leq 1$. The effect of rheology is thus biggest for smaller values of ϕ. Taking $\phi = 1/3$ (i.e., a linear-viscous rheology) then, for the three different combinations of erosion parameters (in same order as Table 1), the relative sizes of the first and second exponents on L in (10) are 15:4, 3:1, and 9:4, respectively. Thus, in controlling the sensitivity of orogen width to changes in precipitation or accretionary flux, the effect of the discharge and drainage-area dependence of fluvial erosion markedly exceeds the effect of variations in the rheology (and hence the topographic form).

These results also point to what would have to come about for the rheology to play a bigger role in the affecting the orogen-width scaling relationship: a lower value of h reflecting less dependence of drainage area on downstream distance, or a lower ratio of m/n, tending to produce rivers of lower concavity (e.g., Whipple, 2004), would both act to increase the importance of rheology.

5.1 Other orogen attributes

In addition to its width, the model orogen can be characterized by two other attributes: the height and the rock uplift rate. These attributes are related to the orogen width via the rheology which gives the topographic form $H \propto L^{\phi}$, and via the conservation of mass, which requires $U = F/L$. Therefore the dependence of H and U on the forcing variables F and P can also be expressed in terms of scaling relationships.

Table 2 gives the exponents on these scaling relationships for the case of $(h, m, n) = (2, 1/2, 1)$, and illustrates the behavior of these orogen attributes for different rheologies. For completeness we also include the cases of a fixed-width orogen and a plateau (which can be emulated by setting $\phi = \infty$ and $\phi = 0$, respectively, in (9)). As ϕ decreases, the orogen width becomes more sensitive to changes in F and P. At the same time, the height becomes less sensitive since it becomes harder to build high topography. There is also a fundamental trade-off between orogen width and rock uplift rate: as the width becomes more sensitive to forcing, the rock uplift rate becomes less sensitive: a large increase in width means the accretionary flux is distributed over a larger area, damping the change in the rock uplift rates.

Although we have included the case of a plateau, the model framework presupposes that the principal outlet drainages reach to near the centre of the orogen. In the case of large plateaux on Earth, interior regions are internally drained and so our model framework does not apply. We explore the effect of the topographic form approaching that of a plateau in Section 7.

5.2 Why does an orogen change so little in response to changes in forcing?

For all of the rheologies considered, the orogen response to changes in climate and tectonic forcing is not very large. All of the exponents in Table 2 have a magnitude less than or equal to one. These exponents can also be interpreted as the factor multiplying the fractional response of that orogen attribute to a fractional change in the forcing. For example, from Table 2, the fractional change in rock uplift rate for a linear-viscous wedge is given by

$$\frac{\Delta U}{U} = \frac{1}{4}\frac{\Delta F}{F} + \frac{3}{8}\frac{\Delta P}{P}. \tag{25}$$

Thus, to bring about dramatic changes in orogen attributes (an order of magnitude, say), extremely large fractional changes in the forcing factors are required. This reflects the strong damping tendency of the tectonic governor. Recalling from (1) that, in equilibrium, $F = Y$, (10) shows that erosional yield is a sensitive function of orogen with. Fundamentally this is a consequence of Hack's law: with increasing orogen width, more of the landscape is incorporated into the erosional domain and the contributing drainage area within an river increases significantly. Therefore the power of the rivers to incise downwards also increases, and so provides a powerful opposing tendency on changes in width.

Although changes in forcing are obviously hard to estimate over geologic time, some factors like average precipitation rates can be partly constrained on physical grounds. For example, all else being equal, the moisture-carrying capacity of the atmosphere limits the precipitation rate. A useful rule of thumb is that an increase of 10^{o}C doubles the moisture content of the air, and 30^{o}C increases it by an order of magnitude (e.g., Roe, 2005). Both correspond to enormous climate changes that might have independent observable evidence. So in evaluating scenarios put forward to explain particular orogen histories, the exponents in Table 2 might be used to assess their physical plausibility. However, the conclusions from such an analysis would be predicated on the validity of the assumed erosion law and rheology. As elaborated on in the discussion, the use of a more general form of the erosion law might allow for a more sophisticated approach in evaluating the impact of climate-change scenarios.

5.3 Feedback analysis

An alternative way to characterize the strength of interactions in a system is through a feedback analysis. Roe et al. (2008) used a model of orographic precipitation to analyze of the strength of precipitation feedback in a critical wedge. We use the same procedure to characterize how changing the governing rheology affects the sensitivity of the orogen properties to the magnitude of the accretionary flux and the precipitation rate.

A formal feedback analysis includes defining gains and feedback factors, which is essential in comparing the relative strength of different interactions in a system, and this in turn depends on defining a reference, no-feedback, case (e.g., Roe, 2009). Roe et al. (2008) only considered a Coulomb-plastic critical wedge, and in order to compare the relative strength of the precipitation and tectonic feedbacks they defined the reference case as a fixed-width orogen. This present study explores the tectonic feedback in more detail, and in using a range of rheologies there is a choice about which reference case to pick. Given we make the initial premise that orogens do have a topographic profile, the most natural reference case is that of a Coulomb critical wedge. With this reference case then, the feedback factors and gains should be interpreted as reflecting how a *departure* from Coulomb-plastic rheology affects the system's response. Secondly, this reference case also allows for a direct comparison with the detailed analysis of the precipitation feedback in Roe et al. (2008), which also took the Coulomb critical wedge as the reference case.

Let ΔL_0 be the change in orogen width for an increment in tectonic forcing, ΔF_0, in the reference case (i.e., for a Coulomb critical wedge). Now let ΔL be the width change to that same increment in tectonic forcing, but with a different rheology (i.e., non Coulomb-plastic). The feedback factor, f_L, and gain, G_L, relate these two length changes:

$$\Delta L = G_L \Delta L_0 = \frac{\Delta L_0}{1 - f_L} \tag{26}$$

Hence the feedback factors and gains quantify the effect that changing the rheology has on the strength of the interactions in the system. In contrast with gains, individual feedback factors for different feedbacks are linearly additive and so can be used to directly compare feedback strengths (e.g., Roe, 2007).

In Appendix A, expressions for the feedbacks factors are derived (see also Roe et al., 2008) for the orogen width, f_L, the orogen height, f_H, and the rock uplift rate, f_U. They are:

$$\begin{aligned} f_L &= \tfrac{n(1-\phi)}{1+hm}, \\ f_H &= \tfrac{(\phi-1)(1+hm-n)}{\phi(1+hm)}, \\ f_U &= \tfrac{n(\phi-1)}{(1+hm)[hm-n(1-\phi)]}. \end{aligned} \tag{27}$$

The calculations for $(h, m, n) = (2, 1/2, 1)$ are presented in Table 3. Because of the choice of $hm/n = 1$, $f_H = f_U$, although this is not always true and in general they can be quite different from each other. In moving from a less- to a more-deformable rheology (i.e., to lower values of ϕ), the feedback on orogen width becomes more positive. That is to say, a more-deformable rheology amplifies the width response to a change in forcing. For the height and rock uplift rates, the effect is the opposite: the more deformable the rheology, the more the height and rock uplift rates are damped. In the limiting case of a plateau, the damping is total.

Roe et al. (2008) found that large precipitation feedbacks were possible if only the leeward (i.e., rain shadow) flank of the orogen were considered. However these large feedbacks were strongly muted when both flanks of the orogen were coupled together. For two-sided wedges Roe et al. (2008) found that, under typical choices of model parameters, the width feedback factor (i.e., f_L) for a precipitation feedback varies from between approximately -0.4 and +0.2. Feedback factors are directly comparable and so from Table 3 it can be seen that changing the rheology from Coulomb-plastic to linear-viscous has about the same effect as including the interaction with orographic precipitation.

