ESS 314

Space Physics by John Booker and Gerard Roe

Introduction

Earth is immersed in an environment that is not empty space, despite what we call the region outside its atmosphere. Our sun is constantly losing large amounts of matter to its surroundings, sometimes extremely violently. The surface of Earth would not be a very hospital to life if it were not for a variety of physical phenomena that protect the surface from most of these effects. This chapter discusses these phenomena. Since this is the starting point for our journey to the planet's surface, it will also be the starting point for a review of basic physics.

Matter

There are four basics forms of matter.

1. Gas – a loose configuration of molecules that interact only by collision. This means that a gas will expand forever unless confined.
2. Liquid – a configuration of molecules that stick to one another, but, which can be easily broken apart by external forces. Like gases, liquids deform permanently under the influence of forces that do not change their volume. Unlike gases, they can form free surfaces that can confine them.
3. Solid – a tight configuration of molecules that require a threshold force in order to break them apart. Solids deform under the influence of external forces, but return to their original configuration if the force is removed and does not exceed the threshold for tearing the molecules apart.
4. Plasma – is a gas whose molecules have been energized sufficiently to strip many, if not all electrons from their atomic nuclei. Thus there are no molecules. Under the influence of electromagnetic fields, a plasma reacts very differently from a gas because positively and negatively charged pieces of the plasma experience different forces. At the low densities typical of interplanetary space, collisions between plasma particles are rare or non-existent and they interact primarily through electromagnetic forces.

Many common substances, such as water exist in the first three forms at temperatures typical of our environment and are therefore very familiar. Gasses and liquids are both termed "fluids" because they share the property of deforming forever under the influence of forces that do not try to change their volume. Plasmas exist inside devices such as florescent lights, in lightning and other electrical sparks and are the primary form of matter in space. So this chapter is primarily about the physics of plasmas.

Basic Physical Laws

Fundamental laws of classical dynamics are:

1. *Conservation of mass* – Einstein generalized this to include the equivalence of energy and mass and the possibility of conversion of mass to energy. This is important to us because it is going on inside the Sun, but it is not important to the other phenomena that we will consider in this course. Thus we will assume that mass is strictly conserved

2. *Conservation of linear momentum* – If m is the mass of an object (assumed constant) and $\mathbf{u}$ is its velocity, the quantity $m\mathbf{u}$ is called its momentum. This law says that the in the absence of force, the momentum does not change with time. More generally

$$\mathbf{F} = \frac{d}{dt}(m\mathbf{u}) = m\mathbf{a}$$

where **F** is the vector sum of all forces acting on an object and **a** is its acceleration. This is known as Netwton's 2nd Law. In a fluid, this must be re-written in terms of quantities per unit volume

$$\mathbf{f} = \rho\mathbf{a} = \rho\frac{D\mathbf{u}}{Dt} = \rho\frac{D^2\mathbf{r}}{Dt^2}$$

where **r** is the instantaneous position of a "fluid particle" and ρ is the fluid density. A tricky point is that the volume used in this law is attached to the molecules of the fluid. No mass is ever allowed to cross its surface. The shape of the volume can deform as the fluid moves, but the mass inside the volume is constant. To remind us that we are discussing a volume that moves with the fluid, we replace d in the time derivative by D when discussing fluids.

3. *Conservation of angular momentum* – Angular momentum is a generalization of linear momentum for motion in a circle and is mass times velocity times the radius of the circle of curvature. It is conserved in the absence of the rotational equivalent of force, which is called torque.

Name (linear)	Expression (linear)	Equivalent name (rotation)	Equivalent expression (rotation)
Distance	x	Angle	θ
Speed	$u = dx/dt$	Angular speed	$\omega = d\theta/dt$
Velocity	$\mathbf{u} = d\mathbf{x}/dt$	Angular velocity	$\mathbf{\Omega} = d\theta/dt$ (see figure)
Acceleration	$\mathbf{a} = d^2\mathbf{x}/dt^2$	Angular acceleration	$\boldsymbol{\alpha} = d^2\theta/dt^2$
Mass	m	Moment of inertia	$I = mr^2$
Force	**F**	Torque	$\mathbf{T} = \mathbf{r} \times \mathbf{F}$

Table showing properties of linear motion and the rotational equivalent

For every property of linear momentum, there is an equivalent property in rotational physics. See the table. Also the laws of linear motion have their rotational equivalent, So linear momentum = m**v**, the angular momentum = I$\mathbf{\Omega}$. Newton's second law for linear motion is given by **F** = m**a**. The rotational equivalent is given by **T**=I$\boldsymbol{\alpha}$.

Figure shows right-hand-rule. The fingers curl in the sense of rotation, and the thumb points in the direction of the vector, $\mathbf{\Omega}$.

