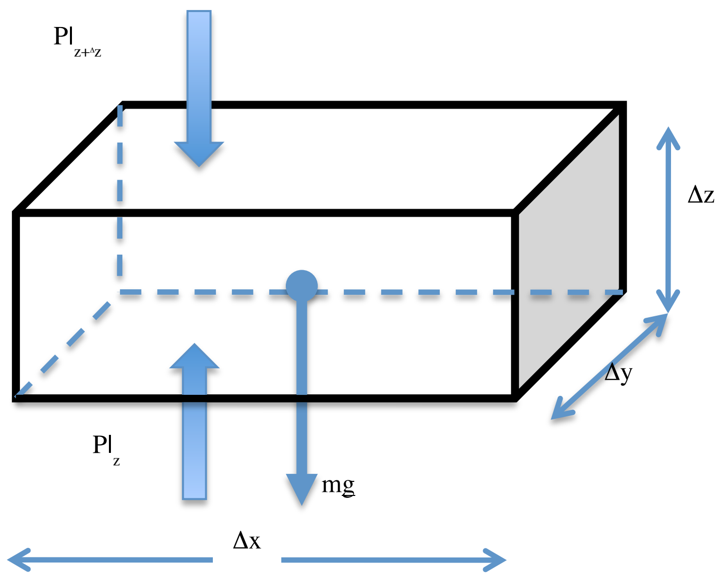
Homework #5 Fluid dynamics – please hand in next Friday.

Question 1: Order of magnitude estimate

According to folklore, every time you take a breath, you are inhaling some of the atoms exhaled in Caesar's last words "et tu, 314?" Is this true? If so, how many?

Question 2: Pressure gradient force



i) The parcel of fluid above has density ρ, has mass m, and is in static equilibrium. P is the pressure, and g is the acceleration due to gravity.

a) Write down the force balance in the vertical direction. Remember that pressure is not a force. And note the direction of the pressures acting on the faces of the parcel.
b) As we have done a bunch in class, give the first order Taylor series expansion of for $P|_{z+\Delta z}$ in terms of $P|_z$.
c) Hence show that the force balance can be written as

$$\frac{dP}{dz} = -\rho g$$

ii) For constant density, we derived in class that the pressure as a function of depth, h, is given by $P = P_s + \rho g h$. Show that this means that the pressure increases by 1 atmosphere for every 10m of depth in the ocean.

iii) For the atmosphere, we derived in class that atmospheric pressure varied as:

$$P = P_s e^{-\frac{z}{H}}$$

where H = RT_s/g. Show that if the whole atmosphere had the same pressure and temperature as it does at the surface, that the total depth of the atmosphere would be H.

Question 3: Forest Fires, survival, and relative (Lagrangian/Eulerian) motion

Some time in the future. It is not looking good for you. You are alone and lost in a parched shrub grassland on a blustery late summer's day with an air temperature soaring to 35 ºC. To soothed your frayed nerves you smoke a cigarette, and plod on. A little while later you look back and to your horror see you have started a fire: a wall of flame stretches across the horizon 2000 m away, and is racing towards you at 10 m s^{-1}. The grass is burning at 235 ºC. Your ignition temperature is considerably less. You need to stay below 55 ºC to survive. As your life flashes before your eyes, one of the highlights you fondly recall is taking ESS314, and so you make some quick calculations:

a) Assume that the temperature gradient between you and the fire is linear. If you stand still and wait for the fire to come to you, how long do you have to live?
b) You wish you'd worked out more, but you reckon you can still manage a respectable sprinting speed of 5 m s^{-1} (away from the fire!). How long do you have now?
c) BONUS POINTS: what is the trick to escaping from the fire alive? You have no blankets. No way of outrunning the flames, just grassland in all directions, and you are a smoker….

Question 4. More Lagrangian vs. Eulerian perspectives.
The last question was a 1-d problem. These questions are in 2-d. It will help to sketch out the situation. You can either solve the problem graphically, or use the equation the equation that links the Eulerian and Lagrangian perspectives:

$$\frac{DF}{Dt} = \frac{\partial F}{\partial t} + \vec{u}.\nabla F$$

a) A ship is steaming northward at a rate of 10 km h^{-1}. The surface pressure increases towards the northwest at the rate of 5 Pa km^{-1}. What is the pressure tendency recorded at the nearby island station if the pressure aboard the ship decreases at the rate of 100 Pa every 3 hr?

b) The temperature at a point 50 km north of a station is 3 ºC cooler than at the station. If the wind is blowing from the northeast at 20 m s^{-1} and the air is being heated by radiation at the rate of 1 ºC h^{-1}, what is the local temperature change at the station?

Question 5: Force balances and flowing ice: ice flow using a nonlinear flow law. In class we talked about a nonlinear flow law. In ice, the strain rate and the stress are related by Glenn's flow law:

$$\frac{du}{dz} = A\sigma^3$$

where the stress is given by:

$$\sigma = \rho g(h - z)\frac{dh}{dx}.$$

By substituting the above expression into Glen's flow law, and integrating it can be shown that the velocity profile as a function of depth, z, is given by

$$u(z) = -\frac{A(\rho g)^3}{4}\left(\frac{dh}{dx}\right)^3\left[(h - z)^4 - h^4\right],$$

where h is the depth of the ice, z is the vertical coordinate, and $z = 0$ is the elevation of the base of the ice sheet. You also need to assume that u(z) = 0 at z = 0.

i) The total mass flux in the horizontal is given by the relationship

$$F(x) = \int_0^h \rho u(z)dz.$$

Do this integral using the expression for *u(z)* given above and show that the mass flux can be written as

$$F(x) = \frac{A\rho(\rho g)^3}{5}h^5\left(\frac{dh}{dx}\right)^3.$$

Note that the flux of ice is a very sensitive function of the ice thickness and the surface slope.

ii) In equilibrium the ice shape is constant, and conservation of mass must also apply. Assuming that there is no melting high up on the ice sheet then the flux of ice past any given point on the ice sheet must equal the total accumulation up-slope of the point. In other words:

$$F(x) = \int_0^x P(x')dx'$$

where P is the accumulation. Let P be spatially uniform. In other words the above expression is simply:

$$F(x) = Px$$

Use the above expression and the answer in part i) to show that

$$h^{\frac{5}{3}}\frac{dh}{dx}=\left(\frac{5}{A\rho}\right)^{\frac{1}{3}}\frac{(Px)^{\frac{1}{3}}}{\rho g}.$$

This can be written as an integral equation:

$$\int_0^h h'^{\frac{5}{3}}\,dh'=\left(\frac{5}{A\rho}\right)^{\frac{1}{3}}\frac{P^{\frac{1}{3}}}{\rho g}\int_L^x x'^{\frac{1}{3}}\,dx'$$

where the limits of the integral incorporate the boundary condition that h = 0 at x = L (where the ice is assumed to drop off into the ocean). Do this integral and hence find an expression for h as a function of x.

Question 6: Atmospheric lapse rates. Death valley lies in the lee of the Sierras, and has a minimum elevation of about 280m below sea level. Repeating the calculations from class that we did for the Cascades, and using the same lapse rate numbers (or those from the notes of a classmate who takes pity on you), about how much of Death Valley's high temperature is attributable to the passage of initially moist air over the Sierras? Assume the air originated at sea level and pick a sensible number for the average height of the mountains upwind of Death Valley.