6 The effect of mass balance patterns

In deriving the solution for the topographic form in Section 4.2, it was demonstrated that the mass balance, $w_b - \dot{e}$ (i.e., the underplating minus the erosion rate), could be neglected. In this section we explore the validity of this assumption using a numerical solution of (20).

As noted in Section 4.2, the typical magnitude of the mass balance can be constrained from mass continuity: w_b and $\dot{e}$ scale as $u_b\delta/L$. However, the pattern of the local mass balance is as yet only poorly constrained from observations or theory (e.g., Willett, 1999b, 2001). There has been some progress recently - Stolar et al. (2007) argue that in fluvially-eroding orogens the local erosion rate should correlate with local relief. In view of the general uncertainty though, we use the scaling to set the amplitude, and bracket the response of the system by considering two simple, but end-member, spatial patterns that are the mirror-opposite of each other.

$$w_b - \dot{e} = \pm \frac{u_b \delta}{L} \left(\frac{x}{L} - \frac{1}{2}\right) \tag{28}$$

This choice of functional form ensures that the integral of the mass balance over the orogen is zero, guaranteeing a steady-state solution for a given L. Note that locally $w_b - \dot{e}$ is not zero, and so in steady state it must be balanced by the divergence or convergence of the vertically-integrated horzontal flux within the orogen.

We consider an orogen with the following scales $L = 40$ km, $\delta = 5$ km, $u_b = 16$ mm yr^{-1}, giving an average uplift rate of 2 mm yr^{-1}. Crustal and mantle densities are assumed to be 2.8×10^3 and 3.3×10^3 kg m^{-3}, respectively. Since we only consider values of 0 or 1 for β, the value of D_0 in (18) is arbitrary except for how it influences the other parameters, but we take it to be 30 km. The ratio of A/τ^* is then chosen such that the height of the orogen is 3 km for the standard case (i.e., not including the mass balance pattern). The value of this ratio must be adjusted as the rheological parameters are varied.

At the toe of the wedge, $u_b h$ is finite and the thickness is zero. From (18), if α is finite then dz_s/dx must go infinity there, and this causes difficulty in achieving numerical convergence. Appendix B gives a horizontal coordinate transformation that was used to ensure that, in the new coordinate frame, the slope at the toe remains finite. A standard numerical ODE-solver was then used to solve the transformed (18). After converting back to the original x-coordinate, the result is the topographic form for a given rheology and mass balance pattern.

We consider three different combinations of the rheology parameters α and β: a linear viscous case, $(\alpha, \beta) = (1, 0)$; an intermediate viscous case, $(\alpha, \beta) = (3, 0)$ (which is also Glen's flow law for ice); and a case approaching that of Coulomb plastic, $(\alpha, \beta) = (30, 1)$. Figure 6a shows the topographic forms using the mass balance and taking the positive sign in (28). From the analytical solutions for these rheologies (24), the exponents characterizing the topographic profiles are 1/3, 3/7, and 30/31, respectively. The validity of neglecting the mass balance in deriving these exponents is confirmed by the profiles, which are almost indistinguishable from the analytical forms. This can also be seen by calculating the topographic form resulting from the opposite pattern in the mass balance, which is done by taking the negative sign in (28). The difference between the two profiles as a function of distance is shown in Figure 6b. The profiles differ by less than 20 m, with differences being largest near the divide and when the linear-viscous rheology is used.

To further illustrate the insensitivity of the topographic form to the mass balance, Figure 7 shows orogen profiles calculated with an imposed a top-hat 'spike' in the mass balance in a 2 km wide strip centered midway along the orogen, compared to the standard case profiles. The amplitude of this imposed spike is 20 cm yr^{-1}, or in other words, one hundred times the average rock uplift rates. The spike has its biggest effect on the profile at the point where it is imposed, but even for a linear-viscous rheology the maximum difference between the profiles is less than 200 m. For the case of $\alpha, \beta = (30, 1)$, the maximum diffrerence is 30 m.

Figure 6b and 7 demonstrate a profound insensitivity of the topographic form to the mass balance, and that the more nonlinear the rheology, the more insensitive the topographic form is. The reasons for this insensitivity are two-fold. Firstly, as argued by Emerman and Turcotte (1983) and in Section 4.2, the flux of material through the orogen is dominated by two terms, the translational flux and the deformation flux, which are both much larger in magnitude than typical values of the local mass balance. Secondly, (18) is a nonlinear diffusion equation, which effectively smears out spatial patterns in the mass balance profile: for rheologies approaching plastic (i.e., $\alpha >> 1$) the exponents on the orogen thickness (h) and surface slope (dz_s/dx) are large numbers. This means a large change in F can be accommodated by only a small change in the shape of the orogen profile (i.e., either h or dz_s/dx).

In the context of orogen development the insensitivity of the orogen profile to precipitation patterns has been demonstrated in the numerical modeling of Stolar et al. (2007). There is a parallel with glaciology because ice is also a power-law viscous fluid. The rheology of ice is typically assumed to be governed by Glenn's flow law (e.g., Paterson, 1994), given by $\alpha = 3$ and $\beta = 0$ in (12). Boudreaux and Raymond (1997) and Roe (2002) showed that the profiles of glaciers and ice sheets are extremely insensitive to the pattern of the mass balance. Roe (2002) derived the solution for the response of an ice-sheet profile to a δ-function spike in the accumulation pattern. This solution can be adapted to the orogen framework presented here by incorporating the basal traction term.

One result from the earlier study is that an ice sheet profile is most sensitive when the δ-function is close to the center of the ice sheet, where the surface slope is small. The reason can also be seen from (18): the smaller the surface slope is, the greater must be the adjustment in the thickness to balance a perturbation in the flux. The increase sensitivity of the topographic form towards the center of the orogen is seen in Figure 6b, and confirmed by moving the location of the mass balance spike (not shown).

Lastly we note that because high exponents in the power-law rheology allow h and dz_s/dx to effectively adjust to perturbations, the orogen will also be robust to changes in model parameters that appear in (18) at the same order as the mass balance: large changes in the flow factor A or in the normalizing (or yield) stress τ^* would be required to cause significant changes in the topographic form.

6.1 The pattern of basal traction

So far we have considered the basal velocity to be uniform. However, in reality it will depend on the details of how material is incorporated into the orogenic wedge (e.g., Willett, 2001). For example, if underplating into the wedge is uniform, the depth of the layer of crustal material scraped off the subducting plate and subsumed into the wedge must vary across the orogen (e.g., Willett, 2001; Figure 3), which will affect how the traction stresses from the subducting slab are transferred to the base of the wedge.

The numerical solution of (18) allows for u_b to be specified as a function of position. To illustrate the effect of spatial variations in u_b, we consider a simple linear decrease across the wedge: $u_b(x) = u_b^*(1 - x/L)$, where u_b^* is a constant. From (18), if u_b goes to zero at the divide, the surface slope must go to zero there. Figure 8 shows this clearly happening in the linear-viscous case. However, as the exponent in the power-law rheology increases, the spatial pattern in u_b has decreasing impact on the topographic form: substituting $u_b(x)$ into (21), it can be rearranged to show that $dz_s/dx \propto (1 - x/L)^{1/\alpha}$ which, for higher powers of α, and except in the vicinity of $x \sim L$, is nearly constant.