4. *Conservation of energy* – Energy comes in several flavors: kinetic, potential, heat, radiation and mass. We have already agreed not to worryabout the equivalence of energy and mass. Taking the dot product of the 1st equation above with the velocity gives

$$\mathbf{u}\cdot\mathbf{F} = \mathbf{u}\cdot\frac{d}{dt}(m\mathbf{u}) = \frac{d}{dt}\left(\frac{1}{2}m\,|\,\mathbf{u}\,|^2\right) = \frac{d}{dt}E_{KE}$$

The quantity E_{KE} in the brackets on the right is called "kinetic" energy. In the absence of force its conservation is obviously a consequence of the conservation of momentum and not a separate law. The right hand side of the above relation is the rate of change of what is often called "work" although it is in a different form than you may have seen it before. The standard definition of the amount work done moving an object a distance δx against a force F is $\delta W = F\delta x$. Dividing this by a small time interval δt gives

$$\frac{\delta W}{\delta t} = F\frac{\delta x}{\delta t}$$

If this time interval is made very small and F is actually the component of a vector force in the direct of the displacement, this becomes

$$\frac{dW}{dt} = F\frac{dx}{dt} = |\,\mathbf{F}\,||\,\mathbf{u}\,| = \mathbf{F}\cdot\mathbf{u} = \mathbf{u}\cdot\mathbf{F} = \frac{d}{dt}E_{PE} = \frac{d}{dt}E_{KE}$$

So the change in kinetic energy must be balanced by a change in work or "potential" energy E_{PE}. Again this is a consequence of the conservation of momentum and not a separate law. This brings us to "heat". Kinetic energy depends on the magnitude of the velocity and not on its direction. The kinetic energy just considered is of an object in which all parts are moving in the same direction. However, this is not the only kind of kinetic energy. All substances that are not at a temperature of absolute zero have internal kinetic energy that is random in nature. Because this motion is random in direction, its average contribution to the large-scale movement of the object cancels out and does not contribute to the kinetic energy that is conserved as a consequence of the conservation of momentum. This thermal energy is, however, conserved and can be passed on to adjacent objects through collisions (or electromagnetic forces in a plasma). A hot solid, for instance, has rapidly vibrating molecules. When this solid is placed next to a cool solid with slower moving molecules, the rapidly vibrating ones in the hot substance will pass some of their momentum to the lower ones at points where they touch. This is a one-way process. A fast molecule can speed up a slow one, but a slow one cannot speed up a fast one. Thus "heat", which is the average random kinetic energy of a substance "flows" from hot regions to cold regions. We will discuss this in more detail later. Finally, electromagnetic radiation can transfer energy from one place to another. In contexts that interest us, radiation becomes of interest when it is converted to heat.

5. Inverse square laws for mass and charge – There are two fundamental forces between small pieces of matter:

   Gravitational: $$\mathbf{F}_g = Gm_1m_2\frac{\hat{\mathbf{r}}}{|\mathbf{r}|^2}$$

   Electrostatic: $$\mathbf{F}_e = \frac{-1}{4\pi\varepsilon_0}q_1q_2\frac{\hat{\mathbf{r}}}{|\mathbf{r}|^2}$$

   where m_1 and m_2 are the masses (Kg), $G = 6.6742\times10^{-11}$ Nm^2/Kg, q_1 and q_2 are the charges of two particles (Coulombs) and $\varepsilon_0 = 8.8542\times10^{-12}$ C^2/Nm^2, $|\mathbf{r}|$ is the distance between the two particles and $\hat{\mathbf{r}}$ is a unit vector pointing from particle one to particle two. The gravitational equation is called Newton's 1st Law.

Except for the constants G and $1/4\pi\varepsilon_0$, gravitation and electrostatic forces have the same form. The electrostatic force is much stronger than the gravitational force, however. For two electrons ($m_e = 9.1094\times10^{-31}$ Kg; $q_e = 1.6022\times10^{-19}$ C)

$$\frac{F_e}{F_g} = 4.17\times10^{42}$$

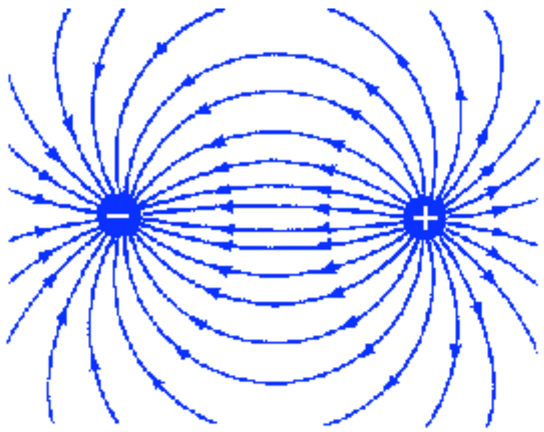
where 10^{42} means 1 followed by 42 zeros (a very big number). Another major difference is that charges can be positive or negative, while mass can only be positive (as far as we know). Because of the negative in front of the constant in the electrostatic force equation, like charges repel each other, while opposite charges attract. For gravity, masses always attract each other. If we define the electric and magnetic "fields" **g** and **E** by

