Homework on diffusion and heat conduction: due next Friday.

1. Order-of-magnitude question

The Earth's surface is about 70% water. Mars's diameter is about half the Earth's, but it has no surface water. Compare the land areas of the two planets.

2. Piles of stones

We talked about how many processes are governed by diffusion, including the gradual degradation of moraines left behind by retreating glaciers. The slow transition from a sharply defined peak to a smooth pile of rubble is one approach people use to date when the glaciers were there, and so infer how climate has changed in the past.

Glacial moraines can be dated by their shape, described by the conservation of mass equation $\partial z / \partial t = -\partial q / dx$, where z is the vertical height of the moraines, x is the horizontal axis, and q is the soil flux. The soil flux is in turn given by $q = -E \partial z / \partial x$, where E is known as the dispersion coefficient (equivalent to the diffusivity). For slopes a few meters tall E is typically around 10^{-2} m^2 yr^{-1} (note the units of time here).

Without solving the equation explicitly, use these values to estimate how the shape of a moraine changes overtime. Assume that the moraine at t=0 has a step shape: that is, for x<0, z=0, for x>0, z = 10m.

a) Using scale analysis (i.e., relationship between length scale, time scale, and dispersion coefficient) how far does the diffusive length scale of the moraine after 200 years? In 10000 years? This is a crude calculation of how far the rocks would have spread.

b) How do these numbers conform to your expectations? What other processes might cause the moraine to spread faster?

3. Pollution.

A careless researcher drops a flask of chlorine gas that shatters and, at a point, results in a concentration of chlorine 1000 times higher than is safe. The point source diffuses according to

$$\frac{\partial n}{\partial t} = \nu \frac{\partial^2 n}{\partial x^2}$$

where n is the concentration of chlorine molecules (the units aren't important but they would be mol m^{-3}).

First assume that the cloud of chlorine diffuses by molecular processes, for which $\nu = 1 \times 10^{-5}$ m^2 s^{-1}. Using scale analysis calculate:

a) How long it takes the cloud to spread 100 m.
b) Assuming this same timescale applies to the exponential decay of the concentration at the point of release, calculate how long it takes for the concentrations to fall to safe levels at that point.
c) In fact, turbulence in the form of eddies and votices mix the atmosphere much more effectively than molecular diffusion does. Measurements suggest that the 'eddy diffusivity' is more like 100 $m^2\ s^{-1}$. Recalculate your answers to part a) and b) using this eddy diffusivity.

4. Defrosting a mammoth

If it takes half an hour in a microwave to defrost a chicken, how long does it take to defrost a wooly mammoth (in the same, very big microwave)? Use scale analysis – you have enough information in the question – just make reasonable guesses about the size of a chicken and the size of a mammoth.

5. Lord Kelvin and the age of the Earth.

In the mid-nineteenth century, a great scientific eminence, William Thompson (later enobled by Queen Victoria for his contributions to science, to become Lord Kelvin) calculated the age of the Earth, at a time when geologists believed in uniformitarianism – that the Earth was immensely old. His contribution caused great consternation and was ultimately proven wrong, but set Geology back a few decades. His argument was based on diffusion. It is often claimed that he was wrong because he had not accounted for radioactive decay in the Earth's crust (even in the Encyclopedia Britannica), but this is not the case. This question walks you through some of the arguments, and asks you to think about why his calculation was wrong.

a) *Diffusive cooling of crustal shell.* The surface temperature of the Earth is 300K and the outer mantle around 1300K, giving an average temperature of 800K. Assuming that the Earth started out as a fiery ball of rock all at 1300K, how long would it take for diffusive cooling to cool the crust (assume a uniform slab 50 km thick)? Use scale analysis. Given your answers in years.

b) *Drain the Earth's interior of heat.* Calculate the amount of heat contained in the Earth using Q = m c T. Assuming the geothermal heat flux has a constant rate of 50 $mW\ m^{-2}$, how long would it take to drain the Earth of this heat? Give your answer in years.

c) What do you think of these numbers? What process within the mantle can transport heat has been left out of these purely diffusive calculations?

Helpful numbers: k_{rock}=2 $W\ m^{-1}\ K^{-1}$; ρ_{rock}=3 x 10^3 $kg\ m^{-3}$; c_{rock}=2 x 10^3 $J\ kg^{-1}\ K^{-1}$; Earth radius = 6400 km.