The results of this and the previous section demonstrate a strong insensitivity of the topographic form to patterns of basal traction and mass balance. This is an important result in that it means that the concept of a 'critical' topographic form can be applied not just to the case of a Coulomb plastic rheology, but also to the wide range of different rheologies we have considered here. Thus, it suggests that provided some 'effective' rheology can be defined for an orogen as a whole, the relative importance of climate and tectonic forcing in setting its attributes can be evaluated through scaling relationships like (8).

7 The approach to a plateau

At depth, higher temperatures means deformation occurs more readily. For larger orogens and plateaux, thermal weakening of the crustal root becomes an important factor in their development (e.g., Stüwe, 2002). The power-law rheology in (12) can be adapted to qualitatively explore how this affects the topographic form. As was noted in Section 5.1 the model framework cannot be applied if substantial areas of the orogen are internally drained, and so the results should probably be interpreted as only applying for large orogens that remain externally drained.

Our goal here is only illustrative, and so there is some flexibility in how to represent this depth-dependent behavior. For example, an Arhenius-type function can be introduced into the flow factor: $A \sim exp(Q/RT)$, where Q is an activation energy and T is the temperature. For our purposes, it also suffices to specify a depth dependence to the normalizing stress, τ^*: the lower τ^*, the more readily the crust deforms. To demonstrate the effect of different rheologies for the upper and lower crust, τ^* is only changed below a given depth:

$$\begin{aligned} \tau^* &= \tau_0^* && (z - z_s) < H_t \\ \tau^* &= \tau_0^* e^{-(\frac{z-zs-Ht}{\Delta H})} && (z - z_s) \geq H_t \end{aligned} \tag{29}$$

where τ_0^* is constant. We take $H_t = 25$ km, and $\Delta H = 2$ km. In an Arrhenius-type temperature dependence, this short e-folding scale would correspond approximately to a Q of 135 kJ mol^{-1} and a geothermal gradient of 20 K km^{-1} (e.g., Willett, 1999b).

Figure 9a show the height of the orogen for three different rheologies, as a function of orogen width and calculated from the numerical solution. It becomes increasingly hard to build the profile above about 4 km in height (or 25 km in total depth): the strong decrease in τ^* means steep surface slopes cannot be supported at depth, and so the orogen profile flattens out.

The results in this paper have presented the strength of the tectonic feedback in terms of ϕ, the exponent governing the critical topographic form. An equivalent value for ϕ for the curves in

Figure 9a can be calculated from the slope of $ln(H)/ln(L)$. These are shown in Figure 9b. For small orogens widths, ϕ is a constant and, as expected, matches the values from the analytical solutions. As the orogen profile nears 4 km in height, however, the flattening out of the profile is reflected in a steep drop in the value of ϕ. For the near Coulomb-plastic case of $(\alpha, \beta) = (30, 1)$ this drop in ϕ occurs over a relatively small change in width of only 20 km. During this transition there is an accompanying large change in orogen sensitivity and feedback strength, which can be calculated from (27), and is shown in Figure 10. For the more deformable rheologies the transition is more gradual.

8 Discussion

A framework has been presented to evaluate the importance of precipitation and accretionary flux in setting the scale and erosion rates of steady-state convergent orogens. The tectonic control on the orogen can be thought of as arising from the relationship between the height and width. A given rheology results in a topographic form for the orogen, which can be combined with an assumption of fluvial erosion to produce scaling relationships for the width, height, and rock uplift rates of the orogen in terms of the accretionary flux, F, and precipitation rate, P. Whereas previous studies considered only Coulomb-plastic wedges with a critical taper angle, we have extended the analysis to incorporate rheologies varying from linear viscous to Coulomb plastic. Orogen development can be reduced to a straightforward flux balance problem. The growth or decay of the orogen is proportional to the imbalance between the accretionary and erosional fluxes, and the system is constrained to always have the same critical topographic form. Large scale steady-state is achieved when erosional and accretionary fluxes balance.

There are two principal results. The first is that for a wide range of assumed rheologies, the topographic form is extremely insensitive to the pattern of erosion rates and underplating, just as in the case of a critical-taper wedge. Our results therefore demonstrate that this concept of a critical topographic form can be extended to other rheologies. The strong tendency for the rheology to maintain of a critical topographic form acts as a powerful tectonic governor on the system, damping the response of the orogen to changes in climate or tectonic forcing. As discussed in Section 5 this is because, in a fluvially-eroding system, the erosional yield is a sensitive function of the area of the landscape incorporated into the erosional domain (and therefore the orogen width).

The fact that the underlying system dynamics does not greatly depend on the rheology is important. It strengthens the conclusions of the growing number of studies that have used conceptual models of the tectonic governor in intepreting the dynamics of real-world settings (as summarized in Whipple 2009). In other words the conclusions of those studies are not vulnerable to uncertainties about what the appropriate rheology is.

The second main result is that for this wide range of rheologies, the sensitivity of the orogen width to precipitation and accretionary flux remains quite similar. The exponents on F and P in the scaling relationships vary by less than a factor of two for all common choices of the erosion parameters. The exponents in the scaling relationships are also all less than one, implying a generally weak sensitivity of orogen attributes to changes in climate or tectonic forcing.

The vertically-integrated horizontal mass flux within the orogen is composed of a translational component due to the basal traction and an oppositely-directed deformational component. The deformation can be thought of as a strongly nonlinear diffusion process whereby large changes in mass flux can be accommodated by only small changes in thickness or surface slope. Significant perturbations to the critical topographic form are therefore only possible during the transient evolution of the system, and then only by changes occurring on time scales that are shorter than that required for the deformational flux to adjust. This timescale can be estimated by a scale analysis of (18) and (19). The more nonlinear the rheology, the more effectively diffusion restores the critical form after a perturbation.

The strongly restorative nature of the nonlinear diffusion governing the deformation suggests that the conclusions of this study ought to be quite robust in natural settings. The range of rheologies explored in this paper is very broad, and so provided there is some tendency to achieve a critical form (i.e., that as the orogen grows upwards, it also grows outwards), it is likely the response will lie within the span we have considered. The analysis was also tentatively extended to the case of a plateau, but the model framework assumes no internally-drained regions. In reality, the preponderance of the erosional fluxes from plateaux come from marginal ranges, where precipitation will also be focussed.

We have assumed erosion occurs fluvially and via a standard formulation of river incision into bedrock. Recent work has explored alternative "tools-and-cover" formulations (Sklar and Dietrich, 1998, 2001; Whipple and Tucker, 2002), which reflect the effects of varying sediment concentrations on the erosive ability of the river. Brandon and Gasparini (2005) argue that such models can all be characterized by effective values of m and n (they find values of m in the range of 0.1 to 0.3).

These effective m and ns allow such an erosion model to be directly transplanted into this present framework. The exercise would be instructive and the purpose would be to understand under what conditions the conclusions of the present analysis would be significantly changed. Tomkin and Roe (2006) have explored the case of glacial erosion - they argue a fully-glaciated orogen is more sensitive to precipitation than the fluvial case, and this conclusion would carry over to the present framework.

Further development of the model rheology is possible. Apart from the depth dependence of the normalizing stress, the rheology has been assumed homogenous within the orogen. The exponent on the power law (12) might also be varied with depth or horizontal distance to provide, for example, a mixed plastic-viscous rheology. Crustal deformation along localized shear zones, whether brittle or viscous, has been neglected. However, they are pinned to material and thus are advected rearwards into the wedge (as can be seen in sandbox experiments including erosion, e.g., Konstantinovskia and Malavieille, 2005). Sand box experiments with surface erosion included show material trajectories that are similar to those in idealized Coulomb-plastic wedges (e.g., Hoth et al., 2006; c.f. Brandon et al., 1998), and numerical models that do not have faults but do have zones of concentrated deformation closely follow our analytical results (Stolar et al., 2006, 2007). It is unlikely therefore that the inclusion of faults would overturn the general picture developed here, however further work in this area would clearly be useful.