$$\mathbf{g} = Gm_2 \frac{\mathbf{r}}{|\mathbf{r}|^3} \qquad \mathbf{E} = \frac{-1}{4\pi\varepsilon_0} q_2 \frac{\mathbf{r}}{|\mathbf{r}|^3}$$

the above force equations become

$$\mathbf{F}_g = m_1\mathbf{g} \qquad \mathbf{F}_e = q_1\mathbf{E}$$

It is convenient to write the equations in this form because they are then valid for gravitational or electric fields due to a superposition of many masses or charges. The electric field of a single charge is a vector that radiates in all directions and decays like r^{-2}. The electric field of a positive and negative charge placed close to one another is called a "dipole" and looks like this:



The magnitude of the field of a dipole decays like r^{-3} when you are at a distance more than a few times the charge separation.

Comparing the gravitational force equation in its field form with Newton's 2nd Law, we see that the gravitational field cannot be distinguished from any other kind of acceleration. On the surface of a planet such as Earth, we often write the force per unit volume

$$\mathbf{f}_g = \rho g\,\hat{\mathbf{z}}$$

because the direction of the vertical unit vector $\hat{\mathbf{z}}$ is almost always defined to be the direction of gravity.

Another important phenomenon occurs when electric charges move: they generate a magnetic field. A collection of moving charges is called an electric current. Most currents are due to moving electrons. By convention a positive current is in the opposite direction to the electron velocity. The magnetic field of a straight line current looks like this:

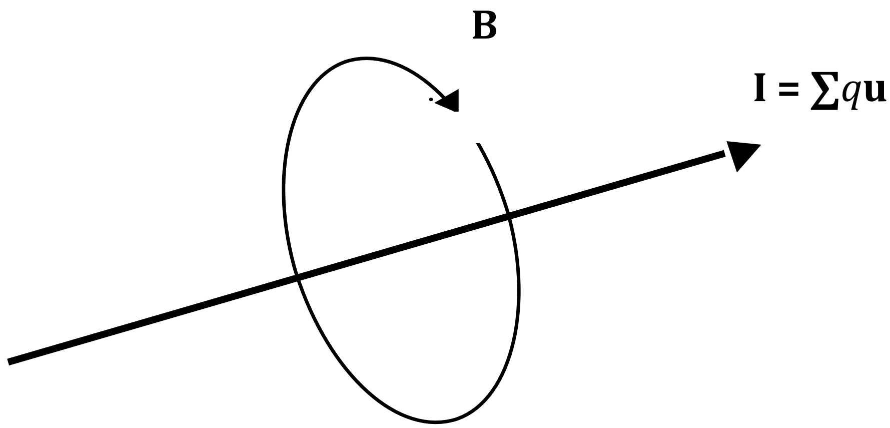
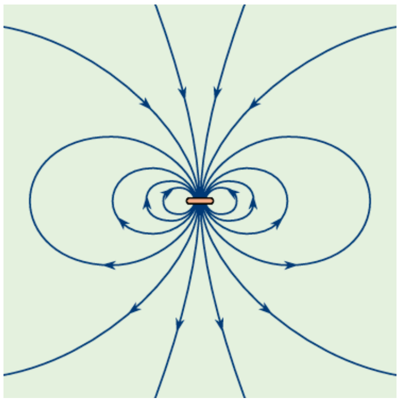
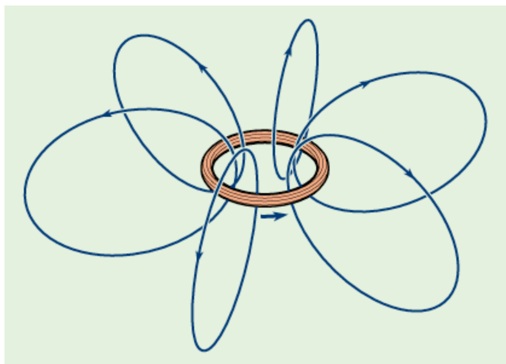


Diagram showing the relationship between a current, I (represented by a sum of moving charges), and the magnetic field they generate. The sense of the field is another application of the right-hand-rule.

