Week 6: Fluids homework, part II. Due on Friday.

Question 1: Back of envelope calculation – Up.
The movie "Up" featured a house floating away attached to a whole load of (presumably) helium balloons. How many helium balloons do you need to lift a typical house? (Hint: think Archimedes' principle).

Question 2: Describe and explain one of the demonstrations you were shown in the fluids lab on Friday Nov 8th.

Question 3: Angular momentum conservation
Using the equations developed in class, and assuming that a parcel of air starts at the equator with no angular momentum except that of the planet.

i) Calculate the latitude at which its velocity would exceed the speed of sound?
ii) At what latitude would its velocity exceed the speed of light?

Question 4: Boston Strong.
Closer Koji Uehara pitched a cutter low and away to Cardinal's lead-off hitter Matt Carpenter on October 28th, inducing a strikeout, and winning the World Series for the Red Sox. Show that the Coriolis force was the least of his worries. Here are some numbers to help: distance from the mound to home plate 59 feet 1 inch (~= 19m). The speed of Uehara's fastball is about 95 mph (~45 ms^{-1}).

(i) calculate the time for the ball to reach the plate.
(ii) calculate the Coriolis acceleration on the ball (Fenway Park is at about 40ºN)
(iii) calculate the sideways velocity due to the Coriolis acceleration as the ball crosses the plate.
(iv) Assuming this Coriolis acceleration was constant, use the formulae for motion under constant acceleration (google them if you need to), and calculate the sideways deflection of the ball.
(v) Make some sensible assumptions similar to the above to calculate the Coriolis deflection of Big Papi's walk-off home run in Game 2 of the ALCS.

Question 5: tornadoes and bathtubs, vortices and whorls.
Here is our friend the equation of motion for a fluid (Navier-Stokes equation to Earthlings), which we have come to know and love.

$$\underset{(1)}{\frac{D\overline{u}}{Dt}} = \underset{(2)}{\frac{d\overline{u}}{dt}} + \underset{(3)}{2\overline{\Omega}\times\overline{u}} + \underset{(4)}{(\overline{u}.\nabla)\overline{u}} = \underset{(5)}{-\frac{1}{\rho}\nabla p} + \underset{(6)}{\overline{g}} + \underset{(7)}{\nu\nabla^2\overline{u}} \qquad (1)$$

The above is a vector equation that has components in the vertical and in the horizontal, which can be treated separately.

For radially-symmetric, horizontal flows (i.e., stuff going round in a circle on a horizontal plane), the acceleration, term (1) above, is imply the centripetal acceleration. Also gravity does not act in the horizontal, and the velocity is only a function of radius, *r*. So eq (1) becomes

$$\underset{\text{(i)}}{\frac{u^2}{r}} + \underset{\text{(ii)}}{2\Omega \sin\theta \times u} = \underset{\text{(iii)}}{-\frac{1}{\rho}\frac{\partial p}{\partial r}} + \underset{\text{(iv)}}{\nu \frac{\partial^2 u}{\partial r^2}} \tag{2}$$

In the above equation (i) is the centripetal acceleration; (ii) is the Coriolis acceleration; (iii) is the acceleration due to the pressure gradient force; and (iv) is the friction force, where ν is the diffusivity.

a) *Tornadoes* and one of the most destructive and powerful phenomena in nature, and have supersonic wind speeds in rare instances. What is the dominant force balance? To assess this, rank the terms in equation (2) in order of their importance. (Hint, use scale analysis like in class, and take the following characteristic values: $\Delta p \sim 100$ mb; $r \sim 1$ km; $\rho = 1$ kg m^{-3}; $\Omega = 2\pi/1$ day; $\theta = 45^{\circ}$; $u \sim 100$ m s^{-1}; $\nu = 100$ m^2 s^{-1}.

b) Using another old friend, the diffusion equation, what is the spin-down time scale for a tornado with these characteristic scales? Hint: remember the link between length scales, diffusivity and timescale: $\delta = \text{sqrt}(\nu\tau)$.

c) *Bathtubs*. Verify by scale analysis that the water emptying out of your bathtub does not care which hemisphere it is in.

d) Roughly, how big a bathtub would you need before it would 'feel' the Earth's rotation, and if it did in which sense would it circulate in Melbourne?

N.B. there is no single right answer to parts (c) and (d), obviously. Just make some sensible assumptions about the length and time scales involved. Remember that the pressure gradient comes from the 'dip' in the surface of the water above the drain-hole