We have only considered the steady-state balances. Whipple and Meade (2006) and Stolar et al. (2006) show in analytical and numerical models that the transient evolution of the system occurs as a sort of quasi-equilibrium in which the critical form is maintained at all times, but that the rate of growth (or decay) of volume is proportional to the flux imbalance between the accretionary and erosional fluxes. Whipple and Meade (2006) and Roe et al. (2008) show further that the characteristic time scale to steady-state is a function of the exponents in the scaling relationship. These results translate directly to the rheologies considered here.

Only one-sided orogens have been considered. Two additional constraints allow the two-sided case to be constructed out of back-to-back one-sided wedges (following Whipple and Meade, 2004). Continuity requires the two wedges to have the same height, and conservation of mass means that the sum of the fluxes accommodated under each of the wedge equals the total incoming accretionary flux. The pattern of strain within the orogen will depend sensitively on the pattern of underplating assumed (e.g., Willett, 1999, 2001). However since the critical topographic form is so insensitive to the pattern of underplating and erosion, the scaling relationships will be essentially the same. This has been demonstrated for the Coulomb-plastic rheology (e.g., Whipple and Meade, 2004; Roe et al., 2008), and the extension to the present study is direct.

9 Summary

A uniform rheology results in a critical topographic form, the consequences of which we have explored here. This critical topographic form means that orogen thickness and width co-vary, and this co-variation is the crux of the orogen dynamics. It implies the action of a tectonic governor, producing a strongly damped system whose basic behaviors can be cleanly characterized in terms of the dominant processes governing erosion, and the large-scale climate gradients. If, as is certain to be the case in reality, there are spatial and temporal variations in rheology, rock erodibility, and climate, then an adherence to the same critical topographic form will not be strictly obeyed. However provided there exists some tendency for thickness and width to co-vary, then the governing principles can be understood through idealized frameworks such as those presented here. A strength of such frameworks is that the assumptions can be varied and evaluated. This and several other works (Whipple and Meade, 2004; 2006; Roe et al., 2006, 2008; Stolar et al., 2006, 2007; Tomkin and Roe, 2006) have shown the fundamental behavior of the system is robust to a wide range of assumptions and conditions.

As this and similar frameworks are refined and evaluated, the key will to sift through which aspects of mountain belts are subject to the vagaries of their individual settings and histories, and which aspects apply more generally, so yielding a greater understanding of the underlying principles governing mountain belt dynamics on Earth.

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Appendix A: Feedback factors and gains

In deriving the expressions for the feedback factors and gains, we follow the analysis of Roe et al. (2008) where more details of this standard method can be found. We can define sensitivity parameters for L, H, and U, which relate a change in these variables to a change in the accretionary flux:

$$\begin{aligned} \Delta L &= \lambda_L \Delta F \\ \Delta H &= \lambda_H \Delta F \ , \\ \Delta U &= \lambda_U \Delta F \end{aligned} \tag{A-1}$$

Obviously, these sensitivity factors are different when a feedback is operating compared to when it is not. The gains and feedback factors characterize this change in sensitivity of the system.

To begin with, the orogenic wedge system obeys the scaling relationship from (8)

$$F \propto L^{1+hm-n} H^n P^m. \tag{A-2}$$

We define the reference case as the Coulomb critical wedge: $H \propto L$. A truncated Taylor series expansion of (A-2) gives

$$\begin{aligned} \Delta F &= \tfrac{\partial F}{\partial L}\Delta L + \tfrac{\partial F}{\partial H}\Delta H \\ &= \tfrac{\partial F}{\partial L}\Delta L + \tfrac{\partial F}{\partial H}\tfrac{\partial H}{\partial L}\Delta L \end{aligned} \ , \tag{A-3}$$

which on substitution from (A-2) and assuming $H \propto L$, gives:

$$\Delta F = \left\{ (1 + hm - n)\frac{F}{L} + n\frac{F}{H}\frac{H}{L} \right\} \Delta L. \tag{A-4}$$

And so for the reference case (denoted by the superscript 0)

$$\lambda_L^0 = \frac{1}{(1 + hm)}\frac{L}{F}. \tag{A-5}$$

In the general case however, $H \propto L^{\phi}$. On substitution of this into (A-2) and (A-3), we get:

$$\begin{aligned}\Delta F &= \left\{(1+hm)\tfrac{F}{L} + n(\phi-1)\tfrac{F}{L}\right\}\Delta L \\ &= \tfrac{1}{\lambda_L^0}\left\{1+\lambda_L^0 n(\phi-1)\tfrac{F}{L}\right\}\Delta L\end{aligned}. \tag{A-6}$$

Therefore, for this general case

$$\lambda_L = \frac{\lambda_L^0}{1-\frac{n(1-\phi)}{(1+hm)}}. \tag{A-7}$$

The gain, G_L, is defined as $\Delta L/\Delta L_0$, and so is equal to λ_L/λ_L^0. The feedback factor is defined via $G_L = 1/(1-f_L)$, and so from (A-7) is given by

$$f_L = \frac{n(1-\phi)}{1+hm}. \tag{A-8}$$

For λ_H, the Taylor series expansion of (A-2) can be written in terms of ΔH:

$$\Delta F = \left\{\frac{\partial F}{\partial H} + \frac{\partial F}{\partial L}\frac{\partial L}{\partial H}\right\}\Delta H \tag{A-9}$$

The reference case λ_H^0 has a similar form to that for orogen width:

$$\lambda_H^0 = \frac{1}{(1+hm)}\frac{H}{F} \tag{A-10}$$

On substituting the critical form $L = H^{\frac{1}{\phi}}$ into (A-9), and following the method above, we get

$$f_H = \frac{(\phi-1)(1+hm-n)}{\phi(1+hm)}. \tag{A-11}$$

The rock uplift rate is related to F and L via $U = F/L$. Roe et al. (2008) showed that the following relationship always holds:

$$G_U = \frac{1+hm-G_L}{hm}. \tag{A-12}$$

Upon substitution from (A-8) and a lot of rearranging, this boils down to

$$f_U = \frac{n(\phi-1)}{(1+hm)[hm-n(1-\phi)]}. \tag{A-13}$$

Appendix B: Coordinate stretching

A horizontal coordinate transformation can be used to remove numerical problems encountered in integrating (18) to calculate the critical topographic form when no analytical solution is possible. Simplified, (18) has the following form:

$$z_s^{\nu} \frac{dz_s}{dx}^{\mu} = c_1 z_s + c_2 F(x) \tag{B-1}$$

where c_1, c_2 are constants, and $\nu = \alpha(1-\beta)+2$, and $\mu = \alpha$.