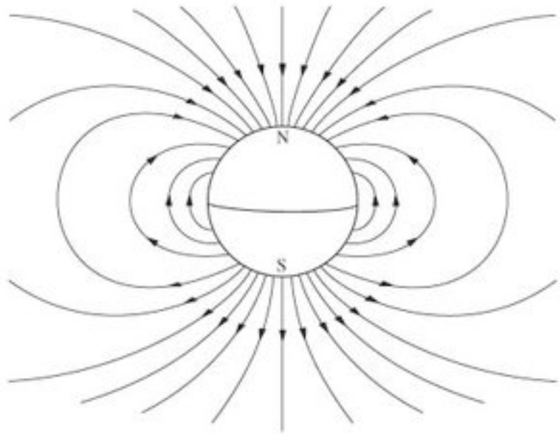
Its magnitude decays like r^{-1} away from the current and its direction is determined by the right-hand rule: point your thumb in the direction of the current and your fingers will curve in the direction of the magnetic field. If the current travels in a circle, the magnetic field inside the loop will be perpendicular to the plane of the loop. Here are a side and angle view:





Outside the loop at a distance large compared to the diameter of the loop, the magnetic field looks like a dipole due to a positive and negative "magnetic charges" or "monopoles" located just above and below the plane of the loop. Magnetic charges have never been isolated in nature and are generally thought to be impossible. Thus magnetic fields are always the result of moving electric charges.

Earth has a magnetic field that is generated by electric currents in its iron core. If there were no Sun, its magnetic field would be an excellent approximation of a dipole and look like this:



The spatial separation of the field lines is proportional to the magnitude of the field, which is clearly stronger as you approach the "poles" where the surface field is vertical. We will return to the effect that the Sun has later.

The electrostatic force equation above is correct only if the charges are at rest. If a charge q_1 moves at a velocity **u** relative to reference frame in which there is a magnetic field **B**, it experiences a force, which is perpendicular to both the magnetic field and the velocity

$$\mathbf{F}_m = q_1 \mathbf{u} \times \mathbf{B}$$

The use of the cross product makes this expression much simpler than it would otherwise be. This force is maximum when **u** and **B** are perpendicular and is zero when they are parallel. Adding the force between the charges and their movement through the magnetic field, the total electromagnetic force can be written

$$\mathbf{F}_{em} = \mathbf{F}_e + \mathbf{F}_m = q_1 (\mathbf{E} + \mathbf{u} \times \mathbf{B})$$

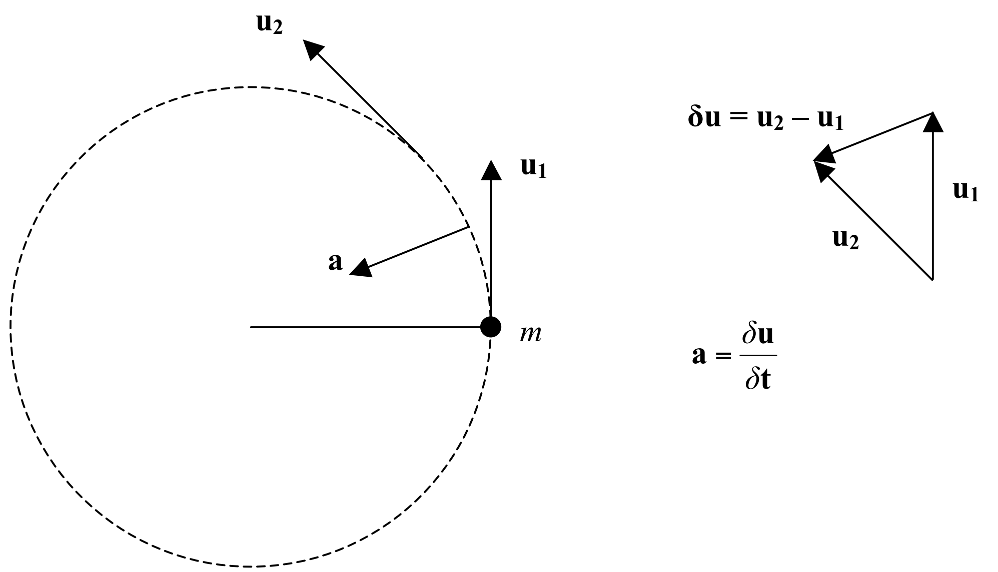
Comparing this with the original electrostatic force equation, we see that it would be correct for both moving and stationary charges if a moving charge experiences an electric field $\mathbf{u} \times \mathbf{B}$. The magnetic force is thus a "relativistic" effect, because it depends on the relative motion of the coordinate systems in which the measurements of the electric and magnetic fields are made.

