Lab 4: Diffusion
ESS 314

Due next Friday.

Please hand in the data sheets (the 2 pages of tables at the end of this handout) and the required graphs, as well as a short write-up of the answers to the questions in this lab, all stapled together.

Physical graph paper is available to be downloaded as a pdf on the class website, or you may use Excel or other plotting software – make sure you follow standard practice of good curve plotting:

- **Label the axes, including units.**
- **Use as much of the plotting area as you can (i.e., minimize empty spaces),**
- **Do not extrapolate between data points using unphysical or inappropriate curve-fitting functions. Ask if you are not sure.**

Introduction

This lab will introduce you to the fundamentals of diffusion. Consider a uniform half-space of material at a fixed temperature T_0. A half-space means surface of the block of stuff sits at z = 0, and we assume it continues downwards forever, and so get to ignore what happens at the bottom.

The top surface temperature will be changed in two ways:

Exercise (1): A sinusoidal oscillation of period τ with total swing (i.e., peak-to-peak amplitude) ΔT about T_0. This is the problem we solved in lectures.

Exercise (2): A step change ΔT (+ or -) at time $\tau = 0$.

The temperature in the half space obeys the 1D diffusion equation:

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2}$$

where t is time, z is the vertical coordinate and the *diffusivity* is

$$\kappa = \frac{k}{\rho c_p}$$

with k the thermal conductivity, ρ the density and c_p the specific heat. We will use two simple Matlab programs to examine the behavior of T for the two exercises, and for various materials.

However, before doing this we can easily guess what is likely to happen using a technique that is widely useful – scale analysis, which was covered in class. Suppose that T varies on a time scale τ. Then

$$\frac{\partial T}{\partial t} \approx \frac{T}{\tau}$$

Likewise, suppose that T varies spatially on a scale δ. Then

$$\frac{\partial^2 T}{\partial z^2} = \frac{\partial}{\partial z}\frac{\partial T}{\partial z} \approx \frac{1}{\delta}\frac{T}{\delta} = \frac{T}{\delta^2}$$

Substituting in the differential equation above we get

$$\frac{T}{\tau} \approx \kappa \frac{T}{\delta^2}$$

T cancels out and it is then simple algebra shows that

$$\delta = \sqrt{\kappa\tau}$$

Thus δ and τ are related to each other. For exercise (1), we expect that an oscillation with period τ will only penetrate about a distance of order δ into the half space. For exercise (2) we expect that the temperature disturbance will have penetrated a distance of order δ at time τ after the step.

Tasks

The Matlab programs are interactive. Data sheets to fill out are attached.

IMPORTANT: When you start a program the parameters are set to default values appropriate for water IF THEY ARE NOT ALREADY SET by a previous run of EITHER of the programs. These values are stored from run to run. If you change them, the new values will be carried on. *To restore default values use the command: "clear global".*

NOTE!! the phase lag is the time between adjacent zero-crossings (of the x-axis) of two sinusoidal curves with the same period.

Exercise 1: The sinusoidal forcing command is "diff_sin"

Data collection phase:

1. Start diff_sin and use the mouse to measure the amplitude and phase lag for several depths in water (the default parameters) at 1 day (86,400) seconds.

You may need to experiment to find the appropriate depths to use. The goal is to span amplitudes from about 0.8 of the surface value to about 0.1 of the surface value. You MAY need to change the scale on the temperature versus depth plot using the menu in order to span a useful range of depths.

2. Write the depths and phase lags in the appropriate boxes of the 2nd table.
   Note that values have already entered for air and aluminum.

Analysis phase:

3. Use the parameters in the 1st table to compute the missing parameters using the formulas given above. Then fill in the boxes for depth/δ in the next three tables.
4. On a single x-y plot (using physical graph paper, or using graphing software), plot amplitude versus depth for water, air and aluminum, at τ = 86,400 s (1 day).
5. On another single x-y plot (using physical graph paper, or using graphing software), plot the amplitude versus depth/δ for water, air and aluminum at τ = 86,400 s (1 day). (Note that δ is different for each material, and you should have calculated the different δs in part 3, above)
6. Now on two more x-y plots (using physical graph paper, or using graphing software), plot phase-lag versus depth and versus depth/δ for the three materials.
7. Compare your graphs. What did you learn?

Exercise 2: The step-function forcing command is "diff_step"

Data collection phase:

1. Run the diff_step program using the defaults for water. Using the mouse pick several values of temperature versus depth after 10,000 s and enter them in the appropriate Table. *Values have already been entered for air and aluminum*.

Analysis phase:

2. Plot on the same graph the amplitude of T vs. depth after 10,000 s for water, air and aluminum.
3. Why do aluminum and air look more like each other than like water?
4. Do the shapes of these curves depend on the magnitude of the surface temperature step?
5. Repeat the plot with depth replaced by depth/δ (make sure you use the value of δ appropriate for each material). What happens?

The data sheets for you to fill out and hand in are on the next two pages……

Table 1: Material properties (calculate κ and δ for each material & timescale)

	Water	Air	Aluminum	Ice
k	0.6	0.024	233	2.2
c_p	4200	1000	963	2100
ρ	1000	1.2	2700	1000
κ				
δ (86,400 s)				
δ (10,000 s)				

Tables 2, 3, and 4: Exercise 1: Temperature-versus-depth for 1-day sinusoidal forcing.

Water τ=86,400 s (1 day) δ=________________

depth	depth/δ	amplitude	Phase lag (seconds)

Air τ=86,400 s (1 day) δ=________________

depth	depth/δ	amplitude	Phase lag (seconds)
0.2		0.77	3.61×10^3
0.4		0.58	7.26×10^3
0.6		0.44	11.2×10^3
0.8		0.34	14.6×10^3
1.2		0.2	22.1×10^3
1.6		0.12	29.4×10^3

Aluminum τ=86,400 s (1 day) δ=________________

depth	depth/δ	amplitude	Phase lag (seconds)
0.5		0.73	4.36×10^3
1.0		0.53	8.67×10^3
1.5		0.39	13.0×10^3
2.0		0.28	17.3×10^3
3.0		0.15	25.9×10^3

Tables, 5, 6, and 7: Exercise 2: temperature-versus-depth 10,000 seconds after a step change in temperature.

Water δ (at τ = 10,000 s) = ________________

Depth	Depth/δ	Temperature

Air δ (at τ = 10,000 s) = ________________

Depth	Depth/δ	Temperature
0.159		0.801
0.337		0.598
0.523		0.402
0.748		0.238
1.0		0.107

Aluminum δ (at τ = 10,000 s) = ________________

Depth	Depth/δ	Temperature
0.331		0.801
0.692		0.598
1.11		0.402
1.50		0.260
2.00		0.128