The boundary condition is that $z_s = 0$ at $x = 0$. For finite $F(x=0)$, and for finite α, this means dz_s/dx must go to infinity at $x = 0$. The goal is to find a coordinate transform $t = t(x)$ that keeps the slope dz_s/dt well behaved at that point. First, we construct the series solution for z_s in the neighborhood of $x = 0$:

$$z_s = x^p(a_0 + a_1 x + a_2 x^2 + ...), \tag{B-2}$$

where a_n are constants to be found, and the first term in the series is of order p, also to be found. Taking the derivative gives

$$\frac{dz_s}{dx} = x^{p-1}(a_0 p + a_1(p+1)x + a_2(p+2)x^2 + ...). \tag{B-3}$$

On substitution of these series expansions into (B-1), the leading order terms on the left- and right-hand side must match. So for finite $F(x=0)$, we must have

$$p\nu + (p-1)\mu = 0, \tag{B-4}$$

or

$$p = \frac{\mu}{\mu+\nu}. \tag{B-5}$$

Therefore in the limit of $x \to 0$, $z_s \sim x^{\frac{\mu}{\nu+\mu}}$ and so $dz_s/dx \sim x^{-\frac{\nu}{\nu+\mu}}$. In the case of a linear viscous wedge $(\alpha, \beta) = (1, 0)$, which gives $(\mu, \nu) = (3, 1)$, and so $dz_s/dx \sim x^{-\frac{3}{4}}$.

The solution for the orogen profile is just

$$z_s(x) = \int_0^x \left(\frac{dz_s}{dx} \right) dx \tag{B-6}$$

The forgoing analysis shows that this is an improper integral with an integrable power-law singularity at the lower boundary. For such an integral, if the integrand, $f(x)$, diverges as $(x - a)^{-\gamma}$, $0 \leq \gamma < 1$, near $x = a$, the standard identity is (e.g., Press et al., 1992):

$$\int_a^b f(x)dx = \frac{1}{1-\gamma} \int_0^{(b-a)^{1-\gamma}} t^{\frac{\gamma}{1-\gamma}} f(t^{\frac{1}{1-\gamma}} + a)dt. \tag{B-7}$$

Therefore we use the change of variable $t = x^{1-\frac{\nu}{\nu+\mu}}$, before numerically integrating to solve for the orogen profile.

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Tables and Figures

			Coulomb Plastic $(\phi = 1)$		Linear viscous $(\phi = \frac{1}{3})$	
m	n	h	F^{γ_1}	P^{γ_2}	F^{γ_1}	P^{γ_2}
$\frac{1}{3}$	$\frac{2}{3}$	2	$\frac{3}{5}$	$-\frac{1}{5}$	$\frac{9}{11}$	$-\frac{3}{11}$
$\frac{1}{2}$	1	2	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{3}{4}$	$-\frac{3}{8}$
1	2	2	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{3}{5}$	$-\frac{3}{5}$

Table 1: Exponents on the orogen-width scaling relationship: $L \propto F^{\gamma_1} P^{\gamma_2}$. The table shows a comparison for two different rheologies (Coulomb plastic and linear viscous), and for three different commonly-assumed erosion processes. The exponents are a measure of the sensitivity of the orogen width to changes in F and P. A linear viscous rheology makes orogen width more sensitive to change in F and P, but not dramatically so. See text for more details.

	Width, L		Height, H		Rock uplift, U	
	F^{γ_1}	P^{γ_2}	F^{γ_1}	P^{γ_2}	F^{γ_1}	P^{γ_2}
Fixed width ($\phi = \infty$)	0	0	1	-$\frac{1}{2}$	1	0
Coulomb plastic ($\phi = 1$)	$\frac{1}{2}$	-$\frac{1}{4}$	$\frac{1}{2}$	-$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
Linear viscous ($\phi = \frac{1}{3}$)	$\frac{3}{4}$	-$\frac{3}{8}$	$\frac{1}{4}$	-$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$
Plateau ($\phi = 0$)	1	-$\frac{1}{2}$	0	0	0	$\frac{1}{2}$

Table 2: Exponents on scaling relationships for different orogen attributes: width, height, and rock uplift. $(h, m, n) = (2, 1/2, 1)$ was assumed for the values shown. The case of a plateau is included for completeness, but as noted in the text, the results derived for this case should be interpreted cautiously.

	f_L	f_H, f_U	G_L	G_H, G_U
Fixed width ($\phi = \infty$)	$-\infty$	$\frac{1}{2}$	0%	200%
Coulomb plastic ($\phi = 1$)	0	0	100%	100%
Linear viscous ($\phi = \frac{1}{3}$)	$\frac{1}{3}$	-1	150%	50%
Plateau ($\phi = 0$)	$\frac{1}{2}$	$-\infty$	200%	0%

Table 3: Feedback factors and corresponding gains for width, height, and rock uplift rates, for different rheologies. The Coulomb plastic rheology is taken as the reference case, and the expressions for the feedback factors are derived in Appendix A. $(h, m, n) = (2, 1/2, 1)$ was assumed for the values shown. Because $hm/n = 1$, $f_H = f_U$, but this is not generally true. The gains are given as percentages and are related to the fractional feedback factors by $G = 1/(1 - f)$. When multiple feedbacks are present, feedback factors add linearly, whereas gains do not. The case of a plateau is included for completeness, but as noted in the text, the results should be interpreted cautiously.

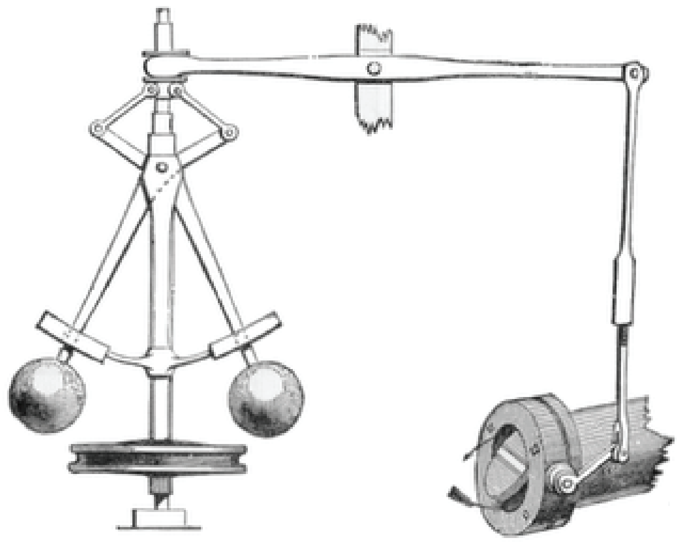

Figure 1: (a) An example of a governor in an early steam engine (Routledge, 1900, available on wikipedia commons). The outward or inward movement of the hinged balls, caused by faster or slower rotation of the belt-driven wheel on its axis, is linked mechanically to a valve on air intake of the steam engine and thus regulating, or governing, the speed of the engine.