Since an electric current is a collection of moving electrons, the total force on a current **I** is just the sum of the forces due to each charge and is $\mathbf{F}_\mathbf{I} = \mathbf{I} \times \mathbf{B}$. If you want to use the current density, **j** instead, the force per unit volume is $\mathbf{f}_\mathbf{j} = \mathbf{j} \times \mathbf{B}$ and the total electromagnetic forces per unit volume are

$$\mathbf{f}_e + \mathbf{f}_\mathrm{j} = \rho_e \mathbf{E} + \mathbf{j} \times \mathbf{B}$$

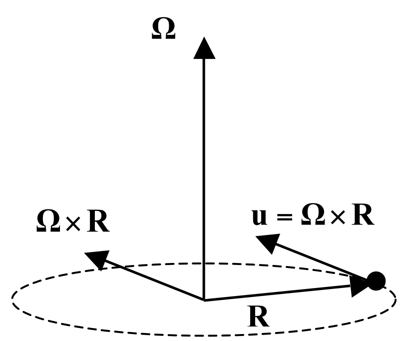
where ρ_e is the electric charge density. The last term on the right is commonly called the "Lorentz" force.
Orbital motion

If we attach a mass m to the end of a string, we can whirl it in a circle as shown in this figure:



The velocity of the mass changes all the time as the mass goes around because its direction changes. Thus it is accelerated. The direction of this acceleration is towards the circle center.

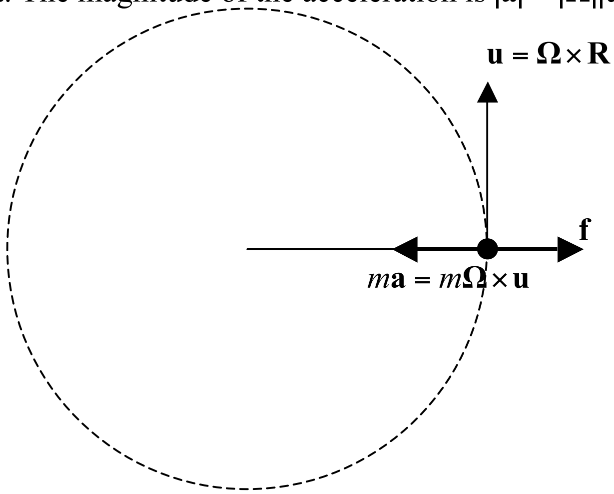
We can use the properties of the cross product to calculate what this acceleration must be. Let $\mathbf{\Omega}$ be a vector whose length is Ω and whose direction is perpendicular to the plane in which the mass circulates. Let $\mathbf{R}$ be the position vector of m from the center of circulation. It is then obvious from this figure:



and the properties of the cross product that $\mathbf{u} = \mathbf{\Omega} \times \mathbf{R}$ and that the magnitude this velocity is ΩR. If $\mathbf{\Omega}$ is constant, the vector acceleration is

$$\mathbf{a} = \frac{d\mathbf{u}}{dt} = \frac{d}{dt}(\boldsymbol{\Omega} \times \mathbf{R}) = \Omega \times \frac{d\mathbf{R}}{dt} = \boldsymbol{\Omega} \times \mathbf{u}$$

Note that the right hand rule for the cross product implies that $\boldsymbol{\Omega} \times \mathbf{u}$ points towards the center of circulation as we expect. The magnitude of the acceleration is $|\mathbf{a}| = |\boldsymbol{\Omega}||\mathbf{u}| = \Omega(\Omega R) = \Omega^2 R$.



Note that in this figure, a "force'" vector **f** is drawn in the opposite direction to the actual force *m***a** that must be applied to the mass to keep it in the circular path. This **f** is the tensile force in the string, which must just balance the force applied to the mass if the length of the string is to remain constant. This **f** is sometimes called the "centripetal" force, while m**a** is usually called the "centrifugal" force. The fact that the two forces are equal in magnitude and opposite in direction for the equilibrium situation is an example of Newton's 3rd ``Law" (for every action there is an equal and opposite reaction), which is not a separate law at all, but another direct consequence of his 2nd Law.

The period of a satellite

The string in the last example is an artifice that is not required if the body circulates under the influence of gravitational or electric fields. The gravitational field outside of a planet (to a very good approximation) the field of a point mass *M* with the same mass as the planet. For a small mass *m* to orbit at a constant distance *R* from the center of the planet, the forces on it must balance, thus

$$G\frac{mM}{R^2} = m\Omega^2 R$$

If we solve this for Ω, we easily see that the orbital period of a satellite is

$$T = \frac{2\pi}{\Omega} = 2\pi\sqrt{\frac{R^3}{GM}}$$

which is independent of the mass of the satellite. Physically, one can think of a satellite as constantly falling towards the planet, but always missing because of its orbital velocity. This relation allows one to determine the mass of the planet from the period of a satellite (artificial or natural). Re-arranging the last equation gives

$$M = \frac{4\pi^2 R^3}{GT^2}$$

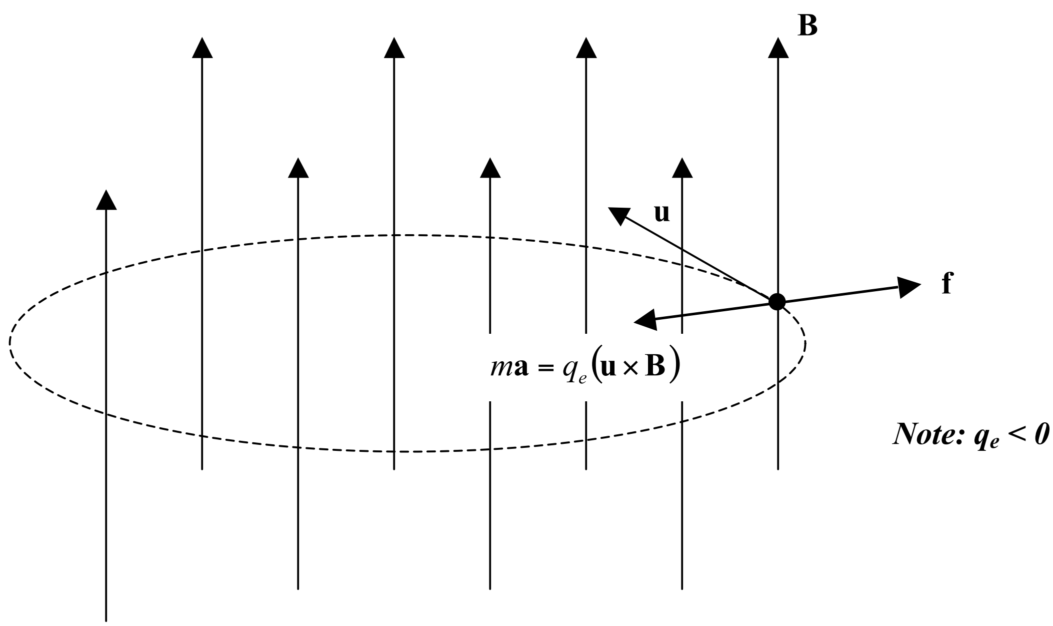
This also allows one to determine the mass of our Sun using the orbital period of any one of its planets and the radius of the orbit. Earth's distance from the Sun is about 150 million km. Thus the mass of the Sun is approximately

$$\frac{4\pi^2 \times (1.5\times 10^8 \times 10^3)^3}{6.667\times 10^{-11} \times (\pi \times 10^7)^2} = \frac{4\times 3.375\times 10^{33}}{6.7\times 10^3} \approx 2\times 10^{30}\, Kg$$

This theory is not the whole story, because there is another equally valid solution in which the path of the satellite is an ellipse with the planet at one of the focii. Earth's orbit about the Sun is actually an ellipse which varies slowly with time. This will be discussed later when we discuss climate variation. Furthermore a circular or elliptical orbit is only approximate for very massive satellites such as Earth's Moon. In this case both Earth and Moon orbit around the center of mass of the Earth-Moon system.

Gyro-motion of a charged particle in a magnetic field

An electron moving with velocity **u** perpendicular to a uniform magnetic field **B** experiences a constant force $q_e(\mathbf{u}\times\mathbf{B})$ that is perpendicular to the velocity and the magnetic field. Just as for the circulating mass on a string or the satellite in a radial gravitational field, this force will cause the electron to follow a curved path:



Note that the charge on an electron is negative. Thus the direction of m**a** is opposite to what you get from applying the right hand rule to the cross product of the velocity and the magnetic field.

The only forces acting on the electron in this case are the centripetal force and the electromagnetic force. Since they must balance, they both must be radial to the center of curvature. Balancing their magnitudes gives

$$q_e\,|\mathbf{u}||\mathbf{B}| = q_e\,\Omega R = m_e \Omega^2 R$$

and thus

$$\Omega = \frac{q_e}{m_e}|\mathbf{B}|$$

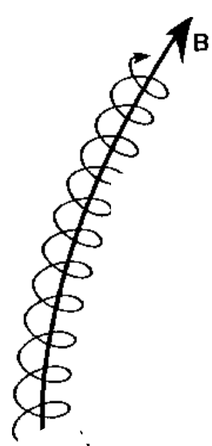
which depends only on the strength of the magnetic field and the charge to mass ratio of the electron. This is called the "gyro" frequency by plasma physicists and the "cyclotron" frequency by atomic physicists. The fact that radius does not enter this relation means that the elliptical solution for a satellite does not exist in this case and the electron orbit will always be circular.