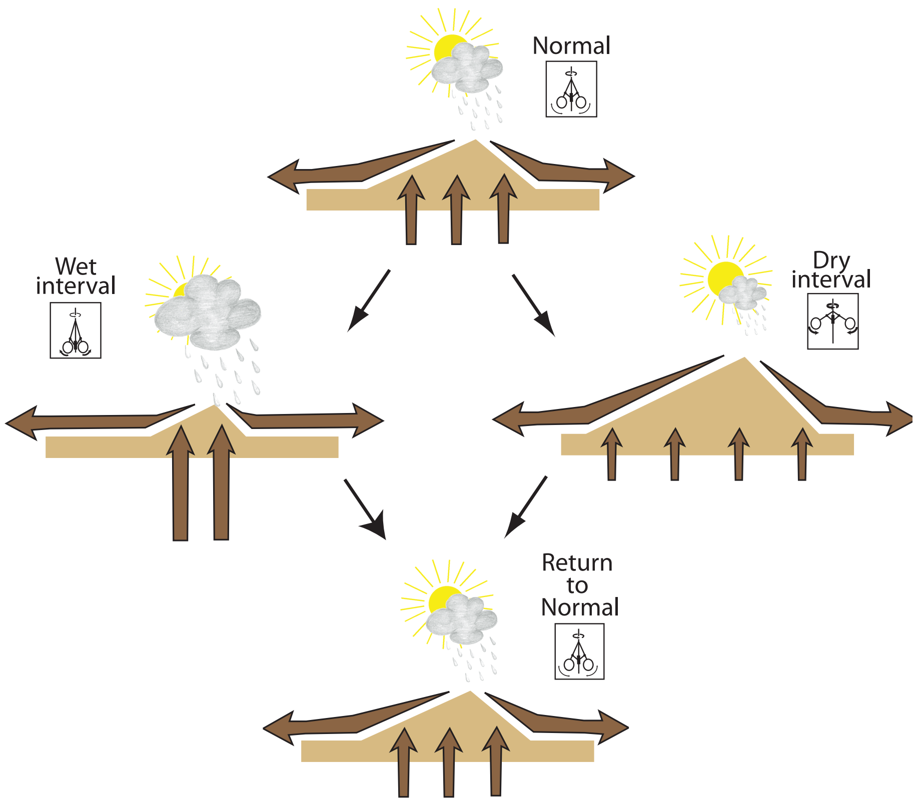

Figure 2: Schematic illustration of the tectonic governor. In equilibrium the size of the orogen is governed by how big it must be for erosion yield to come into balance with the accretionary flux. To the extent that the orogen maintains a critical form, the orogen will shrink in both height and width in response to an interval of a climatically-driven increase in erosion. Thus, rock uplift rates will be locally enhanced, creating a tendency opposing the erosional forcing. If the erosional forcing returns to its previous value, and again, if the orogen maintains a critical form, then the increase in rock uplift will act to restore the orogen to its previous size and shape. For a decrease in climatically-driven erosion, the complementary argument also applies. The purpose of the figure is illustrative so it has been highly idealized - the general argument does not depend on the orogen root or on the material trajectories through the orogen.

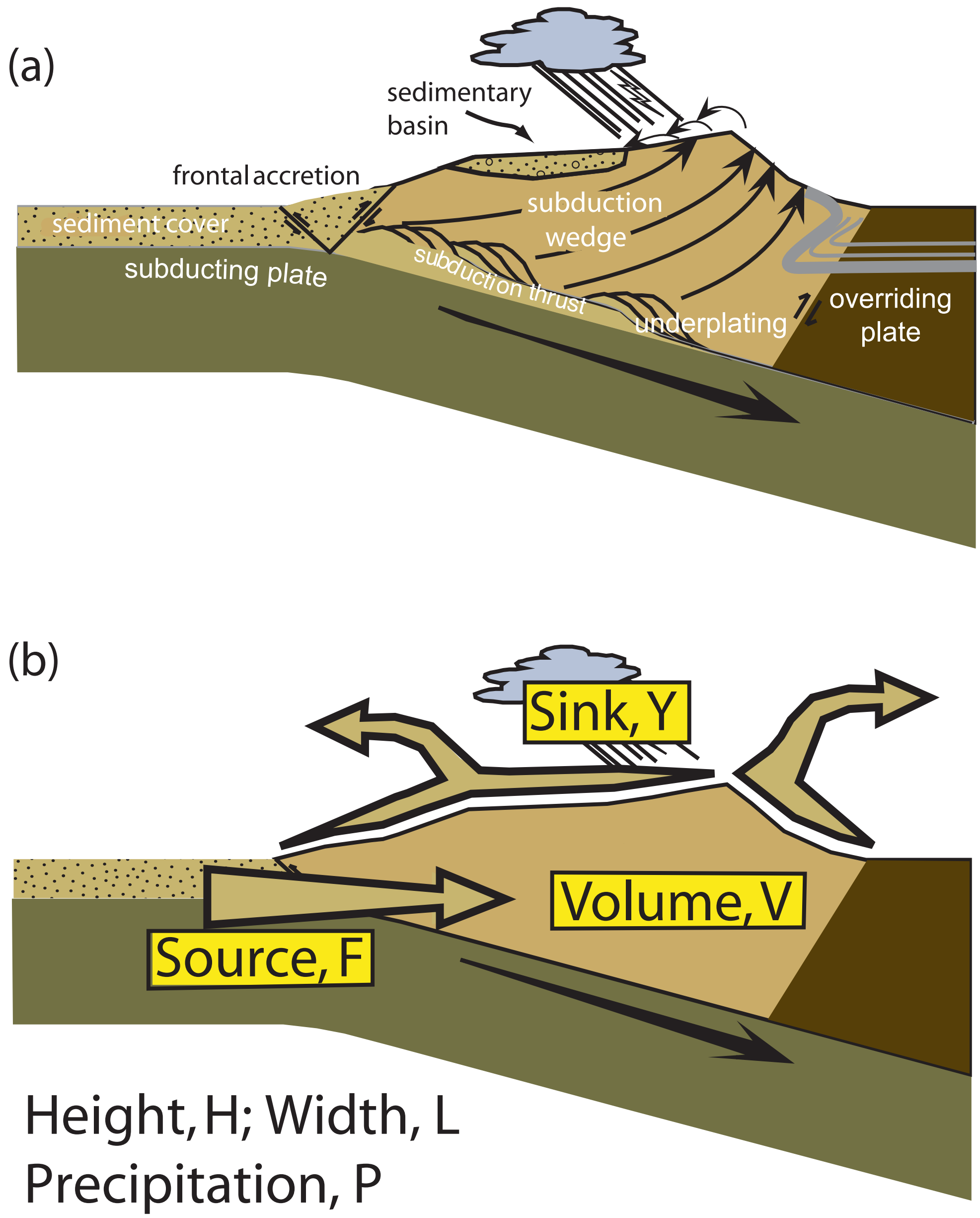

Figure 3: (a) Typical schematic illustration of various processes operating in orogenesis; (b) The same orogen dynamics distilled into a flux-balance problem.

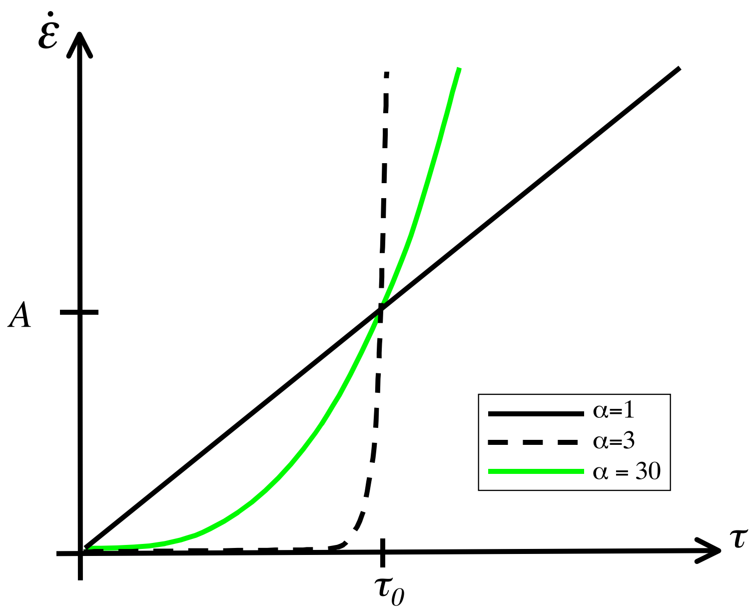

Figure 4: Illustration of different power-law viscous rheologies from (12), for different values of the exponent α. τ_0 is the normalizing stress at which the strain rate $\dot{\epsilon} = A$. As the value of α approaches infinity, the material approaches plastic behavior. For a Coulomb plastic material τ_0 increases linearly with depth within the material.