Substituting this expression for Ω in the force balance above and re-arranging terms leads immediately to the "gyro-radius"

$$R = \frac{m_e\,|\,\mathbf{u}\,|}{q_e\,|\,\mathbf{B}\,|} = \frac{\sqrt{2 m_e E_{KE}}}{q_e\,|\,\mathbf{B}\,|}$$

which depends on the momentum (or the energy) of the electron. A proton would also orbit in a similar way, but would have a much lower frequency and a much larger gyro-radius because of its larger mass. It would also circulate in the opposite direction.

The result just derived has interesting consequences. If the electron's initial velocity is not perpendicular to the magnetic field, its path will be a helix like this



This is because the component of velocity along (parallel to) the magnetic field, $\mathbf{u}_{||}$ creates no electromagnetic force and thus $\mathbf{u}_{||}$ remains constant as the electron orbits in the magnetic field. The acceleration on the electron is entirely perpendicular to the direction of the magnetic field and must be balanced by the centripetal acceleration about the center line of the helix.

Trapping of charged particles in a planetary magnetic field

Things get even more interesting if |**B|** increases in the direction of **B**. An electron spiraling along a field line has a velocity parallel to the field line and a velocity of circulation around the field line (and hence perpendicular to the field line, Thus

$$\mathbf{u} = \mathbf{u}_{||} + \mathbf{u}_{\perp} = \mathbf{u}_{||} + \mathbf{\Omega} \times \mathbf{R}$$

The kinetic energy of this electron is

$$E_{KE} = \frac{1}{2} m_e |\mathbf{u}|^2 = \frac{m_e}{2}\left(|\mathbf{u}_{||}|^2 + (\Omega R)^2\right)$$

which must be conserved. This means that if ΩR increases $|\mathbf{u}_{||}|$ must decrease. The relation between ΩR and |**B|** can be deduced from the conservation of angular momentum. Angular momentum is a generalization of linear momentum and is mass times velocity times the radius of the circle of curvature. Thus as a particle moves from position 1 to a position 2 where the magnetic field strength has changed from $|\mathbf{B}_1|$ to $\mathbf{B}_2|$,

$$m_e \Omega_1 R_1^{\,2} = m_e \Omega_2 R_2^{\,2}$$

Simple manipulation reduces this relation to

$$\left(\frac{R_2}{R_1}\right)^2 = \frac{\Omega_1}{\Omega_2}$$

and thus

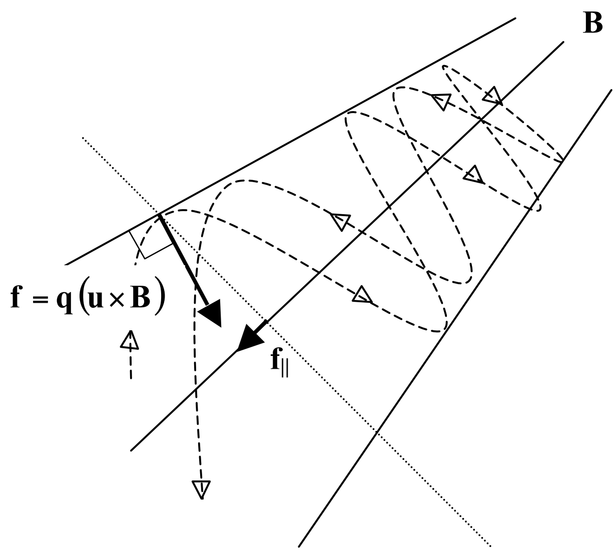
$$\frac{(\Omega_2 R_2)^2}{(\Omega_1 R_1)^2} = \left(\frac{\Omega_2}{\Omega_1}\right)^2 \left(\frac{R_2}{R_1}\right)^2 = \left(\frac{\Omega_2}{\Omega_1}\right)^2 \frac{\Omega_1}{\Omega_2} = \frac{\Omega_2}{\Omega_1}$$

The gyro-frequency relation above then gives

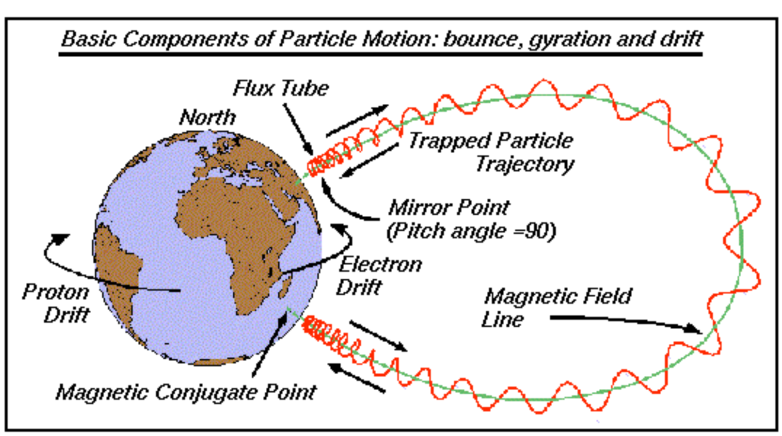
$$\frac{(\Omega_2 R_2)^2}{(\Omega_1 R_1)^2} = \frac{\Omega_2}{\Omega_1} = \frac{|\mathbf{B_1}|}{|\mathbf{B_2}|}$$

If $|\mathbf{B}_2| > |\mathbf{B}_1$ we must have that $\Omega_2 R_2 > \Omega_1 R_1$. So we finally conclude that the conservation of kinetic energy requires that $\mathbf{u}_{||}$ must decrease as $|\mathbf{B}|$ increases. To sum up: As the magnetic field gets stronger, the orbital velocity and the orbital part of the kinetic energy increase. This is done at the expense of the velocity component parallel to the magnetic field. In fact, $\mathbf{u}_{||}$ can actually decrease to 0.

One might think that the electron will just stop its motion along the field when the growing orbital kinetic energy has robbed all the kinetic energy of the motion along the magnetic field. However, this is not the case. The force on the electron is perpendicular to both the magnetic field and the velocity and when the field lines are converging, this force is not perpendicular to the center of the helix as you can see in this figure:

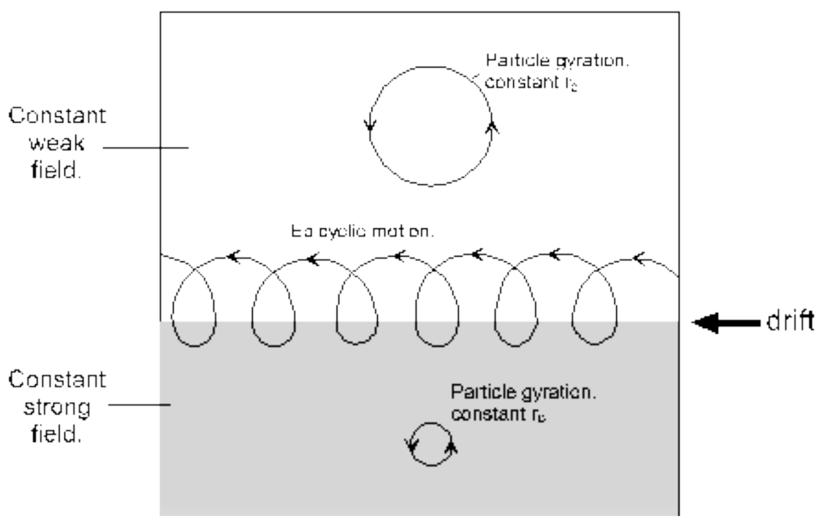


Thus for converging magnetic field lines, there is always a component of the force ($\mathbf{f}_{||}$) that is in the direction of field line divergence. This component is determined by the gyro-velocity and the field convergence and does not go to zero when $\mathbf{u}_{||}$ goes to zero. Thus the electron will start to accelerate back out of the region of converging magnetic field lines. This is called a magnetic "bounce". Because Earth's magnetic field is a dipole with northern and southern polar regions of magnetic field convergence, electrons can have bounce points in both hemispheres and be trapped like this:



As long as an electron spiraling along Earth's field bounces high enough that it is unlikely to collide with an air molecule, the electron will bounce back and forth many times. This is the reason for the existence of the Van Allen radiation belts. If the bounce point is low enough that the electron hits the upper air, it will cause the gas to emit light that is called the “Aurora”.

There is one further effect that we can explain with the same physics. This is east-west drift of trapped particles. If an electron is circulating in a region where the magnetic field magnitude changes in the direction perpendicular to the magnetic field, the radius of gyro motion will change as the electron goes around its orbit. In this illustration, the magnetic field perpendicular to the page. It is weak at the top and so a particle of a given energy will have a large radius of gyration. In the lower (gray) area, the magnetic field is strong and so a particle with the same energy has a much smaller radius of gyration. A particle that spends part of its time in the weak magnetic field and part of its time in the strong magnetic field will have an orbit that has a different radius of gyration in the two regions. The result is that its orbits are not closed and so the particle “drifts”.



This happens around Earth, because the magnetic field decreases with distance from the center of the planet. Thus trapped particles drift east or west depending on their charge (see the next to last figure above). The biggest source of trapped particles is the Sun and they are preferentially injected into Earth’s field on the sunward side. The drift, however, implies that eventually trapped particles are found at all longitudes.