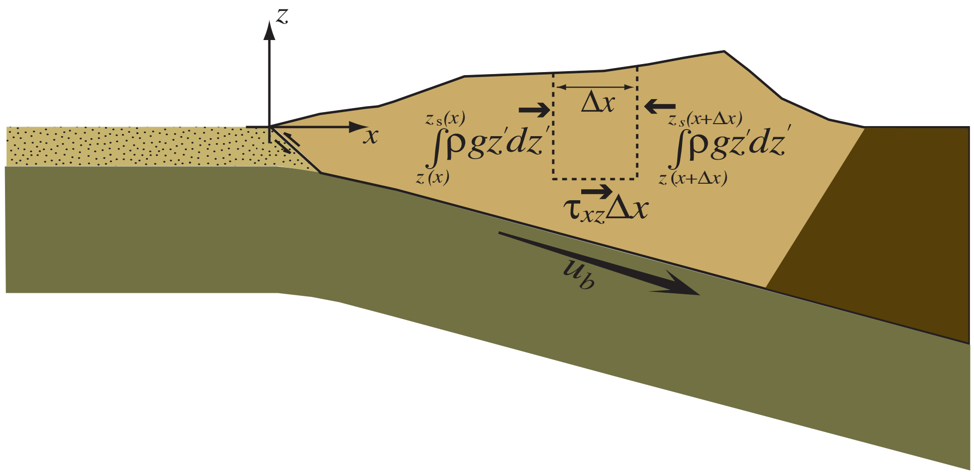

Figure 5: Illustration of the horizontal forces acting on an element within the wedge, shown by the dashed line. The pressure gradient force integrated along the vertical sides of the element is balance by the shear stress along the bottom face of the element. Within this model framework, the longitudinal deviatoric stress components are negligible because of the depth scales are assumed to be much smaller than horizontal scales within the wedge (e.g., Emerman and Turcotte, 1983).

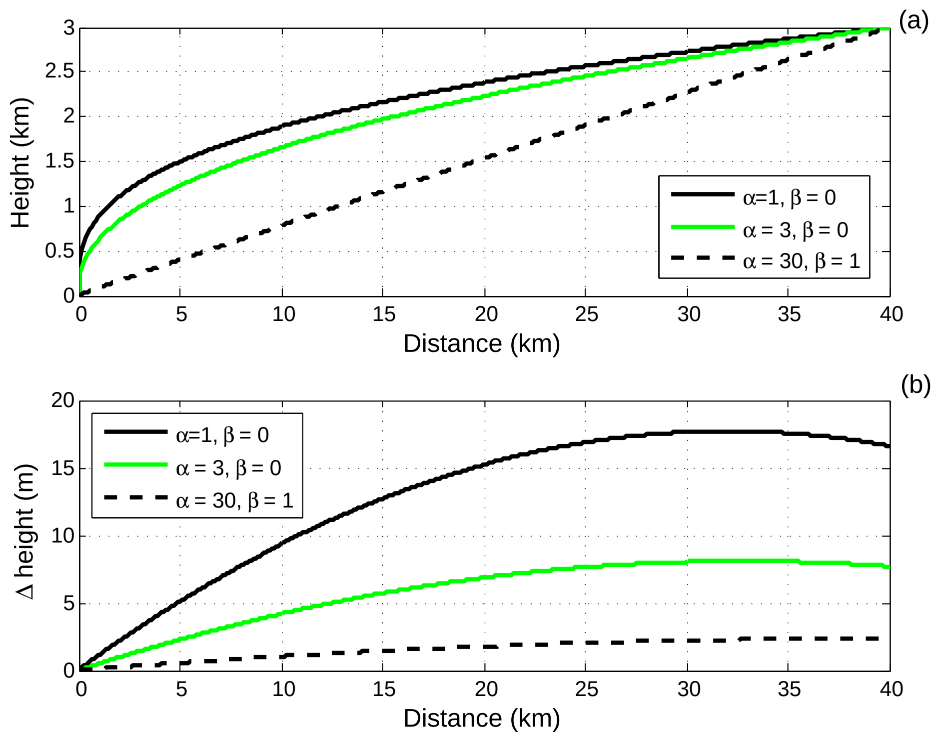

Figure 6: For different rheologies: (a) critical form profiles for the mass balance pattern in (28) using the positive sign; and (b), the change in profile elevation when the sign of mass balance pattern is reversed. Note the change of y-axis scale between (a) and (b). The figure shows that the critical form of the orogen is extremely insensitive to the assumed pattern of the mass balance, with rheologies that are near Coulomb-plastic being the least sensitive.

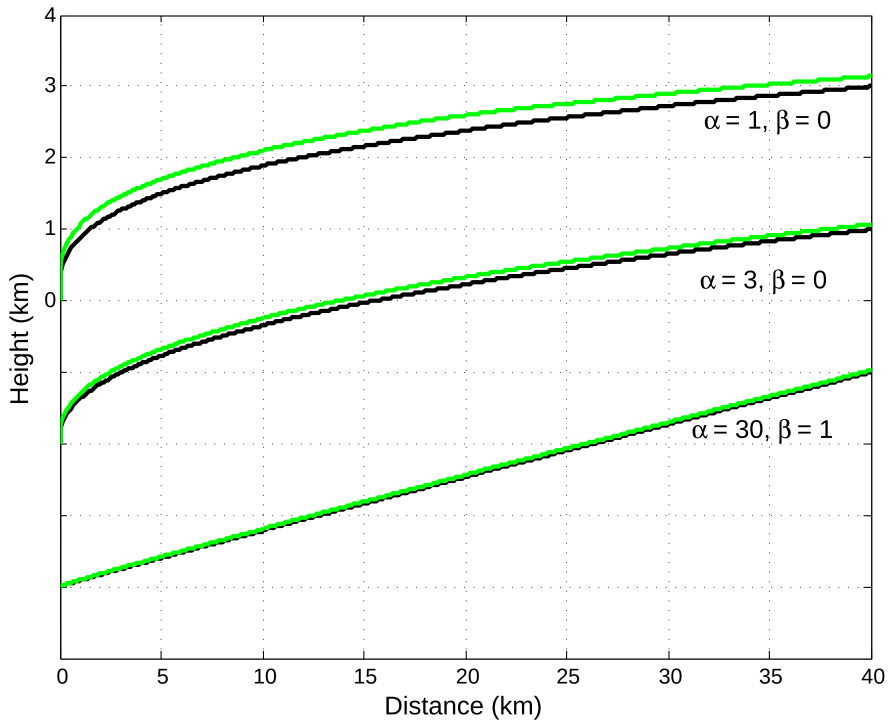

Figure 7: For different rheologies, the response of the critical form to adding a large (20 cm yr^{-1}) spike in rock uplift rates in a 2 km wide swath centered at 20 km. The black shows the critical form for the control case, the gray line shows the critical form including the spike in mass balance. Curves are offset on the y-axis for clarity. Even with this large perturbation to the mass balance there is relatively little change in the critical form. As in Figure 6 near Coulomb-plastic rheologies are least the sensitive.

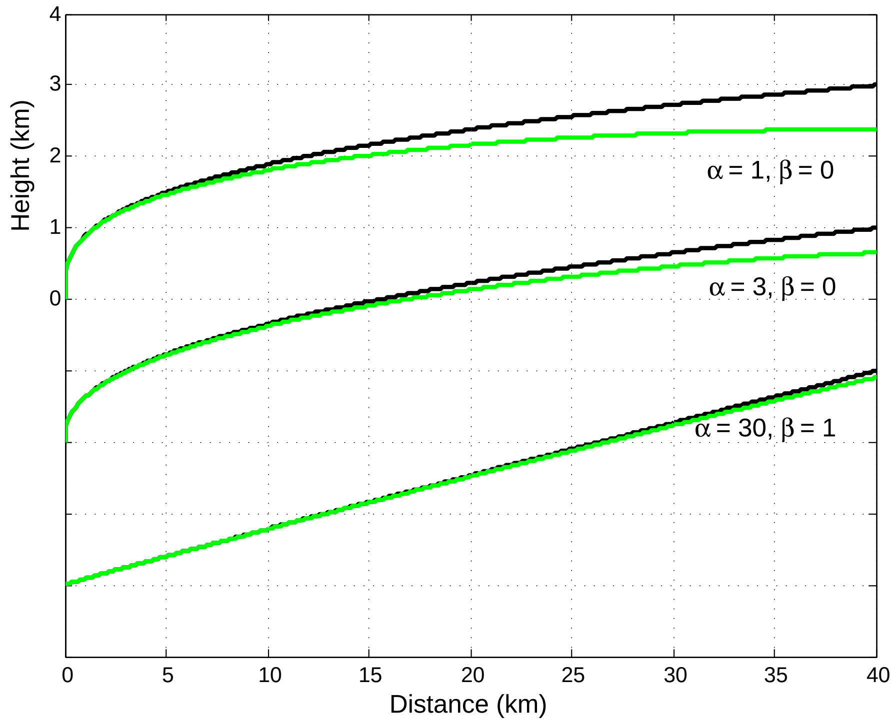

Figure 8: For different rheologies, the response of the critical form to changing the pattern of the basal traction velocity from constant (black lines), to a linear decrease (gray lines). Curves are offset on the y-axis for clarity. The surface slope at the divide must go to zero. For a near Coulomb-plastic rheology, the critical form is almost unaffected.

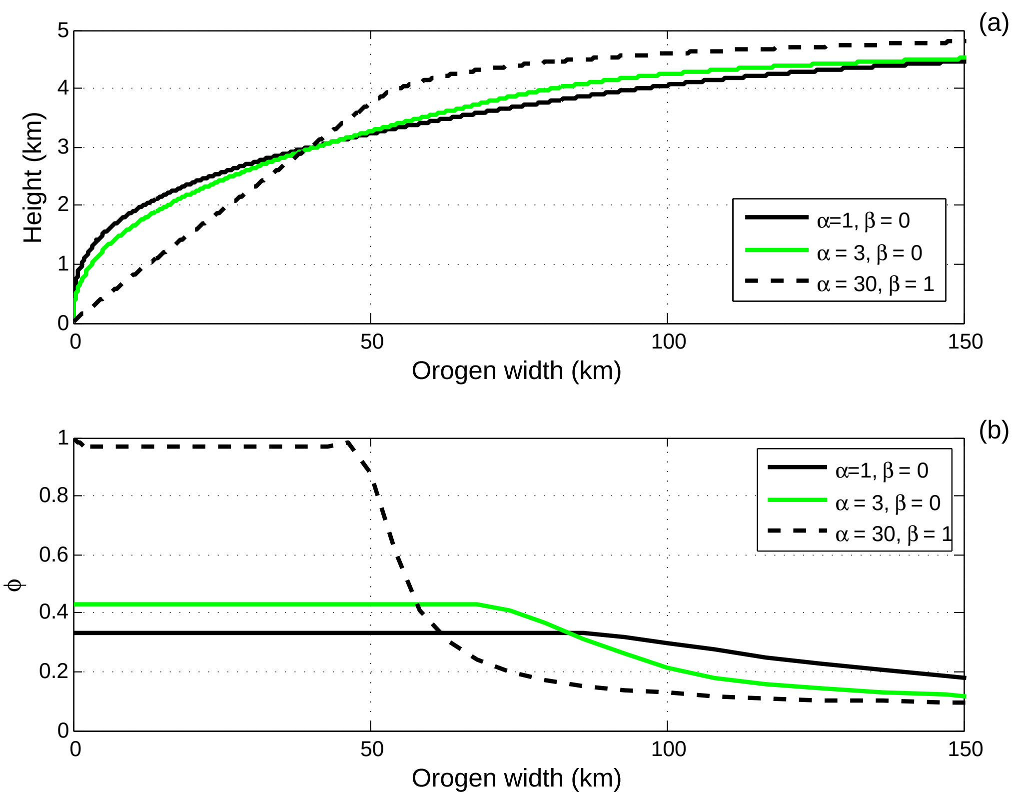

Figure 9: For different rheologies, (a) height of the orogen, H, and (b) value of the height-width power-law relationship, ϕ, as a function of orogen scale, L, during the approach to a plateau. An exponential decrease in the normalizing stress has been assumed at depths exceeding 25 km. Consequently shear stresses within the orogen cannot support topography above approximately 4 km. See (29) and text for more details.

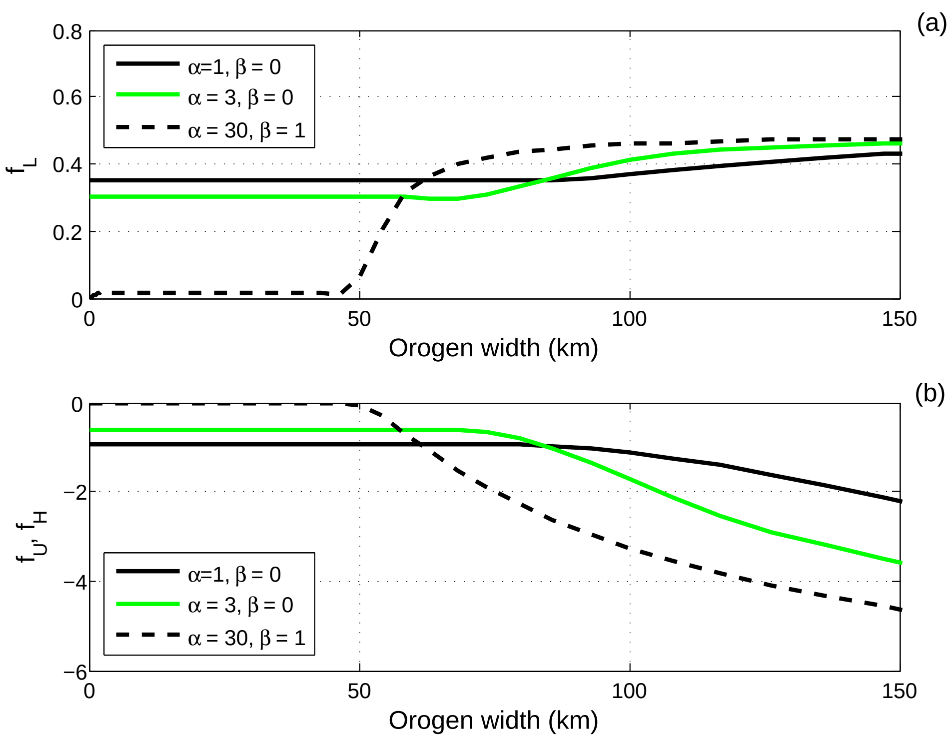

Figure 10: For different rheologies, feedback factors (a) f_L and (b) f_H and f_U, as a function of orogen size, L, during the approach to a plateau. $(h, m, n) = (2, 1/2, 1)$ was assumed for the values shown. Details of calculations as for Figure 9. Because $hm/m = 1$, $f_H = f_U$, but this is not generally true. Note the different scales on the y-axes. For the near Coulomb-plastic rheology, note the strong change in orogen sensitivity between approximately 50 and 70 km in width.